Longevity, Schooling and Lifetime Labor Supply.

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Abstract

Our main goal is to quantitatively investigate trends in labor force participation and schooling attainment of American men born between 1840 and 1930. A recent paper by Hazan (2009) reports time allocation patterns for these cohorts. The main facts are: between the first and the last cohorts, years of schooling dramatically increased and expected total lifecycle work hours significantly decreased. The latter trend is driven by the decline in annual hours of work (the intensive margin)[1] and despite the rise in expected years of working (the extensive margin). We study the contribution of productivity growth (keeping the human capital accumulation function fixed across cohorts) and gains in age-specific survival rates in accounting for trends in schooling and in labor force participation, both at the extensive and intensive margins.

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1 Introduction

Our main goal is to quantitatively investigate trends in labor force participation and schooling attainment of American men born between 1840 and 1930. A recent paper by Hazan (2009) reports time allocation patterns for these cohorts. The main facts are: between the first and the last cohorts, years of schooling dramatically increased and expected total lifecycle work hours significantly decreased. The latter trend is driven by the decline in annual hours of work (the intensive margin)[2] and despite the rise in expected years of working (the extensive margin). Figure 1 summarizes these empirical findings. We use the data compiled in Hazan (2009) to make the plots.

We study the contribution of productivity growth (keeping the human capital accumulation function fixed across cohorts) and gains in age-specific survival rates in accounting for trends in schooling and in labor force participation, both at the extensive and intensive margins.

Many studies emphasize the important role of gains in life expectancy in the process of output per capita takeoff. These studies are relevant to ours because growth occurs as a result of gains in schooling attainment. Such papers include Ehrlich and Lui (1991), Kalemli-Ozcan et al. (2000), Kalemli-Ozcan (2002), De La Croix and Licandro (1999), Boucekkine et al. (2002, 2003), Lagerlöf (2003a, 2003b), Cervellati and Sunde (2005), Soares (2005) and Tamura (2006) among others.[3]

Two theoretical links between declines in mortality to increases in human capital accumulation are commonly used. In the first one, parents make human capital decisions for the children. A decline in child mortality or uncertainty associated with it reduces parental costs of educating each surviving child, inducing greater investments in children's human capital. In the second one, individuals make their own human capital accumulation decisions. The intuition often coincides with that in the original model of Ben-Porath (1967). Gains in life expectancy increase the expected period over which investments in human capital are paid off, consequently encouraging more human capital accumulation. Hazan (2009) also points out that raising longevity in a simple Ben-Porath type model, while increasing schooling attainment, also necessarily leads to the counterfactual prediction of increasing total work hours. Because we employ a model in which individuals make their own schooling decisions, we want to make sure that our model is flexible enough that both real wage growth and gains in survival rates can potentially produce trends observed in the data.

We are not aware of any study that quantitatively investigates changes in expected work hours at both margins, and schooling choice. Vandenbroucke (2009) focuses on the intensive margin. Kopecky (2010) and Kalemli-Ozcan and Weil (2010) focus on the extensive margin, the retirement decision, and do not model schooling attainment as a choice. Restuccia and Vandenbroucke (2010a) focus on the intensive margin, although they also examine schooling.

Many papers, focus on accounting for cross-country differences in schooling, for example, Bils and Klenow (2000) and Córdoba and Ripoll (2010). Restuccia and Vandenbroucke (2010b) investigate educational attainment over time in countries other than the U.S., by taking the U.S. historical relationship between schooling years, real wage growth and longevity

[2] Francis and Ramey (2006) is the main source for hours at the intensive margin employed in Hazan (2009).

[3] Servellati and Sunde (2005), Lagerlöf (2003a, 2003b), Tamura (2006) endogenize mortality.

as given.

2 Model

We employ a lifecycle model. Time is continuous and indexed by t. Cohorts are indexed by the year of birth τ. They are born with zero assets and endowed with 1 unit of time in each period. The date of birth in the model corresponds to age 5 in the data. Cohorts differ in their productivity parameter $w(t)$ and age-specific survival functions $\pi(t-\tau, t)$, which denotes the probability of being alive at age $t-\tau$ faced by cohort born in τ. The separate dependence on τ allows for changes in surivival functions across cohorts. Preferences of the cohort born in τ are given by

$$\int_{\tau}^{T} e^{-\rho(t-\tau)} \pi(t-\tau, \tau) u[c(\tau, t), l(\tau, t)] dt,$$

where T denotes the maximum possible life length.

The cohort born in τ chooses consumption $c(\tau, t)_{t=\tau}^{T}$, length of schooling $S(\tau) \geq 0$, schooling intensity, or time allocated to education during schooling years $s(\tau, t)_{t=\tau}^{\tau+S(\tau)}$, goods investment in education $x(\tau) > 0$, time allocation to leisure $l(\tau, t)_{t=\tau}^{T}$, time allocation to work during working period $n(\tau, t)_{t=\tau+S(\tau)}^{\tau+R(\tau)}$, and retirement age $R(\tau) \geq 0$ to maximize preferences subject to the following constraints:

$$\int_{\tau}^{T} e^{-r(\tau,t)} \pi(t-\tau, \tau) c(\tau, t) dt + p(\tau) x(\tau) = \tag{1}$$

$$= \int_{\tau+S(\tau)}^{\tau+R(\tau)} e^{-r(\tau,t)} \pi(t-\tau, \tau) w(t) h(\tau, t, s(\tau, t), S(\tau), x(\tau)) n(\tau, t) dt,$$

$$[\text{Time constraint}] : \begin{cases} l(\tau, t) = 1 & \text{for } \tau + R(\tau) \leq t \leq T, \\ l(\tau, t) + n(\tau, t) = 1 & \text{for } \tau + S(\tau) \leq t \leq \tau + R(\tau), \\ l(\tau, t) + s(\tau, t) = 1 & \text{for } \tau \leq t \leq \tau + S(\tau), \end{cases} \tag{2}$$

where $h(\tau, t, s(\tau, t), S(\tau), x(\tau))$ is the human capital of this cohort, which depends on the effective time spent in school $\int_{\tau}^{\tau+S(\tau)} s(\tau, t) dt$, non-time investment in education $x(\tau)$, traded at price $p(\tau)$ and time t. Time t matters because individuals may accumulate human capital through their work experience. For simplicity, we constrain the schooling effort to be constant over schooling years, $s(\tau, t) = s(\tau)$ for $t \in [\tau, \tau + S(\tau))$, and hence $\int_{\tau}^{\tau+S(\tau)} s(\tau, t) dt = s(\tau) S(\tau)$. In the budget constraint, we also have $w(t)$ which denotes the exogenously evolving productivity parameter. We also assume that there are perfect annuity markets, so individuals can perfectly insure against mortality risk. We plan to explore sensitivity of our results to relaxing this assumption. The time constraint specifies that leisure is set to 1 upon retirement, that during the work period time endowment is allocated between work and leisure, and during the school period, time endowment is allocated between studying and leisure.

We draw on human capital ingredients from Bils and Klenow (2000), Restuccia and Vandenbroucke (2010) and much of the labor literature and assume that

$$h(\tau, t, s(\tau, t), S(\tau), x(\tau)) = x(\tau)^{\gamma} \left(e^{s(\tau)S(\tau)}\right)^{1-\gamma} e^{\beta_2(t-\tau-S(\tau))+\beta_3(t-\tau-S(\tau))^2+\beta_4(t-\tau-S(\tau))^3+\beta_4(t-\tau-S(\tau))^4},$$

which is defined for $t \geq \tau + S(\tau)$. In the formulation above, $t - \tau - S(\tau)$ represents experience. Including the experience quatric in the human capital production function enables us to closely match the hump-shaped wage-age profile observed in the data. So, different cohorts will face different wage-age profiles due to (1) differences in their $h(\tau, t, s(\tau, t), S(\tau), x(\tau))$ as they choose different educational investments $x(\tau)$, $s(\tau, t)$, $S(\tau)$ and (2) differences in the productivity parameter $w(t)$, which evolves over time and will raise wage profiles for the younger cohorts. In the data, we observe that wages are a convex function of schooling, and log wages are a linear function of schooling. Thus, specifications that assume that human capital is a strictly concave function of scooling are inconsistent with the data, and we choose a linear dependence on schooling.

Since we do not consider any general equilibrium effects, it suffices to discuss the solution to the problem of the cohort born at $t = 0$, for which t refers both to time and age. Solving the model for other cohorts would then simply involve using the appropriate $p(\tau), \pi(t, \tau)$ and $w(t)$. Thus, the cohort born in time 0 and of age t solves

$$\max_{c(t)_{t=0}^{T}, n(t)_{t=S}^{R}, (s,S,R,x) \geq 0} \int_0^S e^{-\rho t} \pi(t) u(c(t), 1-s) \, dt + \int_S^R e^{-\rho t} \pi(t) u(c(t), 1-n(t)) \, dt + \int_R^T e^{-\rho t} \pi(t) u(c(t), 1) \, dt$$

s.t.

$$\int_0^T e^{-r(t)} \pi(t) c(t) \, dt + px = \int_S^R e^{-r(t)} \pi(t) w(t) h(t, s, S, x) n(t) \, dt$$

$$0 \leq n(t) \leq 1 \quad \text{for } S \leq t \leq R$$

Note that $[0, S)$ is the schooling period, $[S, R]$ is working period and $(R, T]$ is the retirement period.

2.1 First Order Conditions

$$[c(t)] : \lambda e^{-r(t)} \pi(t) = \begin{cases} e^{-\rho t} \pi(t) u_1(c(t), 1-s) & t \in [0, S) \\ e^{-\rho t} \pi(t) u_1(c(t), 1-n(t)) & t \in [S, R] \\ e^{-\rho t} \pi(t) u_1(c(t), 1) & t \in (R, T] \end{cases} \tag{3}$$

$$[n(t)] : \lambda e^{-r(t)} \pi(t) w(t) h(t, s, S, x) = e^{-\rho t} \pi(t) u_2(c(t), 1-n(t)), \ t \in [S, R] \tag{4}$$

$$[s] : \lambda \int_S^R e^{-r(t)} \pi(t) w(t) h_s(t, s, S, x) n(t) \, dt \tag{5}$$

$$= e^{-\rho t} \pi(t) u_2(c(t), 1-s), \ t \in [0, S) \tag{6}$$

$$[S] : \lambda \int_S^R e^{-r(t)} \pi(t) w(t) h_S(t, s, S, x) n(t) \, dt \tag{7}$$

$$= e^{-\rho S} \pi(S) [u(c(S), 1-s) - u(c(S), 1-n(S))] \tag{8}$$

$$[R] : \lambda e^{-r(R)} \pi(R) w(R) h(R, s, S, x) n(R) \tag{9}$$

$$= e^{-\rho R} \pi(R) [u(c(R), 1) - u(c(R), 1-n(R))] \tag{10}$$

$$[x] : \int_S^R e^{-r(t)} \pi(t) w(t) h_x(t, s, S, x) n(t) \, dt = p \tag{11}$$

The conditions for optimal consumption, (3), equate the marginal cost of extra consumption with the marginal utility. These marginal utilities might be different for the three phases of life, if utility is non-separable in consumption and leisure. To solve for optimal consumption during $t \in [0, S)$, we use condition (3) for time t and time 0:

$$\frac{e^{-r(t)}}{e^{-r(0)}} = \frac{e^{-\rho t} u_1 (c(t), 1-s)}{e^{-\rho 0} u_1 (c(0), 1-s)}$$

Notice that the above can be solved for consumption at any $t \in [0, S)$ as a function of $r(t)$, s, and $c(0)$.

To solve for optimal consumption during $t \in [0, S)$, we use condition (3) for time t, time S and time R:

$$\begin{aligned} \frac{e^{-r(t)}}{e^{-r(S)}} &= \frac{e^{-\rho t} u_1 (c(t), 1-n(t))}{e^{-\rho S} u_1 (c(S), 1-n(S))} \\ \frac{e^{-r(t)}}{e^{-r(R)}} &= \frac{e^{-\rho t} u_1 (c(t), 1-n(t))}{e^{-\rho R} u_1 (c(R), 1-n(R))} \end{aligned}$$

The above system can be solved for $[c(t), n(t)]$ as functions of $r(t)$, S, R, $c(R)$, $c(S)$, $n(R)$, $n(S)$.

To solve for optimal consumption during $t \in (R, T]$, we use condition (3) for time t and time T:

$$\frac{\lambda e^{-r(t)}}{\lambda e^{-r(T)}} = \frac{e^{-\rho t} u_1 (c(t), 1)}{e^{-\rho T} u_1 (c(T), 1)}$$

Notice that if we know T and $c(T)$, the above can be solved for any $c(t)$, $t \in (R, T]$.

Combining (3) and (4), gives the intratemporal optimality condition for work and consumption during $t \in [S, R]$:

$$\begin{aligned} \frac{e^{-\rho t} \pi(t) u_2 (c(t), 1-n(t))}{e^{-\rho t} \pi(t) u_1 (c(t), 1-n(t))} &= \frac{\lambda e^{-r(t)} \pi(t) w(t) h(t, s, S, x)}{\lambda e^{-r(t)} \pi(t)} \\ \frac{u_2 (c(t), 1-n(t))}{u_1 (c(t), 1-n(t))} &= w(t) h(t, s, S, x) \end{aligned}$$

This condition equates the marginal rate of substitution between leisure and consumption with the relative price (=wage).

2.2 Separable Utility

In this section we assume separabale utility, constant interest rate over time, exogenous schooling effort s during schooling years $t \in [0, S)$. The problem is then:

$$\max_{\substack{c(t)_{t=0}^{T},\\ l(t)_{t=S}^{T}\\ S,R,x}} \int_0^S e^{-\rho t}\pi(t)\left[U(c(t)) + V(1-s)\right]dt + \int_S^R e^{-\rho t}\pi(t)\left[U(c(t)) + V(l(t))\right]dt$$

$$+\int_R^T e^{-\rho t}\pi(t)\left[U(c(t)) + V(1)\right]dt$$

$$s.t.$$

$$\int_0^T e^{-rt}\pi(t)c(t)\,dt + px = \int_S^R e^{-rt}\pi(t)w(t)h(t,s,S,x)\left[1-l(t)\right]dt$$

Equivalently, we can collect all the consumption and leisure terms as follows:

$$\max_{\substack{c(t)_{t=0}^{T},\\ l(t)_{t=S}^{T}\\ S,R,x}} \int_0^T e^{-\rho t}\pi(t)U(c(t))\,dt + \int_0^S e^{-\rho t}\pi(t)V(1-s)\,dt$$

$$+\int_S^R e^{-\rho t}\pi(t)V(l(t))\,dt + \int_R^T e^{-\rho t}\pi(t)V(1)dt$$

$$s.t.$$

$$\int_0^T e^{-rt}\pi(t)c(t)\,dt + px = \int_S^R e^{-rt}\pi(t)w(t)h(t,s,S,x)\left[1-l(t)\right]dt$$

First order conditions with respect to $c(t)$ and $l(t)$:

$$\begin{aligned}
[c(t)] &: e^{-\rho t}\pi(t)U'(c(t)) = \lambda e^{-rt}\pi(t),\ t\in[0,T] \\
[l(t)] &: e^{-\rho t}\pi(t)V'(l(t)) = \lambda e^{-rt}\pi(t)w(t)h(t,s,S,x),\ t\in[S,R]
\end{aligned}$$

Combining the conditions for $c(t)$ and $c(R)$, gives:

$$\begin{aligned}
\frac{e^{-\rho t}U'(c(t))}{e^{-\rho R}U'(c(R))} &= \frac{\lambda e^{-rt}}{\lambda e^{-rR}} \\
e^{-\rho(t-R)}U'(c(t)) &= U'(c(R))\,e^{-r(t-R)} \\
U'(c(t)) &= U'(c(R))\,e^{(\rho-r)(t-R)} \\
c(t) &= U'^{-1}\left[U'(c(R))\,e^{(\rho-r)(t-R)}\right] \equiv g(c_R)
\end{aligned}$$

Thus, consumption at any date age t, can be expressed as a function of consumption at some other date, say R. It can be shown that $g'(R) > 0$. Notice that if $\rho = r$, then $c(t) = c(R)$ $\forall t$.

Combining the conditions for $l(t)$ and $l(R)$, gives:

$$\begin{aligned}\frac{e^{-\rho t}V'(l(t))}{e^{-\rho R}V'(l(t))} &= \frac{\lambda e^{-rt}w(t)\,h(t,s,S,x)}{\lambda e^{-rR}w(R)\,h(R,s,S,x)}\\ V'(l(t)) &= V'(l(R))\frac{w(t)\,h(t,s,S,x)}{w(R)\,h(R,s,S,x)}e^{-r(t-R)}\\ l(t) &= V'^{-1}\left[V'(l(R))\frac{w(t)\,h(t,s,S,x)}{w(R)\,h(R,s,S,x)}e^{(\rho-r)(t-R)}\right]\end{aligned}$$

3 Calibration

3.1 Wage-age profile

Recall that in the model, a wage of an individual born in τ at the time of t (i.e., at the age of $t-\tau$) is given by $\omega(t,\tau)=w(t)\,h(\tau,t,s(\tau,t),S(\tau),x(\tau))$. We normalize $w(0)=1$ and assume that $w(t)$ grows at a constant growth rate: $w(t)=e^{0.01t}$ to match the approximate rate of annual productivity growth of 1%.

Taking the natural log of the wage-age profile in the model gives

$$\begin{aligned}\log\omega(t,\tau) &= \log w(t)+\gamma\log x(\tau)+(1-\gamma)\,\bar{s}(\tau)\,S(\tau)+ \\ &+\beta_1(t-\tau-S)+\beta_2(t-\tau-S)^2+\beta_3(t-\tau-S)^3+\beta_4(t-\tau-S)^4.\end{aligned} \tag{12}$$

We use the Current Population Survey data to estimate the wage-age profile in the model. The identifying assumption we make is that returns to experience are fixed across all cohorts. In the data, we observe individual hourly wages $\omega^{data}(t,\tau)$, which correspond to $\omega(t,\tau)$ in the model. The empirical model that we adopt is then

$$\begin{aligned}\log\omega^{data}(t,\tau) &= \hat{\beta}_0+\hat{\beta}_1 S+\hat{\beta}_2(t-\tau-S)+\hat{\beta}_3(t-\tau-S)^2/100 \\ &+\hat{\beta}_4(t-\tau-S)^3/1000+\hat{\beta}_5(t-\tau-S)^5/10000+ \\ &+\sum_{y=1968}^{2006}\hat{\beta}_y I_{y=t}+\sum_{yborn=1901}^{1974}\hat{\beta}_{yborn}I_{yborn=\tau}.\end{aligned} \tag{13}$$

In other words, we regress annual wages on schooling years, experience quatric, year and cohort dummies. We omitted the first cohort dummy (born in 1900) and the first time dummy (1967). Time dummies pick up any trend in wages due to time, while the resgression constant $\hat{\beta}_0$ plus the coefficient on the relevant cohort represent the cohort-specific level of the wage profile unaccounted for schooling and experience, which our model attributes to this cohort's investment in $x(\tau)$. Even though we make the assumption in the model that productivity grows exponentially, we do not restrict the time effect to be linear in the empirical model, instead using the time dummies. This allows for greater flexibility, and also pick up short run fluctuations. Also, even if we made the restriction, it would not make sense to calibrate productivity growth to the estimated coefficient on the time trend because the time period for which the CPS was conducted does not cover the entire time period we examine in the model.

We now give more details on sample restrictions. Hence, we estimate the empirical model (13) using Current Population Survey for 1968-2007, restricting our attention to 18 to 80 year old white men born between 1900 and 1974, and not living in group quarters. We further restrict our attention to full time full year workers, as we believe that there is less measurement error for hours and earnings of full time workers.

Earnings reported in CPS are last year earnings. To construct hourly wages last year, we need information on total hours worked last year. Beginning in the 1976 survey, CPS provides information on usual weekly hours worked for the prior year, and weeks worked last year. However, prior to the 1976 survey, CPS reported an interval of the number of weeks worked. Following what has now become a standard procedure, we impute the number of weeks worked for respondents in years prior to 1976 by the 1976–1978 survey average weeks worked within weeks worked categories. We impute usual weekly hours worked (prior year) for the respondents in year prior to 1976 using the 1976–1978 survey average usual weekly hours worked within weeks worked categories. Once we obtain the hourly wage estimates, we translate them into year 2000 dollars using the Consumer Price Index. We trim the outliers using the procedure employed in Mulligan and Rubinstein (2008), which basically excludes observations with measured hourly wage below $2.80 per hour and below the 1st-percentile value or above the 98th percentile of the FTFY 18-65 year old male hourly wage distribution.

The measure of education is also inconsistent across survey years. Prior to 1992, we have information regarding complete years of schooling. Beginning in 1992, CPS reports education according to the highest degree attained. Precisely, respondents are classified into one of the following education categories: no school, 1-4 years of schooling, 5-8 years, 9 years, 10 years, 11 years, 12 years no diploma, high school diploma or GED, some college no degree, associate degree/occupational program, associate degree/academic program, bachelors degree, masters degree, professional degree, doctorate degree. To each of these categories (except the obvious "no school" and 9,10, 11, 12 year), we need to assign the average number of schooling years. To impute the appropriate averages, we use the 1991 averages in the categories that we define using the number of schooling years. Precisely, we compute the average years of schooling for people with 1 to 4 grades of schooling, those with 5 to 8 years of schooling, 13-15 years of schooling, and 16+ years of schooling.

We subtract 1 from respondent's age, so it refers to the year for which the earnings are reported. Finally, we construct potential work experience as the max between zero and age (in year of survey) minus years of schooling minus 6.

The results are given in Table 1. Cohort born in 1930 (the last cohort we examine in the paper) is cohort31 in our dataset. Because we also plan to explore the formulation of the wage-age profile in which the age quartic in (12) is replaced by the age quartic, which does not suffer from criticism that $t - \tau - S$ is a poor measure of actual experience, we report the results for the corresponding empirical model estimation in the same table. However, at this point, we only explore the formulation which depends on experience.

Hence, we calibrate the model returns to experience by setting

$$\begin{aligned} \beta_1 &= \hat{\beta}_2 = 0.0661 \\ \beta_2 &= \hat{\beta}_3/100 = -0.303/100 \\ \beta_3 &= \hat{\beta}_4/1000 = 0.0572/1000 \\ \beta_4 &= \hat{\beta}_5/10000 = -0.00393/10000 \end{aligned}$$

Since we do not observe the schooling effort, we could only include years of schooling on the right hand side. Hence, the empirical model identifies the product $(1-\gamma)\,\bar{s}$:

$$(1-\gamma)\,\bar{s} = \hat{\beta}_1.$$

By setting $\bar{s} = 0.4$, since most students study full time when at school, we obtain

$$\gamma = 1 - \hat{\beta}_1/0.4 = 1 - \frac{0.0714}{0.4} = 0.8215.$$

	(1)	(2)
VARIABLES	lnw	lnw
educ_years	0.0714***	0.0654***
	(0.000421)	(0.000165)
exp	0.0661***	
	(0.000687)	
exp2	-0.303***	
	(0.00408)	
exp3	0.0572***	
	(0.00112)	
exp4	-0.00393***	
	(0.000101)	
cohort31	0.282***	0.238***
	(0.0174)	(0.0174)
age		0.198***
		(0.00397)
age2		-0.516***
		(0.0143)
age3		0.0597***
		(0.00219)
age4		-0.00266***
		(0.000121)
Constant	1.204***	-0.955***
	(0.00780)	(0.0394)
Observations	953,185	953,185
R-squared	0.225	0.227

Table 1: The empirical estimates for the wage-earnings profile. We omit reporting coefficients on year dummies (1968 to 2006, recall 1967 is omitted) and coefficients on cohort dummies for cohorts born in years 1901 to 1974 (recall the 1900 cohort is omitted), except for the cohort born in 1930 (the last cohort we investigate). Standard errors are in parenthesis, and we use stars to denote the level of significance: *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

3.2 Survival functions

Next we discuss how we estimate cohort-specific survival functions $\pi(t-\tau, t)$. To ease notation, we drop the dependence on τ and explain how we estimate a survival function for one given cohort, so we write $\pi(i)$ to denote probability of being alive at the age of i. Life tables are reported for discrete ages $i \in \{0, 1, 2, ..., T\}$. For example, $q(i)$ in life tables is the probability that an age i individual dies exactly between ages i and $i+1$. In other words, $q(i)$ is the hazard function in the data. In our model, age is continuous, $a \in [0, T)$. If the data were generated by a continuous time model, we would have

$$\begin{aligned} q(i) &= \Pr(i \leq X \leq i+1 | X \geq i) \\ &= \frac{\int_i^{i+1} f(a)\, da}{\pi(i)} = \frac{F(i+1) - F(i)}{\pi(i)} \\ &= \frac{1 - \pi(i+1) - 1 + \pi(i)}{\pi(i)} = \frac{\pi(i) - \pi(i+1)}{\pi(i)}, \end{aligned}$$

where $F(\cdot)$ and $\pi(\cdot) = 1 - F(\cdot)$ are the model (continuous time) cumulative distribution and survival functions. What we need to fit is not the hazard function, but the survival function $P(i)$, i.e. the probability of being alive at each age i. In the discrete data, the survival function is:

$$P(i) = \prod_{j=0}^{i} [1 - q(j)].$$

Using the model counterpart for $q(i)$ gives

$$\begin{aligned} P(i) &= \prod_{j=0}^{i} \left[1 - \frac{\pi(j) - \pi(j+1)}{\pi(j)}\right] \\ &= \prod_{j=0}^{i} \left[\frac{\pi(j) - \pi(j) + \pi(j+1)}{\pi(j)}\right] \\ &= \prod_{j=0}^{i} \left[\frac{\pi(j+1)}{\pi(j)}\right] \\ &= \frac{\pi(1)}{\pi(0)} \cdot \frac{\pi(2)}{\pi(1)} \cdot ... \cdot \frac{\pi(i)}{\pi(i-1)} \cdot \frac{\pi(i+1)}{\pi(i)} \\ &= \frac{\pi(i+1)}{\pi(0)}. \end{aligned}$$

Since in the continuous time model we must have $\pi(0) = 1$, we conclude that $\pi(i+1)$ is the model's equivalent to the survival function in the data $P(i)$. If the model's survival function is characterized by an unknown vector of parameters θ, then ssion suggests an estimator of θ:

$$\hat{\theta} = \arg\min_{\theta} \sum_{i=0}^{T} [P(i) - \pi(i+1; \theta)]^2$$

Suppose that life starts at age 5. In addition, suppose that we do not have $P(i)$ for all $i \in \{5, 6, ..., T\}$, but instead we have data on the probability of surviving during ages $5-14$ and $15-19$. The continuous time model analogs of these two numbers are:

$$\begin{aligned}
\rho_{5-14} &\equiv \Pr(X \geq 15 | X \geq 5) = \frac{\int_{15}^{T} f(a)\, da}{\pi(5)} = \frac{\pi(15)}{\pi(5)} \\
\rho_{15-19} &\equiv \Pr(X \geq 20 | X \geq 15) = \frac{\int_{20}^{T} f(a)\, da}{\pi(15)} = \frac{\pi(20)}{\pi(15)}
\end{aligned}$$

In this case for $i \geq 20$, we have:

$$\begin{aligned}
P(i) &= \frac{\pi(15)}{\pi(5)} \cdot \frac{\pi(20)}{\pi(15)} \cdot \prod_{j=20}^{i} \left[\frac{\pi(j+1)}{\pi(j)} \right] \\
&= \frac{\pi(15)}{\pi(5)} \cdot \frac{\pi(20)}{\pi(15)} \cdot \frac{\pi(21)}{\pi(20)} \cdot \frac{\pi(22)}{\pi(21)} \cdot ... \cdot \frac{\pi(i)}{\pi(i-1)} \cdot \frac{\pi(i+1)}{\pi(i)} \\
&= \frac{\pi(i+1)}{\pi(5)}
\end{aligned}$$

Since life starts at 5, in the continuous time model we must have $\pi(5) = 1$. The estimator is then modified as follows:

$$\hat{\theta} = \arg\min_{\theta} \left(\sum_{i=0}^{T} [P(i) - \pi(i+1; \theta)]^2 + \left[\rho_{5-14} - \frac{\pi(15)}{\pi(5)} \right]^2 + \left[\rho_{15-19} - \frac{\pi(20)}{\pi(15)} \right]^2 \right) \tag{14}$$

The advantage of the above estimator is that it attempts to fit only the observed values, as opposed to interpolated (made up) data points. Alternatively, the squared deviations of the model from ρ_{5-14} and ρ_{15-19} can be wieghted by 10 and 5, to reflect the number of years that these data represent.

3.2.1 Gompertz-Makeham

Makeham (1860) extended Gompertz (1825) mortality function by adding a constant:

$$\mu(a) = \alpha e^{\beta a} + \zeta$$

The survival function is computed using theorem 1:

$$\begin{aligned}
\pi(a) &= \exp\left[-\int_{0}^{a} \mu(x)\, dx \right] \\
&= \exp\left[-\int_{0}^{a} \left(\alpha e^{\beta x} + \gamma \right) dx \right] \\
&= \exp\left[-\alpha \left(\frac{e^{\beta x}}{\beta} \right)_{0}^{a} - \zeta a \right] \\
&= \exp\left[\frac{\alpha}{\beta} \left(1 - e^{\beta a} \right) - \zeta a \right]
\end{aligned}$$

The parameters of this function, $\theta = \{\alpha, \beta, \zeta\}$, are estimated by the non-linear least squares in (14).

The estimation results are given in Table 2. The fit is very good. Figure 2 illustrates the fit for the 1840 and 1930 cohorts, i.e. the first and the last cohort.

cohort	α	β	ζ
1840	0.000399	0.075698	0.005747
1850	0.000369	0.076305	0.005486
1860	0.000237	0.082357	0.005724
1870	0.000234	0.081872	0.005264
1880	0.000273	0.078952	0.00438
1890	0.000253	0.079679	0.003759
1900	0.000221	0.080966	0.003165
1910	0.000229	0.079575	0.002072
1920	0.00019	0.081278	0.001405
1930	0.000135	0.084559	0.001114

Table 2: Estimation of the survival functions for cohorts 1840 to 1930 using Gompertz (1825) functional form.

4 Results

5 Conclusions

6 Appendix

Often, we are given the hazard function, and want to find the survival function. The following theorem is then used:

Theorem 1

$$\pi(a) = \exp\left[-\int_0^a \mu(x)\, dx\right]$$

Proof. Taking logs of the above:

$$\begin{aligned} \ln \pi(a) &= -\int_0^a \mu(x)\, dx \\ -\ln \pi(a) &= \int_0^a \mu(x)\, dx \end{aligned}$$

Differentiating with respect to a:

$$-\frac{d \ln \pi(a)}{da} = \mu(a)$$

■

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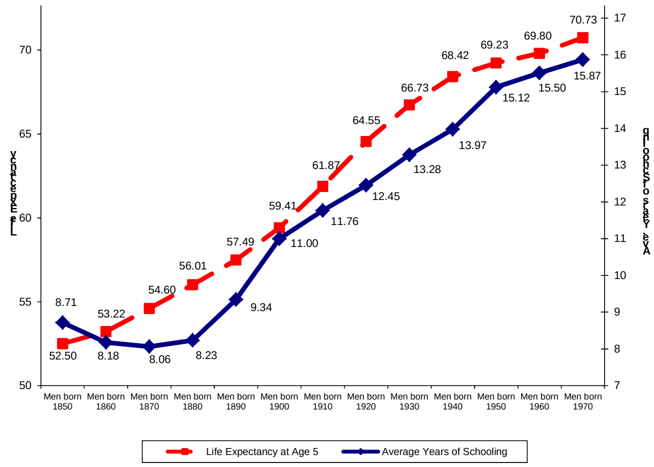

Figure 1. Life expectancy and schooling years for men born in 1840-1970
Source: Figure 1 in Hazan (2009).

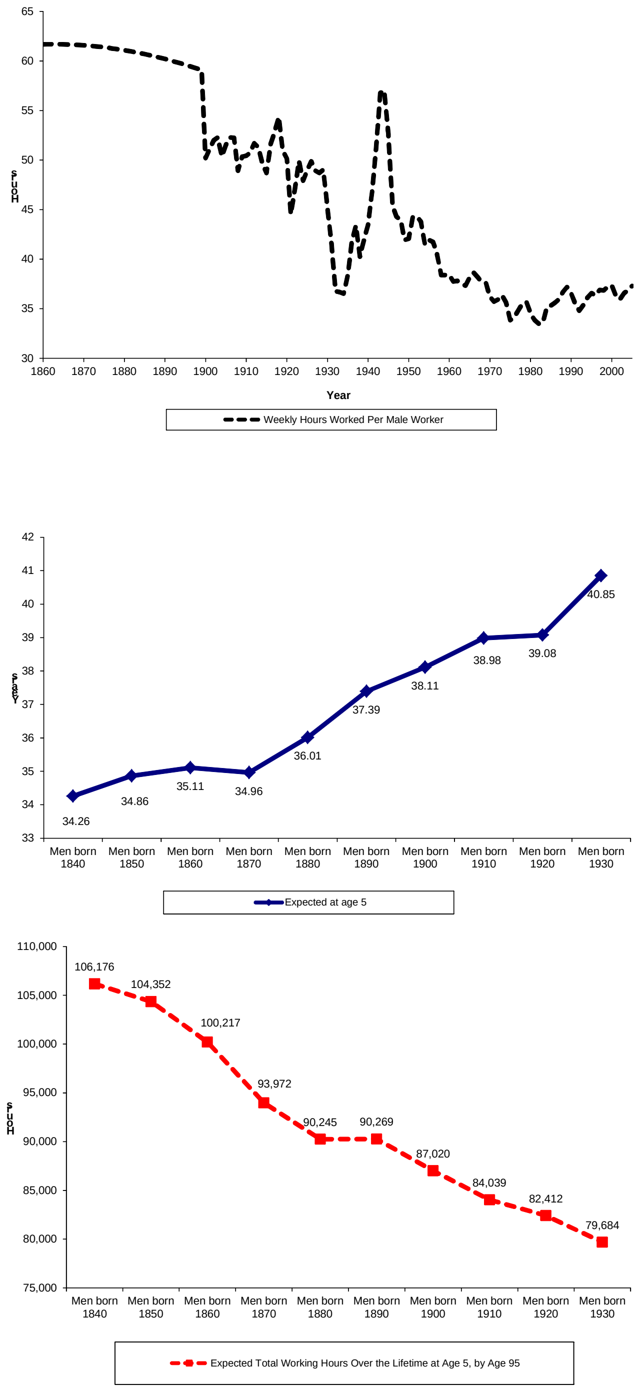

Figure 2. Weekly hours by year and expected (at age 5) years and total hours of work. Source: Figures 4,6,and 8 of Hazan (2009).

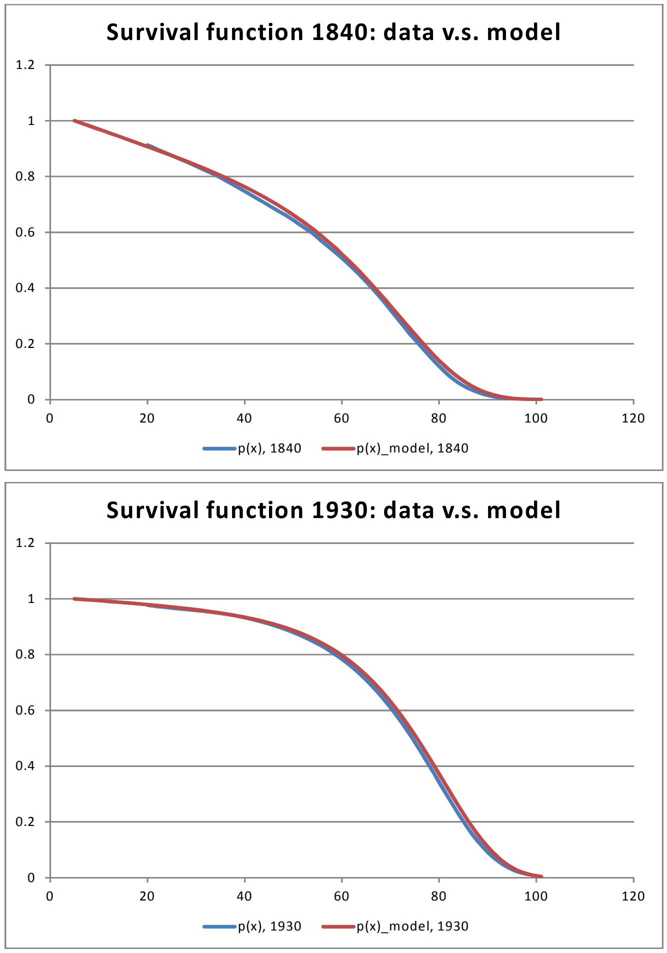

Figure 3. Estimated and empirical survival functions for the 1840 and 1930 cohorts.