Trend Inflation and the New Keynesian Phillips Curve

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Abstract

This paper estimates empirical models for the forward-looking New Keynesian Phillips curve (NKPC) when inflation is allowed to have a stochastic trend. Using the observed CBO output gap as a measure of economic activity, I propose a baseline unobserved components (UC) model as implied by the closed form specification of the forward-looking NKPC. Then, I extend the baseline case to a bivariate UC model of real output and inflation, assuming that the output gap is unobserved. Empirical evidence suggests that the forward-looking NKPC with regime-switching in the variances and covariances of the shocks can provide a good description of postwar US inflation dynamics. The standardized residuals and their squares for the inflation series obtained from estimation of the proposed models are serially uncorrelated. Furthermore, the bivariate UC model displays superior forecasting abilities both in-sample and out-of-sample, relative to univariate benchmark models in the literature that forecast inflation well. Finally, the output gap measure obtained as the byproduct of estimating the bivariate UC model displays reasonably narrow confidence bands and is not significantly different from the CBO output gap.

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1 Introduction

New Keynesian Phillips curves (NKPCs) are widely used as theoretical models for inflation. The forward-looking specification, in particular, is most favored by academics and policymakers because it is derived from an optimizing framework that features rational expectations and nominal rigidities. However, a disconcerting feature of the model is that it is deemed inconsistent with empirical evidence of considerable inflation persistence, and it often cannot find a significant role for the output gap in explaining short-run inflation dynamics (see Fuhrer and Moore, 1995; Fuhrer, 1997; Estrella and Fuhrer, 2002).

The NKPC serves as an important ingredient for monetary policy analysis. Therefore, a large body of empirical literature focuses on estimating the model in an attempt to identify which features of it fail to explain the data[1]. In these studies, the authors generally estimate the NKPC under the assumption that inflation is stationary, which implies that in the long-run, inflation will converge to a deterministic or zero trend. This is a simplifying assumption that enables econometricians to obtain a tractable solution for the New Keynesian pricing model, and is also consistent with optimal monetary policy in a so-called cashless economy (see Goodfriend and King, 2001; Woodford, 2003). However, a number of studies show that the stationarity assumption for inflation can be at odds with the data in real-world economies. For example, Bai and Ng (2001) and Henry and Shields (2003), among many others, find evidence of a unit root in postwar US inflation, which implies that the long-run characteristics of inflation follows a stochastic or time-varying trend[2].

The case for a time-varying trend is largely undisputed among macroeconomists, because in the long-run, inflation is known to move with the growth rate of money supply. In addition, trend inflation in general equilibrium is determined by the long-run target in the Central bank's policy rule, which has arguably varied over time. Nevertheless, it has only been until very recent

[1] A large literature shows that the NKPC faces many issues related to estimation. These include but are not limited to: specification bias (see Rudd and Whelan, 2005; Linde, 2005), model identification (see Mavroeidis, 2005; Nason and Smith, 2008), and dynamic misspecification (see Bardsen et. al, 2004; Dees et. al, 2009).

[2] Due to the limited power of unit root tests, there is no consensus on whether or not inflation is stationary or has a unit root. However, the more recent literature tends to suggest that US inflation switched between unit root and stationary regimes over the postwar period (see Kim, 2000; Leybourne et. al, 2003; Murray et. al, 2009). Although the dates of regime shifts found in these studies do not necessarily coincide, the general impression is that postwar US inflation dynamics exhibited unit root behavior at least during the 1970s. Besides displaying unit root behavior, non-stationarity in inflation has also been detected in the form of long memory (see Hassler and Wolters, 1995; Baillie et. al, 1996), and structural breaks in the average rate of inflation (see Benati, 2003; Levin and Piger, 2004).

that authors incorporate the possibility of a stochastic trend into their empirical models for inflation. These include, for example, the New Keynesian dynamic stochastic general equilibrium (DSGE) models of Belaygorod and Dueker (2005), Milani (2006), and Ireland (2007). In these models, the authors allow trend inflation to evolve as a driftless random walk, for the purposes of analyzing the patterns, causes, and consequences of changes in the Federal Reserve Bank's implicit target for postwar US inflation.

A few recent papers also propose unobserved components (UC) models for inflation with a stochastic trend component that follows a driftless random walk. For example, Stock and Watson (2007), and Kang et. al (2009) estimate univariate UC models to forecast and analyze the statistical properties of inflation. Recently, univariate UC models have been augmented into bivariate ones. Lee and Nelson (2007) propose a reduced form bivariate UC model for inflation and unemployment, where the stochastic trend component corresponds to the public's long-run forecast of inflation as implied by a forward-looking NKPC. Harvey (2008) specifies a bivariate UC model for inflation and output based on a reduced form Phillips curve, where lagged inflation terms in the Phillips curve relation are replaced by a driftless random walk. The empirical model of Cogley and Sbordone (2008) also allows for time variation in the trend rate of inflation, but in contrast to the earlier studies, the authors do not employ the UC modeling approach. Their research focuses on how the presence of stochastic trend inflation can give rise to time-varying NKPC model parameters, and therefore, they resort to a two-step estimation approach where the trend inflation process and the structural parameters of the model are identified separately.

On the theoretical front, the concept of stochastic trend inflation has also been gaining increasing amounts of attention. For example, Woodford (2008) derives a forward-looking NKPC that allows inflation to have a time-varying trend. This model is adapted by Goodfriend and King (2009) to explain how the Central bank's priority for interest rate smoothing and maintaining output at capacity can result in a stochastic trend rate of inflation in the presence of adverse shocks to capacity output.

In this paper, I am interested in investigating the implications of stochastic trend inflation on the empirical performance of the forward-looking NKPC, as research work done on this topic is currently limited. To do so, I bring the theoretical model of Goodfriend and King (2009) to postwar US data, by building upon the modeling approach of Lee and Nelson (2007)[3]. Specifically, I propose a baseline UC model for inflation that is consistent with the closed form solution of the forward-looking NKPC as derived by Woodford (2008). In this specification, I follow the standard approach in the literature and measure real economic activity in the NKPC relation with the CBO output gap. However, the output gap is a latent variable, as its measure depends on an estimate of the unobserved level of potential output. Therefore, I extend the baseline case into a bivariate UC model of inflation and real output, which allows the output gap to be treated as an unobserved variable. To account for heteroskedasticity in inflation, I allow the variances and covariances of the shocks in both the baseline and bivariate UC models to undergo endogenously determined structural breaks.

As a preview of the empirical results, I find that the stochastic trend component of inflation is an important determinant of actual inflation, especially during the 1970s. Trend inflation displays significant variability over the postwar period, although its movements have become much less pronounced since the mid 1980s. In addition, I find that once time variation in trend inflation is taken into account, the forward-looking NKPC turns out to be a good description of postwar US inflation dynamics. Unlike previous versions of the NKPC that are estimated under the stationary restriction, my model is able to fully account for inflation persistence, and can establish a significant link between inflation and the output gap. Also, there is no remaining serial correlation in the models' standardized residuals and their squares for the inflation series, which suggests proper model specification. Furthermore, based on a forecasting exercise, I find that the bivariate UC model can produce superior in-sample and out-of-sample inflation forecasts relative to models in the literature that are generally known for their good forecasting performance. Finally, from estimation of the bivariate UC model, I find that the output gap that is consistent with the forward-looking NKPC displays relatively narrow confidence bands, and contains movements that are remarkably similar to the CBO output gap.

The contents of this paper are organized into 7 sections. Following the introduction section, Section 2 describes the closed form specification of the forward-looking NKPC in the presence of stochastic trend inflation. Section 3 lays out the empirical model based on the UC modeling approach, and describes the specification for each of its components. The estimation approach in

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[3] Note that the main focus of Lee and Nelson (2007) is different from the one in this paper. The primary interest of those authors is to investigate an empirical puzzle related to the slope of the Phillips curve.

the presence of structural breaks is described in Section 4. Section 5 presents and discusses the empirical findings. Diagnostic checks which include tests of serial correlation and an evaluation of inflation forecasts are covered in Section 6. Section 7 concludes.

2 The Closed Form Specification with Stochastic Trend Inflation

The benchmark for the discussion and analysis of inflation dynamics is based on the rational expectations sticky-price models of Taylor (1980), Rotemberg (1982), Calvo (1983), and others. As shown in Roberts (1995), these models imply the linearized forward-looking NKPC of the following form:

$$\pi_t = \beta E_t \pi_{t+1} + k x_t . \tag{1}$$

In this expression, π_t is the current inflation rate, $E_t \pi_{t+1}$ represents the expected inflation rate in the next period conditional on the information set available at time t, and x_t is the output gap. The parameter β denotes the subjective discount factor, and k is a nonlinear function of structural parameters that describe the market structure and pricing mechanisms, such as the frequency of price adjustment and the elasticity of marginal cost with respect to output. Theory suggests that β is close or equal to one, and k is positive. Readers are referred to Woodford (2003) for a detailed derivation of the model and a discussion of its microeconomic foundations.

The majority of empirical studies in the literature rely on (1) or variants thereof for their analyses of inflation dynamics. However, the model is derived from log-linearizing equilibrium conditions of the New Keynesian pricing model around a steady-state characterized by zero inflation, which is generally known to be counterfactual. Cogley and Sbordone (2008) were the first to estimate a version of the forward-looking NKPC that allows for time-variation in the long-run trend rate of inflation. This paper follows in the same spirit, but considers estimation of an alternative specification for the forward-looking model as derived in the theoretical study of Woodford (2008)[4]. The dynamics of inflation are expressed as:

[4] The two models are similar in form, but they are derived under different assumptions on how prices are set in periods when firms do not choose to reoptimize their prices. In Cogley and Sbordone (2008), prices remain

$$\pi_t = \bar{\pi}_t + \beta E_t\left[\pi_{t+1} - \bar{\pi}_{t+1}\right] + kx_t , \qquad (2)$$

where $\bar{\pi}_t$ is the trend rate of inflation at time t. Following the conventional approach, $\bar{\pi}_t$ is defined as the Beveridge-Nelson stochastic trend, $\bar{\pi}_t = \lim_{j\to\infty} E_t\pi_{t+j}$, which is modeled as a driftless random walk (see Beveridge and Nelson, 1981). Then, (2) can be viewed as a generalization of (1). By the law of iterated expectations, we can write $E_t\bar{\pi}_{t+1} = E_t\left[\lim_{j\to\infty} E_{t+1}\pi_{t+1+j}\right] = \left[\lim_{j\to\infty} E_t\pi_{t+1+j}\right] = \bar{\pi}_t$, and (2) becomes:

$$\pi_t = (1-\beta)\bar{\pi}_t + \beta E_t\pi_{t+1} + kx_t . \qquad (3)$$

Hence, under the assumption of zero trend inflation, the above forward-looking specification reduces to the standard model under (1).

As shown, the presence of a stochastic trend distinguishes the model in this paper from the majority of empirical studies on the NKPC. With trend inflation being zero in (1), the time-series properties of inflation are characterized as stationary (i.e. inflation is an $I(0)$ process), such that in the long-run, inflation converges to a deterministic steady-state of zero inflation. In (2) however, inflation has a unit root (i.e. inflation is an $I(1)$ random variable), which implies an absence of any tendency for inflation to settle on a deterministic steady-state level in the long-run. Apart from leading to different descriptions about the long-run behavior of inflation, the presence of stochastic trend inflation also carries important implications for the short-run dynamics of inflation. In the standard case, the model characterizes short-run movements of inflation as small fluctuations around zero, whereas with stochastic trend inflation, these fluctuations are described as deviations from a time-varying trend.

This paper estimates the closed form specification of the forward-looking NKPC, which is obtained by repeated substitutions of the model in (2) under the assumption of rational expectations. This method solves out iteratively for inflation expectations and provides the model-path solution of inflation in the following form:

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unchanged, whereas in Woodford (2008), firms automatically change their prices at the trend inflation rate. This assumption of indexation to trend inflation has also been used in the DSGE models of Yun (1996), Smets and Wouters (2003) and Ireland (2007).

$$\pi_t = \bar{\pi}_t + \beta^{\infty} E_t \left[\pi_{\infty} - \bar{\pi}_{\infty} \right] + k \sum_{j=0}^{\infty} \beta^j E_t x_{t+j} \ . \tag{4}$$

Since $E_t \left[\pi_{\infty} \right] = E_t \left[\bar{\pi}_{\infty} \right] = \bar{\pi}_t$, (4) reduces to:

$$\pi_t = \bar{\pi}_t + k \sum_{j=0}^{\infty} \beta^j E_t x_{t+j} \ . \tag{5}$$

Here, the forward-looking NKPC implies that current inflation is the sum of stochastic trend inflation and the discounted sum of expected present and future values of the output gap. The stochastic trend component captures the $I(1)$ behavior in inflation and corresponds to the permanent counterpart of inflation that represents its underlying long-run rate. The latter component which is determined by the evolution of expected output gaps is assumed to be $I(0)$ with zero mean, and it drives the transitory fluctuations in inflation over the short-run horizon. Hence, it follows that the expression in (5) can be viewed as a trend and cycle decomposition of inflation. This decomposition differs from other trend and cycle decompositions for inflation in the literature, such as that of Stock and Watson (2007). In previous studies, trend and cycle components are identified solely based on statistical relationships. Here, they are specified in a way that is consistent with the forward-looking NKPC, and therefore they contain more structure and economic content.

The closed form specification in (5) is used by Goodfriend and King (2009) in their theoretical analysis of stochastic trend inflation. However, an empirical analysis of the model is lacking. Thus far, studies that analyze the closed form solution of the forward-looking NKPC have only done so under the assumption that the inflation process converges to a steady-state of zero inflation in the long-run. Accordingly, the stochastic trend component $\bar{\pi}_t$ in (5) vanishes, and current inflation depends only on the discounted sum of current and future output gaps (see Gali and Gertler, 1999; Gali and Lopez-Salido, 2001; Sbordone, 2002, 2005; Kurmann, 2005).

The specification in (5) should help clarify the two main empirical issues associated with the forward-looking NKPC, namely: (i) its failure to account for considerable inflation persistence in the data, and (ii) its inability to find a positive and statistically significant estimate of k, which links the short-run dynamics of inflation to fluctuations in real economic activity. According to (5), if the presence of stochastic trend inflation is ignored as in past studies, then the forward-looking specification forces inflation persistence to be explained entirely by the discounted sum of expected current and future output gaps. This is problematic, as both theory and widespread empirical evidence show that the amount of persistence that the output gap imparts onto inflation is rather small (see Fuhrer, 2005). In other words, regardless of the persistence in the output gap process, it follows that the amount of persistence that is passed on from the output gap to inflation would not be sufficient to match with the high levels of inertia in actual inflation data[5]. The inclusion of stochastic trend inflation should help reconcile this persistence problem, because trend inflation contributes as a highly persistent component of inflation. It should also help the forward-looking model find a significant link between inflation and the output gap, as the short-run dynamics of the model are likely to become better specified once movements in the long-run rate of inflation are properly taken into account.

3 Unobserved Components Model Specification

Lee and Nelson (2007) and Cogley and Sbordone (2008) present estimation methodologies for the forward-looking model that incorporates stochastic trend inflation. In the former study, the authors estimate a reduced form model in inflation and unemployment with stochastic trends and related stationary cycles. These results are then used to obtain indirect inference on the NKPC model parameter k. In the latter study, the authors rely on a two-step procedure to estimate the model via Bayesian methods. The first step gives inference on the time-varying trend, then conditional on those results, the second step yields the estimates of the NKPC model parameters.

In contrast to the aforementioned approaches, I introduce an econometric methodology that allows for direct estimation of the forward-looking NKPC that does not resort to estimating trend

[5] Accordingly, a number of authors such as Gali and Gertler (1999) and Christiano et. al (1995) add lagged inflation terms to the forward-looking specification which results in a hybrid NKPC that fits better with the data. However, the presence of lagged inflation terms in the NKPC has been criticized for its lack of convincing microfoundations (see Rudd and Whelan, 2007; Cogley and Sbordone, 2008). An alternative approach that has been used to improve the fit of the forward-looking NKPC is to replace the output gap with a more persistent driving process, such as labor income share (see Gali and Gertler, 1999; Sbordone, 2002, 2005). However, the use of labor income share has been criticized, with the most important reason being that its observed movements are countercyclical in contrast to what is predicted by theory. The use of labor income share in the NKPC relation also delivers awkward implications for the conduct of monetary policy (see discussions in Rotemberg and Woodford, 1999; Neiss and Nelson, 2002; and Rudd and Whelan 2002, 2007).

inflation and the model parameters separately. The idea behind this approach is based on the observation that neither the trend nor cycle components of inflation are observed, and thus, they should be treated as latent state variables in an UC model for inflation. This direct estimation approach should lead to more accurate and reliable estimates, as it exploits the dependence structure between different model components. In addition, it allows trend inflation to be endogenously determined within the model, which is different from the usual approach of specifying trend inflation as an exogenous random process (see Smets and Wouters, 2003; Ireland, 2007; Cogley and Sbordone, 2008).

The rest of this section describes the UC model specification for the forward-looking NKPC based on the closed form formulation in (5). I propose two UC model specifications. In the first one, the output gap that drives the cycle component of inflation is assumed to be a perfectly observable variable, which is the standard assumption used in the literature when estimating NKPCs. This model is referred to as the baseline UC model. The second specification extends the baseline UC model to include an additional UC model for real output. This results in a bivariate UC model that allows the output gap to enter the NKPC relation as an unobserved variable, rather than replacing it by measured proxies. A byproduct from estimation of the bivariate UC model is that it produces estimates of the output gap that are consistent with the forward-looking NKPC.

3.1 Baseline UC Model with Observed Output Gap

The baseline UC model is based on the following specification for inflation:

$$\pi_t = \bar{\pi}_t + k\sum_{j=0}^{\infty} E_{t-1}x_{t+j} + z_t, \qquad z_t \sim i.i.d.N(0,\sigma_z^2), \tag{6a}$$

which follows from three slight modifications of (5). First, following Gali and Gertler (1999) and Kurmann (2005), as well as other studies that assess the empirical performance of closed-form NKPCs, β is restricted to unity[6]. Second, the subscript on the expectations operator in the cycle

[6] This is the upper bound of theoretically admissible values for the discount factor. Calibrating the discount factor to other values that is used in the literature such as 0.99 does not alter the empirical results in this paper.

component is changed from time t to $t-1$. This is because it should be more realistic to assume that firms make forecasts about current and future output gaps at any date t based on past information available in I_{t-1} rather than in I_t. Last, an irregular component z_t is included, to capture the effect of any determinants of current inflation that are not explained by the New Keynesian pricing theory. The irregular component is assumed to be $i.i.d.$ (independently and identically distributed), normally distributed with a mean of zero and variance σ_z^2, and is assumed to be uncorrelated with all other disturbance terms in the model.

A complete specification of the baseline UC model requires describing the 'laws of motion' of the trend and cycle components of inflation. As is standard in the literature, the stochastic trend component $\bar{\pi}_t$ is assumed to follow a driftless random walk:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + e_t, \qquad e_t \sim i.i.d.N(0, \sigma_e^2) . \tag{6b}$$

This specification reflects that trend inflation is a source of stochastic or unpredictable (random) variation in inflation. In addition, since the random walk model implies $E\left[\bar{\pi}_{t+j}\right] = \bar{\pi}_t$ for any future date $t+j$, it is consistent with the Beveridge-Nelson interpretation of stochastic trend which defines the long-run forecast of inflation to be the latent trend at time t. Note that according to (6a), the best prediction of remote future inflation is only the permanent random walk component, because over a long enough horizon, transitory fluctuations in the cycle and irregular components of inflation are expected to approach their zero means.

To obtain an empirically operational expression for the infinite sum term that drives the cycle component of inflation in (6a), I employ the VAR projection method of Campbell and Shiller (1987). This method was first introduced in the context of calculating fundamental stock prices from forecasts of discounted future dividends, and has been subsequently used to evaluate closed form specifications of the NKPC. The idea behind this approach is that expectational elements in the cycle component of inflation are unknown, and needs to be forecasted in terms of available information from a reduced form VAR.

Thus, the first step is to determine an appropriate reduced form VAR for the output gap. Following Harvey (1985), Watson (1986), and Clark (1987), I assume that the dynamics of the output gap x_t are described as an autoregressive (AR) process of order 2:

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + v_t , \quad v_t \sim i.i.d.N\left(0,\sigma_v^2\right), \tag{6c}$$

where x_t is stationary and ergodic. When put into state-space form, this AR(2) representation implies the reduced form (univariate) VAR, $X_t = AX_{t-1} + V_t$, with $X_t = \left[x_t \quad x_{t-1}\right]'$, $X_{t-1} = \left[x_{t-1} \quad x_{t-2}\right]'$, $A = \begin{bmatrix} \phi_1 & \phi_2 \\ 1 & 0 \end{bmatrix}$, and $V_t = \left[v_t \quad 0\right]'$.

Given that X_{t-1} is a subset of the market's full information set at time $t-1$ (i.e. $X_{t-1} \subseteq I_{t-1}$), we can use the reduced form VAR in (6c) to compute multiperiod forecasts of the output gap conditional on information X_{t-1}. By the law of iterated expectations, it follows that $E\left[E_{t-1}x_{t+j} \mid X_{t-1}\right] = E\left[E\left[x_{t+j} \mid I_{t-1}\right] \mid X_{t-1}\right] = E\left[x_{t+j} \mid X_{t-1}\right] = e_1' A^{j+1} X_{t-1}$, with $e_1' = \left[1 \quad 0\right]$ acting as a 1×2 selection vector that singles out the forecast for the output gap. These multiperiod forecasts are then used to construct the following empirically operational expression for the infinite sum term:

$$\begin{aligned} \sum_{j=0}^{\infty} E_{t-1}x_{t+j} &= E\left[x_t \mid X_{t-1}\right] + E\left[x_{t+1} \mid X_{t-1}\right] + E\left[x_{t+2} \mid X_{t-1}\right] + \ldots \\ &= e_1' A X_{t-1} + e_1' A^2 X_{t-1} + e_1' A^3 X_{t-1} + \ldots \\ &= e_1' A (I_2 - A)^{-1} X_{t-1} \end{aligned} \tag{6d}$$

where I_2 is a 2×2 identity matrix.

Gathering equations (6a)-(6d), the full specification of the baseline UC model is:

$$\pi_t = \bar{\pi}_t + k\sum_{j=0}^{\infty} E_{t-1}x_{t+j} + z_t$$
$$\bar{\pi}_t = \bar{\pi}_{t-1} + e_t$$

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + v_t$$

$$\sum_{j=0}^{\infty} E_{t-1} x_{t+j} = e_1' A(I_2 - A)^{-1} X_{t-1}$$

with $e_1' = [1 \quad 0]$, $A = \begin{bmatrix} \phi_1 & \phi_2 \\ 1 & 0 \end{bmatrix}$, and $X_{t-1} = [x_{t-1} \quad x_{t-2}]'$.

The system has three shocks, with two that are specifically related to the forward-looking NKPC. These are e_t and v_t, and they are allowed to be correlated. Hence, the variance-covariance matrix associated with the baseline UC model is:

$$Cov(z_t, e_t, v_t) = \begin{bmatrix} \sigma_z^2 & 0 & 0 \\ 0 & \sigma_e^2 & \sigma_{ev} \\ 0 & \sigma_{ev} & \sigma_v^2 \end{bmatrix}. \qquad (6e)$$

3.2 Bivariate UC Model with Unobserved Output Gap

To allow the output gap to enter the NKPC relation as an unobserved variable, the baseline UC model for inflation is extended to a bivariate setting by adding an UC model for real output. Other bivariate UC models for inflation and output include those of Kuttner (1994), Gerlach and Smets (1999), Apel and Jansson (1999) and Basistha and Nelson (2007). However, these bivariate UC models differ substantially from the one studied here. This is because these bivariate specifications treat the inflation process as $I(0)$, and they do not account for heteroskedasticity in the inflation process as is done in this paper. Furthermore, the inflation equation in previous bivariate UC models are based on reduced form expressions, rather than a structural model that is consistent with theory. The bivariate UC model of Basistha and Nelson (2007) is an exception, because they also specify an UC model for inflation that is consistent with the forward-looking NKPC. However, instead of mapping inflation expectations into a long-run trend component as I do, they measure expectations of inflation with survey data and an arbitrary lagged inflation term.

The bivariate UC model augments the baseline UC model by including the following UC model for real output:

$$y_t = \tau_t + x_t \tag{7a}$$

$$\tau_t = \delta_0 D_t + \delta_1 (1 - D_t) + \tau_{t-1} + w_t \tag{7b}$$

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + v_t \tag{7c}$$

Here, y_t is the equilibrium level of real output, τ_t is the stochastic trend that represents the permanent component of y_t, and x_t is the transitory or cyclical component of real output.

The trend component τ_t is assumed to follow a random walk process with drift δ. According to Perron and Wada (2009), it may be important to include a one-time break in δ to account for the productivity slowdown that occurred in the first quarter of 1973. To incorporate this one-time break, this paper follows the approach of Basistha (2006) and Mitra and Sinclair (2008), by specifying a series of dummies D_t that takes on the value of 1 for dates from the start of the sample to 1973:Q1, and 0 otherwise. Thus, δ_0 and δ_1 are the rates of growth in the periods before and after the productivity slowdown in 1973:Q1 respectively. The inclusion of D_t does not force a one-time break on the output drift δ, but simply allows it to happen. This point is worth mentioning since a number statistical tests have failed to find significant evidence in favor of a change in the deterministic part of the trend function for US real output. The specification in (7b) lets the data decide, because this failure may possibly be due to a lack of power.

The transitory component of output x_t corresponds to the deviations of actual real output from its capacity level. Hence, it can be interpreted as a measure of the output gap, which as before, is assumed to follow a stationary AR process of order 2. This unobserved measure of the output gap drives the cycle component of inflation, and therefore it serves as the important link in the bivariate UC model of inflation and real output.

In the baseline UC model, the infinite sum term in the inflation cycle is computed according to the expression in (6d). However, since the output gap is unobserved in the bivariate setting, the cycle component would now depend on an expected rather than observed measure of the lagged output gaps. To be more explicit, I rewrite (7c) in state-space form, $X_t^* = AX_{t-1}^* + V_t$, where $X_t^* = [x_t \quad x_{t-1}]'$, $X_{t-1}^* = [x_{t-1} \quad x_{t-2}]'$, $A = \begin{bmatrix} \phi_1 & \phi_2 \\ 1 & 0 \end{bmatrix}$, and $V_t = [v_t \quad 0]'$. Then, assuming that $X_{t-1}^* \subseteq I_{t-1}$, we have:

$$\begin{aligned} \sum_{j=0}^{\infty} E_{t-1} x_{t+j} &= E\left[x_t \mid X_{t-1}^*\right] + E\left[x_{t+1} \mid X_{t-1}^*\right] + E\left[x_{t+2} \mid X_{t-1}^*\right] + \ldots \\ &= E\left[x_t \mid X_{t-1}^*\right] + AE\left[x_t \mid X_{t-1}^*\right] + A^2 E\left[x_t \mid X_{t-1}^*\right] + \ldots \\ &= (I_2 - A)^{-1} E\left[x_t \mid X_{t-1}^*\right]. \end{aligned} \tag{7d}$$

Hence, as illustrated by the above expression, the infinite sum term now depends on an estimate of $E\left[x_t \mid X_{t-1}^*\right]$, which is defined as the expected value of the current output gap conditional on information X_{t-1}^*, rather than simply information X_{t-1}. The two information sets, X_{t-1} and X_{t-1}^*, both contain two lags of the output gap, but since the output gap terms in X_{t-1}^* are determined by estimation of the bivariate UC model, X_{t-1}^* also contains information about real output growth and inflation.

The full specification for the bivariate UC model is a collection of (6a), (6b), (7a), (7b), (7c), and (7d):

$$\pi_t = \bar{\pi}_t + k \sum_{j=0}^{\infty} E_{t-1} x_{t+j} + z_t$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + e_t$$

$$y_t = \tau_t + x_t$$

$$\tau_t = \delta_0 D_t + \delta_1 (1 - D_t) + \tau_{t-1} + w_t$$

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + v_t$$

$$\sum_{j=0}^{\infty} E_{t-1} x_{t+j} = (I_2 - A)^{-1} E\left[x_t \mid X_{t-1}^*\right]$$

with $A = \begin{bmatrix} \phi_1 & \phi_2 \\ 1 & 0 \end{bmatrix}$, and $X_{t-1}^* = [x_{t-1} \quad x_{t-2}]'$.

As before, all shocks in the system other than that of the irregular component z_t are allowed to be correlated. The generalized variance-covariance matrix to be estimated is:

$$Cov(z_t, e_t, w_t, v_t) \equiv \begin{bmatrix} \sigma_z^2 & 0 & 0 & 0 \\ 0 & \sigma_e^2 & \sigma_{ew} & \sigma_{ev} \\ 0 & \sigma_{ew} & \sigma_w^2 & \sigma_{vw} \\ 0 & \sigma_{ev,S_t} & \sigma_{vw} & \sigma_v^2 \end{bmatrix}. \quad (7e)$$

Here, it is emphasized that by allowing the shocks to the inflation trend to be correlated with those of the output trend and cycle components, I create additional channels in which the dynamics of real output may have additional implications for inflation, other than through their related cycles.

4 Estimation Methodology in the Presence of Structural Breaks

It is generally accepted that during the postwar period, US inflation has undergone two permanent and significant structural changes due to shifts in the monetary policy regime. The first regime shift has been found to occur around the beginning of the 1970s, surrounding the period associated with the collapse of the Bretton Woods system and severe oil price shocks. The second regime shift has been documented to take place in the early to mid 1980s. It is generally found to occur some time in between the Volcker disinflation and the Great Moderation, which is a term used to describe the substantial decline in the volatility of many macroeconomic aggregates that took place in the mid 1980s.

To account for these changes in US inflation dynamics, the variances and covariances of the shocks that are related to the inflation equation in both the baseline and bivariate UC models are allowed to undergo two endogenously determined structural breaks. Accounting for heteroskedasticity in this way is especially relevant in the context of modeling trend inflation. In UC models for inflation, Ball and Cechetti (1990), Stock and Watson (2007), Kim (1993), Kang et. al (2009), and many others, find that the variance of permanent shocks to inflation was significantly higher during the 1970s to mid 1980s compared to other periods. In addition, according to Kang et. al (2009), the long-run trend component of inflation is related to a measure of inflation persistence. Therefore, the papers of Levin and Piger (2004) and Cogley and Sargent (2005) that document higher inflation persistence during the 1970s to mid 1980s relative to the other periods provides further evidence that the variance of permanent shocks to US inflation has indeed undergone two significant structural changes over the postwar period.

Heteroskedasticity in UC models for inflation are typically captured by Markov switching parameters (see Kang et al., 2009) or are modeled as stochastic volatility (see Cogley and Sargent, 2005; Stock and Watson, 2007). However, Stock and Watson (2007) show that their stochastic volatility approach towards modeling gradual changes in inflation variability yields quantitatively and qualitatively similar estimates to the same model where shocks undergo rapid changes. In addition, Kang et. al (2009) show that their UC model for inflation with Markov switching parameters is comparable to one where those same parameters undergo structural breaks. Therefore, I choose to capture heteroskedasticity in inflation with structural break parameters, which should simplify the estimation procedure without altering the empirical results.

The structural break parameters θ_{S_t} are $\theta_{S_t} = \left\{\sigma^2_{z,S_t}, \sigma^2_{e,S_t}, \sigma_{ev,S_t}\right\}$ for the baseline UC model, and $\theta_{S_t} = \left\{\sigma^2_{z,S_t}, \sigma^2_{e,S_t}, \sigma_{ew,S_t}, \sigma_{ev,S_t}\right\}$ for the bivariate UC model. The dynamics of θ_{S_t} are described as:

$$\theta_{S_t} = \theta_1 S_{1t} + \theta_2 S_{2t} + \theta_3 S_{3t} \tag{8}$$

$$S_{mt} = \begin{Bmatrix} 1 & if & S_t = m \\ 0 & otherwise & \end{Bmatrix}, \qquad m = 1, 2, 3,$$

where m denotes the three regimes under consideration. The latent state variable S_t follows a first-order Markov switching process with nonrecurring states. The state transition probability matrix is given by:

$$\Pi = \begin{pmatrix} p_{11} & 0 & 0 \\ 1 - p_{11} & p_{22} & 0 \\ 0 & 1 - p_{22} & 1 \end{pmatrix}, \tag{9}$$

where $p_{ij} = \Pr[S_t = j \mid S_{t-1} = i]$, and $i, j = 1, 2, 3$. The last regime acts as the absorbing state, and the expected duration of regime m is given by $1/(1 - p_{mm})$. Therefore, each structural break point is estimated to be the sum of the expected durations of past and current regimes.

The rest of this section describes how the baseline and bivariate UC models are put into state-space form, so they can be estimated by maximum likelihood (ML) with the Kim filter (see Kim, 1994; Kim and Nelson, 1999). The Kim filter is an extension of the Kalman filter (see Harvey, 1989) combined with the Hamilton filter (see Hamilton, 1989), and is used to provide inference on UC models with regime switching.

4.1 Baseline UC Model with Observed Output Gap

The baseline UC model (6a)-(6e) with two structural breaks can be written as:

$$\pi_t = \bar{\pi}_t + k \sum_{j=0}^{\infty} E_{t-1} x_{t+j} + z_t \tag{10a}$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + e_t \tag{10b}$$

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + v_t \tag{10c}$$

$$\sum_{j=0}^{\infty} E_{t-1} x_{t+j} = e_1' A (I_2 - A)^{-1} X_{t-1} \tag{10d}$$

$$\begin{bmatrix} z_t \\ e_t \\ v_t \end{bmatrix} \sim i.i.d.N \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2_{z,S_t} & 0 & 0 \\ 0 & \sigma^2_{e,S_t} & \sigma_{ev,S_t} \\ 0 & \sigma_{ev,S_t} & \sigma^2_v \end{bmatrix} \right) \tag{10e}$$

with $e_1' = [1 \quad 0]$, $X_{t-1} = [x_{t-1} \quad x_{t-2}]'$, $A = \begin{bmatrix} \phi_1 & \phi_2 \\ 1 & 0 \end{bmatrix}$, and the dynamics of $\theta_{S_t} = \{\sigma^2_{z,S_t}, \sigma^2_{e,S_t}, \sigma_{ev,S_t}\}$ as defined in (8) and (9).

Estimation of the model involves putting the unobserved counterpart of the model, (10a) and (10b), into state-space form, then maximizing a joint log-likelihood function that involves the equations for inflation π_t, and the output gap process x_t. However, since $\bar{\pi}_t$ does not enter into this joint log-likelihood function directly, it would be cumbersome to allow σ_{ev}, the covariance between the shock to the inflation trend $\bar{\pi}_t$, and the shock to the output gap process x_t, to be non-zero.

To overcome this difficulty, the model is transformed with the control function approach. Consider the following variance-covariance structure:

$$\begin{bmatrix} v_t^* \\ e_t \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{S_t}\sigma_{e,S_t} \\ \rho_{S_t}\sigma_{e,S_t} & \sigma_{e,S_t}^2 \end{bmatrix} \right). \tag{11}$$

In the above specification, $v_t^* \equiv \sum_{v,S_t}^{-1/2} v_t$ is a vector of standardized error terms, with v_t being the shock to the output gap process x_t. ρ_{S_t} denotes the correlation between e_t, the disturbance term to trend inflation, and v_t^*, in each of the three regimes. Based on a Cholesky decomposition of the above variance-covariance matrix, the error term e_t is decomposed into a part that is correlated with v_t, and a part that is not:

$$e_t = \gamma_{S_t} v_t^* + \varepsilon_t, \qquad \varepsilon_t \sim N(0, (1-\rho_{S_t})\sigma_{e,S_t}^2). \tag{12}$$

Here, ε_t is the new disturbance term that is uncorrelated with v_t^* and all other shocks in the system, with $\gamma_{S_t} = \rho_{S_t}\sigma_{e,S_t}$.

With (12), (10b) can be rewritten as:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \gamma_{S_t} v_t^* + \varepsilon_t \tag{10b$'$}$$

and (10e) becomes:

$$\begin{bmatrix} z_t \\ \varepsilon_t \\ v_t \end{bmatrix} \sim i.i.d.N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{z,S_t}^2 & 0 & 0 \\ 0 & \sigma_{\varepsilon,S_t}^2 & 0 \\ 0 & 0 & \sigma_v^2 \end{bmatrix} \right). \tag{10e$'$}$$

Therefore, in the transformed model, the correlation between the shocks to trend inflation and the output gap is simply taken care of through the term $\gamma_{S_t} v_t^*$ in (10b′).

To estimate the transformed model, the unobserved counterpart of the baseline UC model, (10a) and (10b), is cast into the following state-space form:

Measurement equation: $Y_t = HB_t + k\sum_{j=0}^{\infty} E_{t-1}x_{t+j} + V_t, \quad V_t \sim i.i.d.N(0, R_{S_t})$ (13a)

$$\pi_t = \bar{\pi}_t + k\sum_{j=0}^{\infty} E_{t-1}x_{t+j} + z_t$$

$$R_{S_t} = \sigma^2_{z,S_t}.$$

Transition equation: $B_t = D_{S_t}\Lambda_t + FB_{t-1} + \xi_t, \quad \xi_t \sim i.i.d.N(0, Q_{S_t})$ (13b)

$$\bar{\pi}_t = \gamma_{S_t} v_t^* + \bar{\pi}_{t-1} + \varepsilon_t$$

$$Q_{S_t} = \sigma^2_{\varepsilon,S_t}.$$

Then, to estimate the baseline UC model, the state-space model above is jointly estimated with the AR(2) specification for the output gap (10c), by ML using the Kim filter. For details on the joint estimation methodology, readers are referred to Appendix A.

4.2 Bivariate UC Model with Unobserved Output Gap

The bivariate UC model, (8a)-(8f), with two endogenously determined structural breaks can be written as:

$$\pi_t = \bar{\pi}_t + k\sum_{j=0}^{\infty} E_{t-1}x_{t+j} + z_t \tag{14a}$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + e_t \tag{14b}$$

$$y_t = \tau_t + x_t \tag{14c}$$

$$\tau_t = \delta_0 D_t + \delta_1(1 - D_t) + \tau_{t-1} + w_t \tag{14d}$$

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + v_t \tag{14e}$$

$$\sum_{j=0}^{\infty} E_{t-1} x_{t+j} = (I_2 - A)^{-1} E\left[x_t \mid X_{t-1}^* \right] \tag{14f}$$

$$\begin{bmatrix} z_t \\ e_t \\ w_t \\ v_t \end{bmatrix} \sim i.i.d.N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{z,S_t}^2 & 0 & 0 & 0 \\ 0 & \sigma_{e,S_t}^2 & \sigma_{ew,S_t} & \sigma_{ev,S_t} \\ 0 & \sigma_{ew,S_t} & \sigma_w^2 & \sigma_{vw} \\ 0 & \sigma_{ev,S_t} & \sigma_{vw} & \sigma_v^2 \end{bmatrix} \right) \tag{14g}$$

with $X_{t-1}^* = \left[x_{t-1} \quad x_{t-2} \right]'$, $A = \begin{bmatrix} \phi_1 & \phi_2 \\ 1 & 0 \end{bmatrix}$, and the dynamics of $\theta_{S_t} = \left\{ \sigma_{z,S_t}^2, \sigma_{e,S_t}^2, \sigma_{ew,S_t}, \sigma_{ev,S_t} \right\}$ as defined in (8) and (9).

The bivariate UC model above can be cast into the following state-space form:

Measurement equation: $Y_t = HB_t + K\sum_{j=0}^{\infty} E_{t-1} x_{t+j} + V_t, \qquad V_t \sim i.i.d.N(0, R_{S_t})$ (15a)

$$\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \bar{\pi}_t \\ \tau_t \\ x_t \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} k \\ 0 \end{bmatrix} \sum_{j=0}^{\infty} E_{t-1} x_{t+j} + \begin{bmatrix} z_t \\ 0 \end{bmatrix}$$

$$R_{S_t} = \sigma_{z,S_t}^2 .$$

Transition equation: $B_t = \Lambda_t + FB_{t-1} + G\xi_t, \qquad \xi_t \sim i.i.d.N(0, Q_{S_t})$ (15b)

$$\begin{bmatrix} \bar{\pi}_t \\ \tau_t \\ x_t \\ x_{t-1} \end{bmatrix} = \begin{bmatrix} 0 \\ \delta_0 D_t + \delta_1 (1 - D_t) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{\pi}_{t-1} \\ \tau_{t-1} \\ x_{t-1} \\ x_{t-2} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_t \\ w_t \\ v_t \end{bmatrix}$$

$$Q_{S_t} = \begin{bmatrix} \sigma_{e,S_t}^2 & \sigma_{ew,S_t} & \sigma_{ev,S_t} \\ \sigma_{ew,S_t} & \sigma_w^2 & \sigma_{vw} \\ \sigma_{ev,S_t} & \sigma_{vw} & \sigma_v^2 \end{bmatrix}.$$

Then, the state-space model above can be estimated by ML with the Kim filter. During the estimation step, an estimate for the infinite sum term (14f) can be easily computed. This is because to obtain inferences on the unobserved components, the Kim filter produces an estimate of $B_{t|t-1}^{(i,j)}$, defined as the forecast of the unobserved state vector B_t conditional on information I_{t-1}, $S_t = j$ and $S_{t-1} = i$, with $i, j = 1, 2, 3$. With an estimate of $B_{t|t-1}^{(i,j)}$, (14f) is computed as:

$$\sum_{j=0}^{\infty} E_{t-1} x_{t+j} = e_1'(I_2 - A)^{-1} C B_{t|t-1}^{(i,j)}, \tag{16}$$

where $e_1' = \begin{bmatrix} 1 & 0 \end{bmatrix}$ is a 1×2 selection vector that singles out the forecast of the output gap, and $C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ is a 2×4 selection matrix that singles out the 2×1 matrix $\begin{bmatrix} x_{t|t-1}^{(i,j)} & x_{t-1|t-1}^{(i,j)} \end{bmatrix}'$ from $B_{t|t-1}^{(i,j)}$. Readers are referred to Appendix B for an outline of the estimation methodology.

5 Data and Empirical Results

5.1 Data Description

Inflation is the annualized log-difference of the quarterly gross domestic product (GDP) chain-type price index. The annualized log-difference of quarterly consumer price index (CPI) for all urban consumers, and the personal consumption expenditure (PCE) chain-type price index are also used as a measure of inflation for robustness checks. The real output measure used for estimation of the bivariate UC model is quarterly data of annualized RGDP, and the output gap measure used for estimation of the baseline UC model is the CBO's estimate of the output gap. The CBO output gap is chosen because compared to other commonly used measures of the output gap in empirical studies of the NKPC, it is estimated in a way that is most consistent with economic theory. This should allow for a meaningful comparison between the results obtained from estimation of the baseline and bivariate UC models, because the output gap that is produced from the bivariate UC model is also constructed in a way that is in accordance with theory, namely, the New Keynesian pricing theory.

The data is taken primarily from the Federal Reserve Economic Database (FRED). All series span 1952:Q1 to 2007:Q3 and are seasonally adjusted with base year 2000. The beginning of the sample is chosen to avoid large swings in inflation resulting from the Korean war, and the end of the sample marks the quarter prior to the 2007 recession.

5.2 Parameter Estimates

5.2.1 Baseline UC Model with Observed Output Gap

As a preliminary study, the baseline UC model is first estimated without any structural breaks. Parameter estimates are reported in Table 1 with standard errors in parentheses. The parameter estimates have magnitudes that fall within the range reported in the literature, and are estimated with reasonable accuracy. An interpretation of the results is as follows. First, the sum of the AR coefficients that describe the output gap process is 0.91, suggesting that the CBO output gap is highly persistent. The standard deviation estimates for the output gap process and inflation trend, σ_v and σ_e, reveal that these processes have been quite volatile over the postwar period. As for the standard deviation of the irregular component, σ_z, its point estimate is of reasonable magnitude. Last, the estimate of γ is insignificant, suggesting no correlation between the disturbance terms to the output gap process and inflation trend.

The parameter of interest is the slope of the Phillips curve k. Similar to earlier results in the literature, I find that the forward-looking NKPC fails to find a statistically significant estimate of k. From an economic perspective, this result is disconcerting because it implies that there is no short-run tradeoff between inflation and real economic activity. This result contradicts with empirical evidence of a sacrifice ratio, and is also unsettling for the basic story on how monetary authorities affect inflation through its influence on aggregate demand. Hence, I conclude that the performance of the baseline UC model with no structural breaks is not entirely satisfactory.

Next, I investigate the empirical performance of the baseline UC model with two structural breaks. The estimation results are reported in Table 2. From the estimates of the transition probabilities p_{11} and p_{22}, two structural breaks in the variances and the covariances of the shocks are found to occur in 1968:Q4 and 1984:Q3. These estimated break dates closely coincide with those found in earlier studies, and lends support to the traditional view that the behavior of inflation is closely tied with the monetary regime.

The estimates of the parameters that describe the output gap process, namely the sum of its AR coefficients and the standard deviation of its shocks, are similar to those obtained from estimation of the model without structural breaks. The estimates of γ are statistically insignificant in all three regimes, suggesting no correlation between the disturbance terms to the output gap process and the inflation trend. From the estimates of σ_z, the irregular component was most volatile during the second regime, which coincides with the period that is commonly referred to as the Great Inflation. This finding suggests that during the Great Inflation, there was a greater portion of movements in actual inflation that could not be explained by the forward-looking NKPC.

Similarly, trend inflation is found to be most volatile during the Great Inflation. The standard deviation estimates of its shocks are of comparable magnitude to those obtained from the univariate UC model of Kang et. al (2009). These estimates suggest that since the mid 1980s, movements in US trend inflation has become relatively stable and subdued. An economic interpretation for this result could be that long-run inflation expectations have become better anchored since then, which is in line with the conclusions drawn by Mishkin (2007). However, although trend inflation has become less volatile and in a sense more ‘anchored’, I find that the degree in which they are anchored is still imperfect. This is because the standard deviation estimate of trend inflation in the third regime is positive and statistically significant. This finding suggests that inflation expectations may still respond to news about the economy or recent developments in the actual rate of inflation, as has been found to be the case in the empirical studies of Levin et. al (2004) and Gürkaynak et. al (2005).

An interesting result that emerges from the inclusion of structural breaks in the variances and covariances of the shocks, is that now, the baseline UC model can find a significant slope estimate for the forward-looking NKPC. Thus, the baseline UC model with heteroskedasticity provides a reasonably good description of postwar US inflation. The slope parameter k is estimated to be 0.02, implying a flat sloped Phillips curve. This point estimate is smaller in magnitude than those usually reported in the literature, and it implies that an output gap of 1% (i.e. output in excess of potential) brings a 0.08% per quarter increase in the annualized inflation rate, which translates to an increase of 0.34% if maintained over an entire year. Negative output gaps yield symmetric effects on inflation.

5.2.2 Bivariate UC Model with Unobserved Output Gap

The bivariate UC model is first estimated without the inclusion of structural breaks, with empirical results that are reported in Table 3. As shown, the parameter estimates that describe the inflation equation, σ_e, σ_z, and k, are similar to the ones obtained from the baseline UC model with no structural breaks. Without the inclusion of heteroskedasticity however, it is noted again that the forward-looking NKPC fails to find a significant point estimate of k.

Estimation of the bivariate UC model allows us to investigate the dynamics of output, along with the implications that it has on the behavior of inflation. First, I focus on the parameter estimates that describe the output equation. From the standard deviation estimates σ_w and σ_v, movements in US output trend and cycle are large and volatile, even after allowing for correlation among their shocks. The covariance of their shocks σ_{vw} is statistically significant[7], with an implied correlation of -0.66. This is lower than the one obtained from Morley et. al's (2003) univariate UC model for output (-0.91) but is of comparable magnitude to the one from Basistha and Nelson's (2007) bivariate UC model for inflation and output (-0.75). As for the estimates of the trend growth rates δ_0 and δ_1, they imply an annual growth rate of around 4.2% before the productivity slow down, and 2.8% thereafter. These estimates are roughly in line with those reported by Perron and Wada (2009). However, in contrary to their results, I find that allowing for a one-time break in output trend drift did not result in a non-stochastic trend for equilibrium real output.

Turning now to analyze the dynamics of the output gap, the sum of the AR coefficients that describe the bivariate UC model's estimate of the output gap is of equal magnitude to the one that governs the CBO output gap. It is also of the same magnitude to the sum of the AR coefficients that describe the output gap process of Basistha and Nelson (2007). These authors

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[7] In the original UC models for output as proposed by Harvey (1985), Watson (1986), and Clark (1987), correlation between trend and cycle components are restricted to zero. However, zero correlation is argued to be counterfactual. Several economic interpretations for the existence of non-zero correlation can be found in Clark (1987), Gali (1999), and Zamowitz and Ozyildirim (2006).

also estimate an output gap that is consistent with the forward-looking NKPC, although their estimate of the output gap is backed out from a bivariate UC model which assumes that inflation is $I(0)$. Therefore, the finding of an equal sum of AR coefficients despite the output gaps being obtained from quite disparate empirical models suggests that the output gap measure consistent with the forward-looking NKPC is highly persistent. In addition, since the univariate UC model of Morley et. al (2003) finds less persistent output gap dynamics, the findings in this paper strengthens Basistha and Nelson's (2007) argument that information in inflation can help identify output gap persistence.

I also find movements in output trend and cycle components to have important implications for the dynamics of trend inflation. From the sign on σ_{ew}, there exists a negative tradeoff between the long-run co-movements of US inflation and real output. Also, the sign on σ_{ev} implies that shocks to trend inflation and the output gap process are positively correlated. Although it is not clear from my model why this might be the case, there are some possible explanations. For example, this non-zero covariance could reflect the Federal Reserve Bank's strong priority for maintaining the level of output at its capacity. As shown in the theoretical macroeconomic model of Goodfriend and King (2009), preference for output gap stabilization coupled with a series of adverse shocks that widens the output gap could cause monetary authorities to give up its nominal anchor for inflation. This, as a result, translates into an inflation target that randomly drifts. Conversely, this correlation may reflect the effect that the nominal funds rate may have on both economic activity and long-run inflation expectations. As shown in Kiley (2008), a tighter monetary policy, usually implemented to reduce aggregate output, can also have the effect of lowering long-run inflation expectations.

Next, the bivariate UC model is estimated with the inclusion of two structural breaks, with estimation results that are reported in Table 4. I also estimated the model with CPI inflation and PCE inflation data, and those results are reported in Table 5 and 6 respectively. There are slight quantitative differences among the estimates that are obtained from different inflation measures, but the qualitative conclusions that can be drawn are the same.

As shown in Table 4, the two structural break dates are estimated to be 1968:Q2 and 1984:Q3. All other parameter estimates that describe the dynamics of inflation are broadly similar to the results obtained from the baseline UC model with two structural breaks. It is also worth mentioning that with the inclusion of two structural breaks, the slope of the forward-looking NKPC k becomes statistically significant. This was also the case when estimating the baseline UC model, and therefore this result highlights the importance of allowing for heteroskedasticity in inflation when fitting an empirical model to postwar US inflation data.

The estimates describing US output dynamics are roughly similar to those obtained from the bivariate UC model without structural breaks. However, with the inclusion of two structural breaks, the output trend and cycle components are now no longer correlated because the estimate of σ_{vw} is statistically insignificant. The cross-equation covariance terms σ_{ew} and σ_{ev}, have the same signs as before, and they imply roughly the same degree of correlation between inflation trend and output trend, and inflation trend and output cycle, across the three regimes respectively. These estimates are statistically significant for most of the postwar period, except for the third regime. This result implies that since the mid 1980s, inflation trend is no longer correlated with output trend or cycle. A plausible interpretation for this finding could be that the Federal Reserve Bank's inflation target has become better anchored since then, as it responds less to the shocks that affect capacity output and aggregate demand.

From the empirical results presented here, both the performance of the baseline and bivariate UC models with the inclusion of two structural breaks are satisfactory. In what follows, these models will simply be referred to as the baseline and bivariate UC models, with the implicit understanding that they include two structural breaks in the variances and covariances of the shocks.

5.3 Estimates of Inflation Trend and Cycle

The estimated inflation trend and cycle components from the baseline and bivariate UC models are similar. Therefore, I only present and discuss the empirical results from estimation of the latter model.

A plot of latent trend inflation along with actual inflation is displayed in Figure 1a. It is evident that trend inflation is highly persistent, and closely tracks the observed movements in actual US inflation. As the figure illustrates, trend inflation varies, thus highlighting the importance of including a stochastic trend component when modeling inflation dynamics. In addition, although variability in trend inflation declined dramatically since the mid 1980s, movements in the underlying trend are still apparent.

The trajectory of trend inflation in Figure 1a is roughly similar to other estimates of trend inflation obtained from the models of Tinsley and Kozicki (2002), Ireland (2007), and Clark and Davig (2008). Tinsley and Kozicki estimate a multivariate VAR with shifting end points to derive an "anchor of long-run inflation expectations", Ireland estimates the Federal Reserve Bank's long-run inflation target in the context of a DSGE model, and Clark and Davig use information from inflation, inflation expectations from survey data, and the federal funds rate to extract an unobserved trend rate of inflation in a VAR framework. However, while displaying similar movements, the estimates produced here are accompanied by a measure of uncertainty, that arises naturally from estimation of the model with the Kim filter. This measure of uncertainty is useful, as it can be used to assess the level of confidence one should attach with trend inflation estimates at any given date.

As shown in Cogley and Sbordone (2008), trend inflation estimates can be incredibly uncertain. These authors report wide 90% confidence bands associated with their estimates of time-varying trend. However, as displayed in Figure 1b, the 90% confidence interval associated with the estimates of trend inflation produced by the bivariate UC model is quite narrow, especially when compared to those of Cogley and Sbordone (2008). In the empirical model of Cogley and Sbordone, trend inflation is assumed to be an exogenous process, and it is obtained from estimating a multivariate reduced form VAR. However, trend inflation is modeled endogenously in my model, and from the empirical results discussed earlier, I find that trend inflation is indeed not isolated from other shocks in the system. Therefore, one conclusion that can be drawn from this analysis is that allowing for joint interaction between trend inflation and economy wide shocks is important, and doing so should have the benefit of delivering more accurate and reliable estimates for the stochastic trend component.

Turning to an analysis of short-run inflation dynamics, Figure 2 displays the estimates of the inflation cycle, along with a measure of the inflation gap. The inflation gap is often used to describe the short-run behavior of inflation, and it is calculated as the difference between actual inflation and its latent trend. As shown, the cycle component consistent with the forward-looking NKPC with a stochastic trend component can track the overall movements in the inflation gap reasonably well.

5.4 A Comparison of Output Gaps

From the empirical results discussed in Section 5.2, both the CBO output gap and the bivariate UC model's estimate of the output gap are significant driving variables for inflation once heteroskedasticity in the inflation series is taken into account. Both of these output gap measures are obtained from quite distinct methods: the CBO output gap from estimation of a large-scale multisector growth model, and the unobserved output gap from estimation of a bivariate UC model that is consistent with the forward-looking NKPC. One other notable difference between these two output gap measures is that the CBO output gap is a two-sided estimate, as it is constantly revised after new information becomes available in the data. On the other hand, the bivariate UC model's estimates is a one-sided measure, that only relies on (revised) data up until date t. Yet, the estimated parameters that describe their dynamics such as the sum of the AR coefficients and the standard deviation of their shocks are remarkably similar.

Just how similar are these two output gap measures? Figure 3a presents a plot of the two output gaps. From a quick glance, both output gap measures contain movements that are largely similar. Some noteworthy differences occur in the late 1990s into the early 2000s, and the period of the mid 2000s. In the former period, the CBO output gap is positive and large, whereas the bivariate UC model's output gap is small and about zero on average. Inflation was relatively stable despite the booming economy during this period, and therefore the measure from the bivariate UC model suggests that potential output was growing at a similar rate as real output. As for the period of the mid 2000s, unlike the CBO's measure, the bivariate UC model's estimate of the output gap points to an economy that was overheated.

To better quantify the differences between the two output gap series, the 95% confidence interval associated with the bivariate UC model's estimates are plotted along with measures of the CBO output gap in Figure 3b. As shown, apart from the mid 1970s to early 1980s period where the CBO output gap dips slightly below the lower 95% confidence band, the CBO output gap is well contained within the confidence set. This result suggests that overall, the differences between these two output gap series are not statistically significant at the 5% level. This finding strengthens Kuttner's (1994) case of using the bivariate UC modeling approach as an effective shortcut method to obtain estimates of the output gap in place of a comprehensive supply-side analysis, such as the one used by the CBO. In addition, as the bivariate UC model is able to produce a one-sided estimate of the output gap that is comparable to a revised two-sided measure with relatively narrow confidence bands, these results suggests that the bivariate UC modeling approach may be able to provide some payoffs for the tasks of real-time business cycle measurement and inflation forecasting.

6 Diagnostic Checks

This section describes the two criterions that are used to evaluate the baseline and bivariate UC models' ability to fit the data. First, I test for serial correlation in the models' standardized residuals and their squares for the inflation series. Then, I evaluate the bivariate UC model's forecasting performance.

6.1 Tests for Serial Correlation

Serial correlation in a model's residuals often signals model misspecification, and serial correlation in their squares implies that there could be remaining ARCH effects that are left unexplained by the model. Hence, I test for serial correlation in the baseline and bivariate UC models' standardized residuals and their squares for the inflation series.

The p-values associated with the Q-statistics under the null hypothesis of no serial correlation up to lag 8 are reported in Tables 7a and 7b for the baseline and bivariate UC models respectively. The results in the first columns of these tables show that under the span of the full sample, the models may be misspecified.

However, upon examining the filtered probabilities in Figure 4a and 4b, which denotes the probability of being in each of the three regimes at any given date t, I notice that the transition dynamics between the second to third regimes is relatively gradual. As shown, the transition from the first to second regimes occurred roughly during 1970:Q1-1972:Q4, whereas the transition period from the second to third regimes took place over 1985:Q1-1989:Q4. It is also true that these two structural changes are quite different in nature. In the early 1970s, the observed oil price shocks and economic recessions that occurred during that time made the shift from the low to high inflation variability regimes in some ways an anticipated one. On the contrary, the cause or source of the remarkable decline in the level and variability of inflation in the mid 1980s period was unobserved and, to date, remains largely unknown[8]. During the transition period after a regime change, a number of studies have shown that persistent learning dynamics in macroeconomic aggregates can occur. This is because in the presence of limited or imperfect information about the structure of the economy, households and firms tend to make persistent mistakes regarding the Central bank's policy objectives (see Erceg and Levin, 2003; Gaspar et. al, 2006; Milani 2007; Bullard and Singh, 2009). Therefore, it is highly plausible that persistent learning dynamics of agents that occurred during the transition period between regimes in the mid to late 1980s could have contaminated the serial correlation test results for the full sample.

For this reason, tests of serial correlation for both empirical models are carried out again, but this time I set aside the sample period that corresponds to the transition period between the second to the third regimes. Upon exclusion of the 1985:Q1-1989:Q4 residual series, the p-values associated with the Q-statistic as reported in columns 2 and 3 of Tables 7a and 7b suggest that there is no evidence of remaining serial correlation in both the models' standardized residuals and their squares.

The main conclusion that can be drawn from this analysis is that the forward-looking NKPC that allows for stochastic trend inflation and regime switching in the variances and covariance of the shocks is properly specified, and can provide a good description of postwar US inflation dynamics. Under the stationary restriction for the inflation series, the forward-looking model struggles to capture inflation persistence, but this is no longer the case here, as the time-varying trend component absorbs the persistence that is not meant to be captured by the short-run dynamics of price adjustments. Through the use of different empirical models that allows for some form of structural shift or time variation in the long-run trend component of inflation, Cogley and Sbordone (2008), and Kim and Kim (2009) report similar results.

[8] This period is known as the Great Moderation and is associated with a large decline in overall macroeconomic volatility. To this date, the origins of the Great Moderation remain unknown. Various hypotheses have been put forth, which include but are not limited to: improved monetary policy, changes in the structural features of the economy, and less severe shocks disturbing the economy (see Owyang et al., 2007, for a survey).

6.2 Forecasting Performance

The forecasting performance of a model is often viewed as a useful metric for evaluating its empirical relevance. Therefore, another diagnostic check that I use to evaluate whether the forward-looking NKPC with stochastic trend inflation can provide a good fit to the data is via an assessment of the bivariate UC model's ability to forecast inflation.

The practice of inflation forecasting has been studied extensively in the literature as inflation forecasts are used as a necessary guide for monetary policymaking. The conventional wisdom is that the Phillips curve is a useful tool for forecasting inflation. Recently however, a number of empirical studies have challenged this idea. Atkeson and Ohanian (2001), were the first to formally point out that since the mid 1980s, four-quarter-ahead out-of-sample Phillips curve-based forecasts have been less accurate than the following simple benchmark:

$$\pi^4_{t+4|t} = \pi^4_t = \frac{1}{4}\left(\pi_t + \ldots + \pi_{t-3}\right). \tag{17}$$

In the above expression $\pi^4_{t+4|t}$ is the four-quarter-ahead inflation forecast for the average rate of inflation π^4_t, made using data through time t. In other words, the Atkeson and Ohanian model (AO model, hereafter) implies that the forecast of inflation over the next four quarters will be the same as the value of four-quarter inflation today.

Based on more comprehensive analyses, Fisher et. al (2002), Stock and Watson (2003, 2007), Brave and Fisher (2004), and many others confirm the AO result[9]. Stock and Watson (2007) show that while the basic AO result still holds, a univariate UC model for inflation with a stochastic trend and stochastic volatility (UC-SV model, hereafter) is able to outperform the AO model at some forecasting horizons. In other words, these authors put forth a more general conclusion, that is, Phillips curve-based forecasts using various activity indicators (unemployment, output gaps, and output growth) have all failed to improve upon the univariate AO and UC-SV benchmarks since the mid 1980s. This finding in a way implies that the Phillips curve relation may no longer hold as a relevant model for inflation dynamics, as economic

[9] See Stock and Watson (2008) for a survey of this literature.

content through the Phillips curve relation adds no marginal predictive content to univariate specifications.

With the above results in mind, I investigate the forecasting performance of the bivariate UC model against two univariate benchmarks, to determine whether the forward-looking NKPC can provide a satisfactory description of the data. The first benchmark is the AO model. The second one is called the univariate UC model, which is the univariate counterpart of the bivariate UC model:

$$\begin{aligned} &\pi_t = \bar{\pi}_t + z_t \\ &\bar{\pi}_t = \bar{\pi}_{t-1} + e_t \\ &\begin{bmatrix} z_t \\ e_t \end{bmatrix} \sim i.i.d.N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2_{z,S_t} & 0 \\ 0 & \sigma^2_{e,S_t} \end{bmatrix} \right), \end{aligned} \tag{18}$$

where the dynamics of the structural break parameters $\theta_{S_t} = \left\{ \sigma^2_{z,S_t}, \sigma^2_{e,S_t} \right\}$ follow (8) and (9).

The reason why I include the above benchmark model into the forecasting exercise is because the univariate UC model should exhibit comparable forecasting abilities to Stock and Watson's UC-SV model. The only aspect in which these two models differ, is in the way that they handle heteroskedasticity. The UC-SV model allows for stochastic volatility, while the univariate UC model allows for two structural breaks in the variances and covariances of the shocks.

The three models are used to forecast three inflation measures: the chain-weighted GDP deflator, CPI inflation, and the chain-weighted PCE inflation. In the forecasting exercise, I compute both one-quarter-ahead in-sample inflation forecasts based on the full sample period 1952:Q1-2007:Q3, and four-quarter-ahead out-of-sample inflation forecasts for the sample periods 1993:Q1-2000:Q4 and 2001:Q1-2006:Q3. For out-of-sample forecasting, I use a recursive procedure. This means that for a forecast made at date t for date $t+4$, all estimation is made with data beginning in 1952:Q1 through date t. Then, the procedure is repeated again with the same starting date but an expanding data window.

The accuracy of the inflation forecasts are evaluated with the conventional root mean squared error (RMSE) statistic. The RMSE statistic calculated over the time period t_1 to t_2 is defined as:

$$RMSE_{t_1,t_2} = \sqrt{\frac{1}{T}\sum_{t=t_1}^{t_2}\left(\pi_{t+h}^{h} - \pi_{t+h|t}^{h}\right)^2} \quad (19)$$

with $T = t_2 - t_1 - 1$. Note that $h = 1$ for in-sample one-quarter-ahead inflation forecasts, and $h = 4$ for out-of-sample four-quarter-ahead inflation forecasts.

Table 8 reports the RMSEs associated with the inflation forecasts. Based on the results, the bivariate UC model outperforms both univariate benchmarks for the in-sample and two out-of sample forecasting periods. One exception is for the out-of-sample period 1993:Q1-1999:Q4, where the univariate UC model performs slightly better than the bivariate UC model when forecasting PCE inflation.

I pay special attention to the out-of-sample forecasts, since out-of-sample rather than in-sample fit is a more commonly used metric of the model's forecasting performance. To assess whether differences in the out-of-sample predictive accuracy between the bivariate UC model and the two univariate benchmarks are statistically significant, I rely on the modified Diebold Mariano test statistic. The original Diebold Mariano test statistic is a t-statistic associated with the null hypothesis that the mean squared errors of the two forecasts being compared is zero (see Diebold and Mariano, 1995). The modified version as derived in Harvey et. al (1997) attempts to correct for the poor size property of the original test statistic in small samples.

The p-values associated with the modified Diebold Mariano test statistics are reported in Table 9. Due to the small sample sizes of the out-of-sample periods, I also included the p-values associated with the forecasts computed for the combined out-of-sample period 1993:Q1-2006:Q3. Based on these results, the null hypothesis of equal predictive accuracy between the bivariate UC model and the two univariate benchmarks is rejected at the 10% significance level for all inflation measures during the out-of-sample period 2000:Q1-2006Q3. This result suggests that in this period, the bivariate UC model delivers superior inflation forecasts relative to those obtained from the two univariate benchmarks. However, the same conclusion cannot be reached for the out-of-sample period 1993Q1-1999Q4. Nevertheless, it does not rule out the case that for this period, the bivariate UC model performed at least as well as the two univariate benchmarks.

Overall, the bivariate UC model as implied by the forward-looking NKPC with a stochastic trend component produces good in-sample and out-of-sample predictions for inflation relative to the AO specification, and a univariate UC model that is similar to the UC-SV benchmark. As both the AO and UC-SV models are known to be leading forecasting models for inflation during the recent decade, the evidence presented here helps strengthen the case that the forward-looking NKPC can provide a good description of postwar US inflation dynamics.

7 Conclusion

It is relatively well accepted that the forward-looking NKPC provides a poor account of postwar US inflation dynamics. I provide an empirical model and estimation methodology to show that the forward-looking NKPC can be reconciled with the data once inflation is allowed to have a stochastic trend. The empirical results help advance existing knowledge about the underlying workings of inflation, and can also provide policymakers and academics with more confidence and alternative directions when using the forward-looking model for assessing practical policy questions and monetary policy analysis.

This paper argues that the poor performance of the forward-looking NKPC found in earlier studies is due to a neglect of a stochastic trend component. I show that once time variation in trend inflation is taken into account, the short-run properties of the model becomes better specified. This conclusion has potentially important implications for the widely used New Keynesian DSGE macroeconomic models, because estimation of these models rests on the assumption that the long-run properties of the economy are constant. To the extent that this assumption is violated, the models will face difficulty in capturing the long-run behavior of the economy. Furthermore, misspecification of these long-run properties would alter the short-run implications of the model as well, and also cause bias in any estimates of the parameters that are governing those dynamics.

The empirical findings in this paper may also have important implications for the conduct of monetary policy. I find a flat-sloped Phillips curve, which implies that the estimated impact of real economic activity onto actual inflation is rather small. At the same time, I find an important role for a stochastic trend component in explaining actual inflation. As trend inflation is highly persistent, movements in the long-run component of inflation should be taken very seriously. With trend inflation being an important determinant of inflation, even small variations in its underlying factors can translate into a persistent source of pressure onto actual inflation.

The econometric methodology presented in this paper also contributes to recent developments in structural time-series modeling for use in output gap measurement and inflation forecasting. Although the research work presented here is still preliminary, its initial results are encouraging. The output gap obtained as the byproduct of estimating the bivariate UC model has relatively narrow confidence bands, and also contains movements that are similar to the CBO output gap. In addition, the bivariate UC model also provides some gains in inflation forecasting relative to good univariate benchmark models that are used in the literature. Since the bivariate UC model is grounded in theory, parsimonious, relatively simple to implement in real-time, and also attaches a measure of uncertainty with its forecasts, the econometric approach described here can be seen as an attractive alternative to forecasting with, for instance, the tightly parameterized DSGE class of models.

There are however, a few ways in which the model can be improved for the use in real-time output gap measurement and inflation forecasting. The success of the bivariate UC model largely stems from the inclusion of random walk trend components and structural break parameters, which allows the model to quickly adjust to changes in economic conditions and monetary policy regimes. However, the number of structural breaks to be included in the model is determined ex post, and therefore the model cannot adequately deal with problems that arise when forecasting takes place on the onset or in the midst of an unknown structural break. The issue of forecasting in the presence of regime changes is an important research topic that is still in its initial stages of being explored. Nevertheless, one simple modification that could be made to the model that would allow it to perform better with regime changes is to replace structural break parameters with Markov switching ones. Then, for instance, the three regimes could be attributed to low, medium, and high inflation volatility regimes, and the model would produce predictions for the output gap and inflation conditional on the probabilities of being in any one of those states. To obtain significant improvements in the forecasting ability of the model however, it would most likely require more economic content to be endowed within the trend component of inflation.

Therefore, recent research efforts that aim to gain a better understanding about the underlying determinants of trend inflation should prove to be a promising start in this direction.

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A Estimation of the Baseline UC Model

The joint log-likelihood function to be maximized is:

$$
\begin{aligned}
&\ln L\left(\lambda, \theta_1, \theta_2, \theta_3, p_{11}, p_{22}, p_{33}\right) \\
&= \sum_{t=1}^{T} \ln\left(f\left(Y_t, X_t \mid I_{t-1}\right)\right) \qquad (20) \\
&= \sum_{t=1}^{T} \ln\left(\sum_{j=1}^{3}\sum_{i=1}^{3} f\left(Y_t, X_t \mid I_{t-1}, S_t = j, S_t = i\right) \Pr[S_t = j, S_{t-1} = i \mid I_{t-1}] \right) \\
&= \sum_{t=1}^{T} \ln\left(\sum_{j=1}^{3}\sum_{i=1}^{3} f(Y_t \mid X_t, I_{t-1}, S_t = j, S_t = i) f(X_t \mid I_{t-1}, S_t = j, S_t = i) \Pr[S_t = j, S_{t-1} = i \mid I_{t-1}] \right)
\end{aligned}
$$

where $\lambda = [k \quad \phi_1 \quad \phi_2 \quad \sigma_v^2]$ and $\theta_m = [\gamma_m \quad \sigma_{z,m}^2 \quad \sigma_{\varepsilon,m}^2]$, $m = 1, 2, 3$.

The individual densities are given by:

$$
f\left(Y_t \mid X_t, I_{t-1}, S_t = j, S_{t-1} = i\right) = \left(2\pi f_{t|t-1}^{(i,j)}\right)^{-1/2} \times \exp\left(-\frac{1}{2} \frac{\left(Y_t - Y_{t|t-1}^{(i,j)}\right)^2}{f_{t|t-1}^{(i,j)}} \right) \qquad (21)
$$

$$
f\left(X_t \mid I_{t-1}, S_t = j, S_{t-1} = i\right) = \left(2\pi\sigma_v^2\right)^{-1/2} \times \exp\left(-\frac{1}{2} \frac{v_t^2}{\sigma_v^2} \right)
$$

where $Y_{t|t-1}^{(i,j)}$ is the prediction of π_t conditional on past information I_{t-1}, $S_t = j$, and $S_{t-1} = i$, and can be obtained from the Kim filter algorithm. $f_{t|t-1}^{(i,j)} = E\left[\left(Y_t - Y_{t|t-1}^{(i,j)}\right)^2\right]$ is the conditional variance of the forecast error associated with π_t conditional on I_{t-1}, $S_t = j$, and $S_{t-1} = i$.

The joint probability that $S_t = j$ and $S_{t-1} = i$ conditional on information I_{t-1} is:

$$
\Pr\left(S_t = j, S_{t-1} = i \mid I_{t-1}\right) = \Pr\left(S_t = j \mid S_{t-1} = i\right) \Pr\left(S_{t-1} = i \mid I_{t-1}\right) \qquad (22)
$$

where $\Pr(S_t = j \mid S_{t-1} = i)$ is the transition probability that enters into the state transition probability matrix in (9). $\Pr(S_{t-1} = i \mid I_{t-1})$ is the probability that $S_{t-1} = i$ conditional on past information I_{t-1} and is obtained from the Kim filter.

B Estimation of the Bivariate UC Model

The state-space model as described in (15a) and (15b) can be estimated by ML with the Kim filter algorithm. The Kim filter algorithm is a recursive procedure that evaluates the following equations:

$$
\begin{aligned}
B_{t|t-1}^{(i,j)} &= \Lambda_t + F B_{t-1|t-1}^{(i)} \\
P_{t|t-1}^{(i,j)} &= F P_{t-1|t-1}^{(i)} F' + G Q^{(j)} G' \\
Y_{t|t-1}^{(i,j)} &= H B_{t|t-1}^{(i,j)} + K e_1' \left(I_2 - A \right)^{-1} C B_{t|t-1}^{(i,j)} \\
f_{t|t-1}^{(i,j)} &= H P_{t|t-1}^{(i,j)} H' + R^{(j)} \\
B_{t|t}^{(i,j)} &= B_{t|t-1}^{(i,j)} + P_{t|t-1}^{(i,j)} H \left[f_{t|t-1}^{(i,j)} \right]^{-1} \left(Y_t - Y_{t|t-1}^{(i,j)} \right) \\
P_{t|t}^{(i,j)} &= \left(I_4 - P_{t|t-1}^{(i,j)} H' \left[f_{t|t-1}^{(i,j)} \right]^{-1} H \right) P_{t|t-1}^{(i,j)}
\end{aligned}
\qquad (23)
$$

where $B_{t|t-1}^{(i,j)} = E\left[B_t^{(i,j)} \mid I_{t-1} \right]$ is the expectation of B_t conditional on I_{t-1}, $S_t = j$ and $S_{t-1} = i$; $B_{t-1|t-1}^{(i)}$ is the expectation of B_{t-1} conditional on I_{t-1} and $S_{t-1} = i$; $P_{t|t-1}^{(i,j)} = E\left[\left(B_t - B_{t|t-1} \right) \left(B_t - B_{t|t-1} \right)' \right]$ is the mean squared error matrix of $B_{t|t-1}^{(i,j)}$ conditional on I_{t-1}, $S_t = j$ and $S_{t-1} = i$; $Y_{t|t-1}^{(i,j)} = E\left[Y_t^{(i,j)} \mid I_{t-1} \right]$ is the expectation of Y_t conditional on I_{t-1}, $S_t = j$ and $S_{t-1} = i$; $Y_t - Y_{t|t-1}^{(i,j)}$ is the conditional forecast error of Y_t based on I_{t-1}, $S_t = j$ and $S_{t-1} = i$; $f_{t|t-1}^{(i,j)} = E\left[\left(Y_t - Y_{t|t-1}^{(i,j)} \right) \left(Y_t - Y_{t|t-1}^{(i,j)} \right)' \right]$ is the conditional variance of the forecast error associated with Y_t , $B_{t|t}^{(i,j)} = E\left[B_t^{(i,j)} \mid I_t \right]$ is the expectation of B_t based on information available at time t, given $S_t = j$ and $S_{t-1} = i$; and $P_{t|t}^{(i,j)} = E\left[\left(B_t - B_{t|t}^{(i,j)}\right)\left(B_t - B_{t|t}^{(i,j)}\right)'\right]$ is the mean squared error of the matrix B_t conditional on I_t, $S_t = j$ and $S_{t-1} = i$. The term $e_1'(I_2 - A)^{-1} C B_{t|t-1}^{(i,j)}$ in the third equation is as defined in (16).

As explained in Kim (1994), the $M \times M$ posteriors of $B_{t|t}^{(i,j)}$ and $P_{t|t}^{(i,j)}$ are collapsed to a single posterior point $B_{t|t}^{j}$ and $P_{t|t}^{j}$ respectively, for tractability purposes:

$$B_{t|t}^{j} = \frac{\sum_{i=1}^{M} \Pr\left[S_t = j, S_{t-1} = i \mid I_t\right] B_{t|t}^{(i,j)}}{\Pr\left[S_t = j \mid I_t\right]}$$
$$P_{t|t}^{j} = \frac{\sum_{i=1}^{M} \Pr\left[S_t = j, S_{t-1} = i \mid I_t\right] \left\{ P_{t|t}^{(i,j)} + \left(B_{t|t}^{j} - B_{t|t}^{(i,j)}\right)\left(B_{t|t}^{j} - B_{t|t}^{(i,j)}\right)' \right\}}{\Pr\left[S_t = j \mid I_t\right]} \tag{24}$$

where

$$\begin{aligned}
&\Pr(S_t = j, S_{t-1} = i \mid I_t) \\
&\quad = \Pr\left[S_t = j, S_{t-1} = i \mid I_t\right] \\
&\quad = \Pr\left[S_t = j, S_{t-1} = i \mid I_{t-1}, Y_t\right] \\
&\quad = \frac{f\left(S_t = j, S_{t-1} = i, Y_t \mid I_{t-1}\right)}{f\left(Y_t \mid I_{t-1}\right)} \\
&\quad = \frac{f\left(Y_t \mid S_t = j, S_{t-1} = i, I_{t-1}\right) \Pr\left[S_t = j, S_{t-1} = i \mid I_{t-1}\right]}{f\left(Y_t \mid I_{t-1}\right)}
\end{aligned} \tag{25}$$

and $\Pr[S_t = j \mid I_t] = \sum_{i=1}^{3} \Pr[S_t = j, S_{t-1} = i \mid I_t]$ with $i, j = 1,2,3$. $\Pr(S_t = j, S_{t-1} = i \mid I_{t-1})$ is as defined in (22). The density function conditional on I_{t-1}, $f\left(Y_t \mid I_{t-1}\right)$, is given by:

$$f\left(Y_t \mid I_{t-1}\right) = \sum_{j=1}^{3} \sum_{i=1}^{3} f\left(Y_t \mid S_t = j, S_{t-1} = i, I_{t-1}\right) \Pr\left[S_t = j, S_{t-1} = i \mid I_{t-1}\right]. \tag{26}$$

The conditional density $f(Y_t \mid S_t = j, S_{t-1} = i, I_{t-1})$ is obtained from the prediction error decomposition:

$$f\left(Y_t \mid S_t = j, S_{t-1} = i, I_{t-1}\right) = \left(2\pi\right)^{-1} \mid f_{t|t-1}^{(i,j)} \mid^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\left(Y_t - Y_{t|t-1}^{(i,j)}\right)' f_{t|t-1}^{(i,j)-1}\left(Y_t - Y_{t|t-1}^{(i,j)}\right)\right\}. \qquad (27)$$

The bivariate UC model is then estimated by maximizing the following log-likelihood function:

$$\begin{aligned} &\ln L(\lambda, \theta_1, \ldots, \theta_M, p_{11}, \ldots, p_{MM}) \\ &= \sum_{t=1}^{T} \ln(f(Y_t \mid I_{t-1})) \end{aligned} \qquad (28)$$

where $\lambda = [k \quad \phi_1 \quad \phi_2 \quad \delta_0 \quad \delta_1 \quad \sigma_v^2 \quad \sigma_{vw}]$ and $\theta_m = \left[\sigma_{z,m}^2 \quad \sigma_{e,m}^2 \quad \sigma_{ev,m}\right]$, $m = 1, 2, 3$.

Table 1. Parameter estimates for the baseline UC model without structural breaks

Parameters	Estimates (s.e.)
The Phillips curve slope and AR coefficients for the CBO output gap	
k	0.012 (0.009)
ϕ_1	1.223 (0.065)
ϕ_2	-0.312 (0.065)
The standard deviations and the correlations of shocks	
σ_v	0.830 (0.040)
σ_z	0.678 (0.065)
σ_e	0.622 (0.078)
γ	0.026 (0.074)
Log-likelihood	-576.454

Note: Standard errors are in parentheses.

Table 2. Parameter estimates for the baseline UC model with two structural breaks

Parameters	Estimates (s.e.)		
Transition probabilities			
p_{11}	0.985 (0.015) → Break date: 1968:Q4		
p_{22}	0.984 (0.016) → Break date: 1984:Q3		
The Phillips curve slope and AR coefficients for the CBO output gap			
k	0.021 (0.009)		
ϕ_1	1.214 (0.065)		
ϕ_2	-0.289 (0.065)		
The standard deviations and the correlations of shocks			
σ_v	0.831 (0.040)		
	Regime 1	*Regime 2*	*Regime 3*
σ_z	0.531 (0.092)	0.826 (0.221)	0.598 (0.060)
σ_e	0.441 (0.103)	1.089 (0.247)	0.256 (0.064)
γ	-0.077 (0.102)	0.045 (0.143)	-0.056 (0.133)
Log-likelihood	-555.166		

Note: Standard errors are in parentheses.

Table 3. Parameter estimates for the bivariate UC model without structural breaks

Parameters	Estimates (s.e.)
The Phillips curve slope, output trend drifts, and AR coefficients for the unobserved gap	
k	0.016 (0.012)
δ_0	1.037 (0.046)
δ_1	0.710 (0.020)
ϕ_1	1.218 (0.077)
ϕ_2	-0.307 (0.074)
The standard deviations and covariances of shocks	
σ_w	0.819 (0.215)
σ_v	1.034 (0.141)
σ_{vw}	-0.564 (0.304)
σ_z	0.733 (0.072)
σ_e	0.551 (0.093)
σ_{ew}	-0.452 (0.138)
σ_{ev}	0.387 (0.148)
Log-likelihood	-375.103

Note: Standard errors are in parentheses.

Table 4. Parameter estimates for the bivariate UC model with two structural breaks (GDP inflation)

Parameters	Estimates (s.e.)		
Transition probabilities			
p_{11}	0.985 (0.015) → Break date: 1968:Q2		
p_{22}	0.985 (0.015) → Break date: 1984:Q3		
The Phillips curve slope, output trend drifts, and AR coefficients for the unobserved gap			
k	0.026 (0.013)		
δ_0	1.015 (0.051)		
δ_1	0.722 (0.027)		
ϕ_1	1.238 (0.101)		
ϕ_2	-0.327 (0.094)		
The standard deviations and covariances of shocks			
σ_w	0.620 (0.208)		
σ_v	0.890 (0.151)		
σ_{vw}	-0.271 (0.229)		
	Regime 1	*Regime 2*	*Regime 3*
σ_z	0.605 (0.084)	0.971 (0.171)	0.649 (0.059)
σ_e	0.352 (0.119)	0.856 (0.201)	0.164 (0.070)
σ_{ew}	-0.218 (0.097)	-0.524 (0.187)	-0.101 (0.054)
σ_{ev}	0.161 (0.093)	0.472 (0.205)	0.085 (0.067)
Log-likelihood	-356.241		

Note: Standard errors are in parentheses.

Table 5. Parameter estimates for the bivariate UC model with two structural breaks (CPI inflation)

Parameters	Estimates (s.e.)		
Transition probabilities			
p_{11}	0.987 (0.013) → Break date: 1971:Q2		
p_{22}	0.984 (0.016) → Break date: 1986:Q3		
The Phillips curve slope, output trend drifts, and AR coefficients for the unobserved gap			
k	0.040 (0.019)		
δ_0	0.979 (0.052)		
δ_1	0.732 (0.037)		
ϕ_1	1.151 (0.095)		
ϕ_2	-0.255 (0.082)		
The standard deviations and covariances of shocks			
σ_w	0.567 (0.236)		
σ_v	1.051 (0.259)		
σ_{vw}	-0.393 (0.373)		
	Regime 1	*Regime 2*	*Regime 3*
σ_z	0.771 (0.122)	1.364 (0.236)	1.193 (0.102)
σ_e	0.530 (0.182)	1.192 (0.286)	0.171 (0.104)
σ_{ew}	-0.265 (0.108)	-0.654 (0.281)	-0.069 (0.099)
σ_{ev}	0.125 (0.138)	1.037 (0.339)	0.179 (0.120)
Log-likelihood	-439.824		

Note: Standard errors are in parentheses.

Table 6. Parameter estimates for the bivariate UC model with two structural breaks (PCE inflation)

Parameters	Estimates (s.e.)		
Transition probabilities			
p_{11}	0.987 (0.013) → Break date: 1971Q2		
p_{22}	0.975 (0.025) → Break date: 1981Q2		
The Phillips curve slope, output trend drifts, and AR coefficients for the unobserved gap			
k	0.023 (0.012)		
δ_0	1.023 (0.050)		
δ_1	0.703 (0.029)		
ϕ_1	1.134 (0.065)		
ϕ_2	-0.246 (0.064)		
The standard deviations and covariances of the shocks			
σ_w	0.535 (0.208)		
σ_v	1.063 (0.150)		
σ_{vw}	-0.384 (0.269)		
	Regime 1	*Regime 2*	*Regime 3*
σ_z	0.815 (0.078)	0.673 (0.393)	0.894 (0.072)
σ_e	0.272 (0.050)	1.327 (0.387)	0.270 (0.073)
σ_{ew}	-0.145 (0.068)	-0.709 (0.338)	-0.143 (0.069)
σ_{ev}	0.188 (0.075)	0.925 (0.422)	0.220 (0.094)
Log-likelihood	-390.561		

Note: Standard errors are in parentheses.

Table 7a. Tests for serial correlation in the standardized residuals and their squares for the inflation series obtained from estimation of the baseline UC model

		Sample period	
Lag	*1954 Q3-2007Q3*	*1954 Q3-1984Q4*	*1990 Q1-2007Q3*
Standardized Residuals			
1	0.852	0.994	0.916
2	0.331	0.702	0.532
3	0.529	0.834	0.723
4	0.008	0.299	0.087
5	0.012	0.392	0.150
6	0.020	0.492	0.223
7	0.032	0.575	0.229
8	0.029	0.680	0.123
Squares of Standardized Residuals			
1	0.350	0.993	0.123
2	0.218	0.278	0.277
3	0.361	0.381	0.425
4	0.333	0.546	0.146
5	0.434	0.676	0.143
6	0.167	0.487	0.207
7	0.178	0.548	0.295
8	0.214	0.532	0.252

Note: Reported are p-values associated with the Q-statistic under the null of no serial correlation.

Table 7b. Tests for serial correlation in the standardized residuals and their squares for the inflation series obtained from estimation of the bivariate UC model

		Sample period	
Lag	*1954 Q3-2007Q3*	*1954 Q3-1984Q4*	*1990 Q1-2007Q3*
Standardized Residuals			
1	0.746	0.999	0.719
2	0.477	0.908	0.441
3	0.684	0.951	0.644
4	0.012	0.337	0.101
5	0.017	0.446	0.170
6	0.031	0.564	0.252
7	0.045	0.636	0.239
8	0.041	0.733	0.109
Squares of Standardized Residuals			
1	0.305	0.854	0.209
2	0.380	0.598	0.398
3	0.548	0.664	0.551
4	0.586	0.805	0.099
5	0.693	0.892	0.096
6	0.132	0.437	0.148
7	0.164	0.527	0.208
8	0.216	0.548	0.233

Note: Reported are p-values associated with the Q-statistic under the null of no serial correlation.

Table 8. RMSEs for in-sample and out-of-sample inflation forecasts

Inflation measure	Forecasting Models		
	AO model	*Univariate UC model*	*Bivariate UC model*
One-step ahead in-sample forecasts, 1952Q1-2007Q3(223 observations)			
GDP	1.173	1.047	1.049
CPI	1.752	1.585	1.541
PCE	1.353	1.191	1.157
Four-step-ahead out-of-sample forecasts, 1993Q1-1999Q4 (28 observations)			
GDP	0.405	0.403	0.375
CPI	0.732	0.701	0.682
PCE	0.679	0.638	0.673
Four-step-ahead out-of-sample forecasts, 2000Q1-2006Q3 (27 observations)			
GDP	0.561	0.581	0.501
CPI	1.086	1.051	0.921
PCE	0.700	0.701	0.640

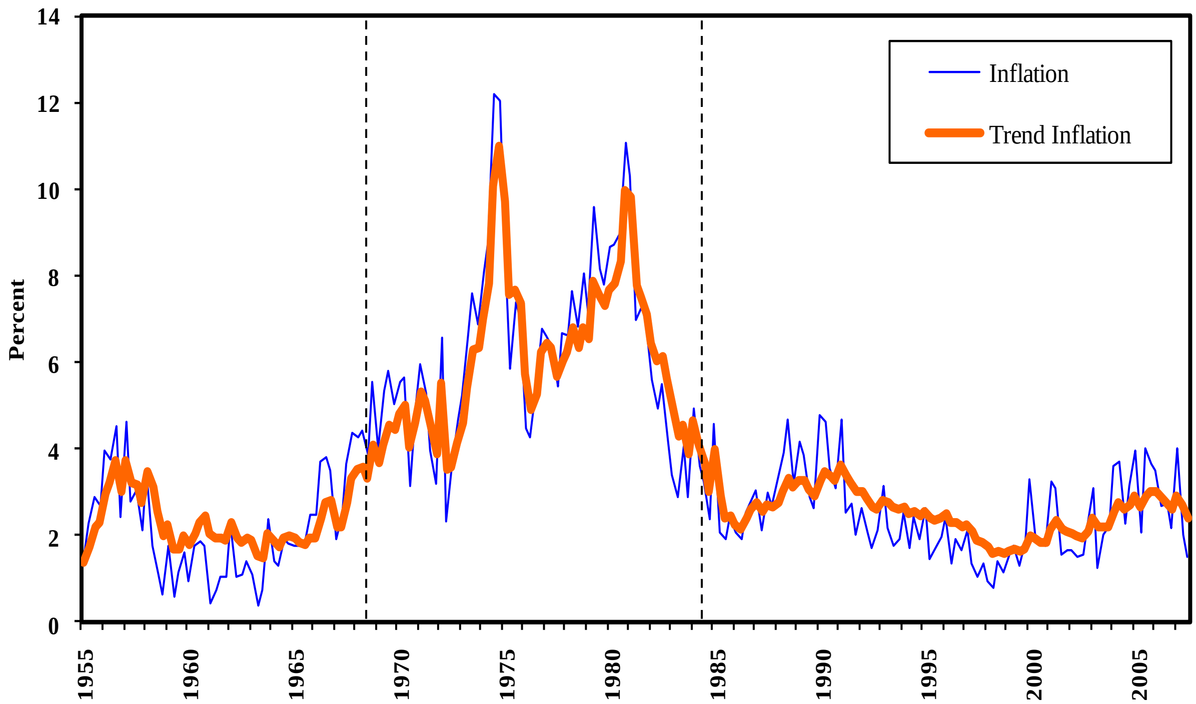
Note: Reported are the root mean squared errors from inflation forecasts.

Table 9. Out-of-sample forecast performance evaluation

Inflation Measure	Competing Models	
	Bivariate UC vs. AO	*Bivariate UC vs. Univariate UC*
Four-step-ahead out-of-sample forecasts, 1993Q1-1999Q4 (28 observations)		
GDP	0.257	0.236
CPI	0.256	0.378
PCE	0.470	0.311
Four-step-ahead out-of-sample forecasts, 2000Q1-2006Q3 (27 observations)		
GDP	0.082	0.012
CPI	0.004	0.002
PCE	0.014	0.027
Four-step-ahead out-of-sample forecasts, 1993Q1-2006Q3 (55 observations)		
GDP	0.080	0.028
CPI	0.016	0.024
PCE	0.230	0.386

Note: Reported are the p-values associated with the modified Diebold Mariano test statistic under the null of equal predictive accuracy.

Figure 1a. Actual Inflation and Trend Inflation



Note: The vertical dotted lines denote the two estimated structural break dates in 1968:Q2 and 1984:Q3.

Figure 1b. Trend Inflation Estimates and Its 90% Confidence Interval

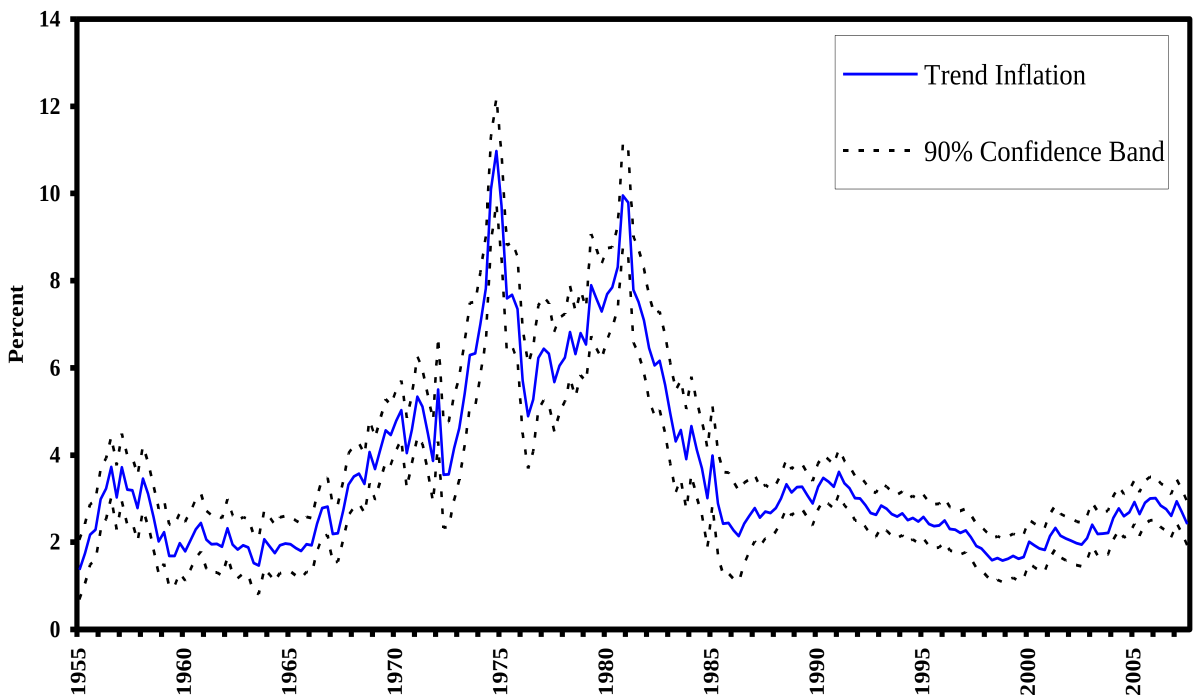


Figure 2. Inflation Cycle and Inflation Gap

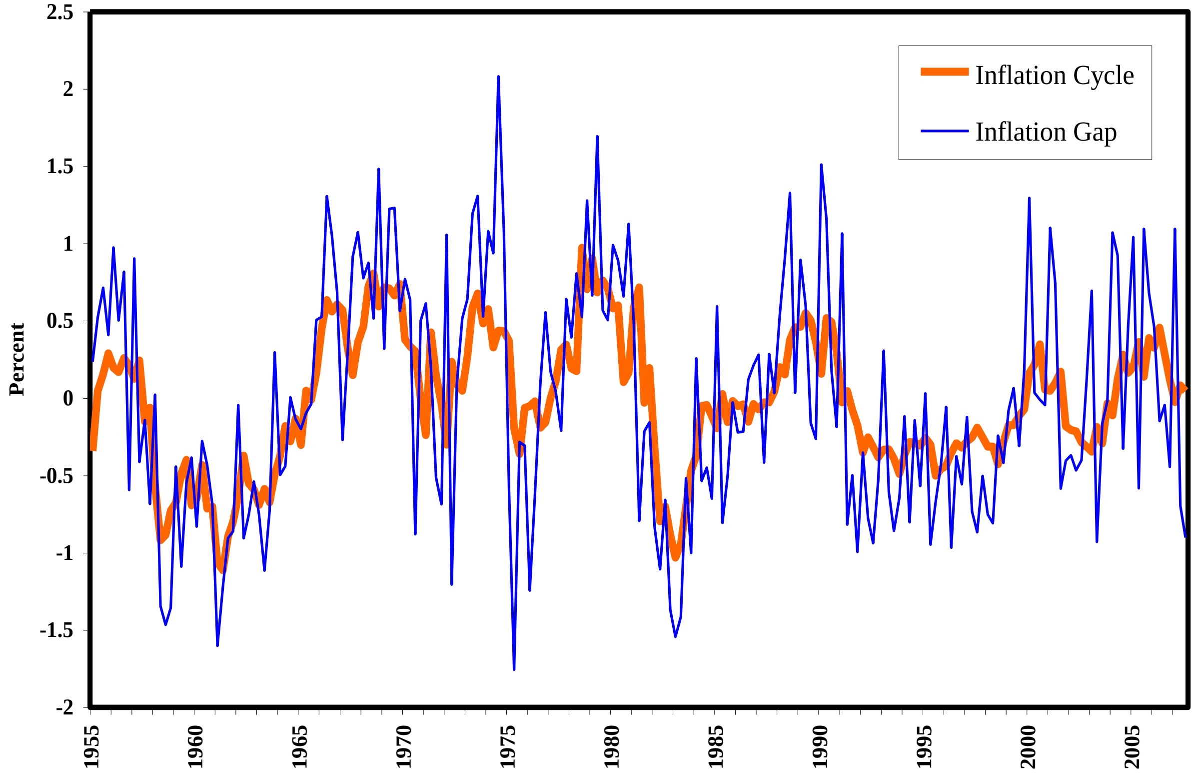
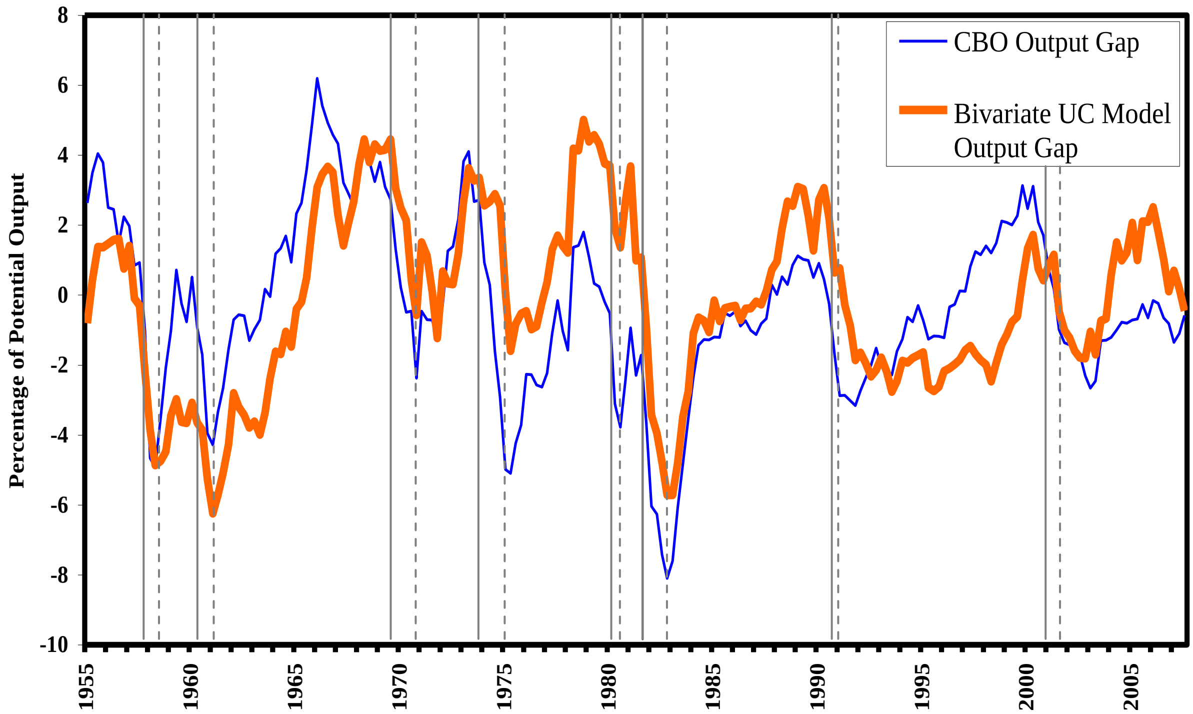


Figure 3a. CBO and Bivariate UC Model Output Gaps



Note: The vertical solid and dotted lines represents NBER business cycle peaks and troughs respectively.

Figure 3b. The CBO Output Gap and the 95% Confidence Band of the Bivariate UC Model's Output Gap

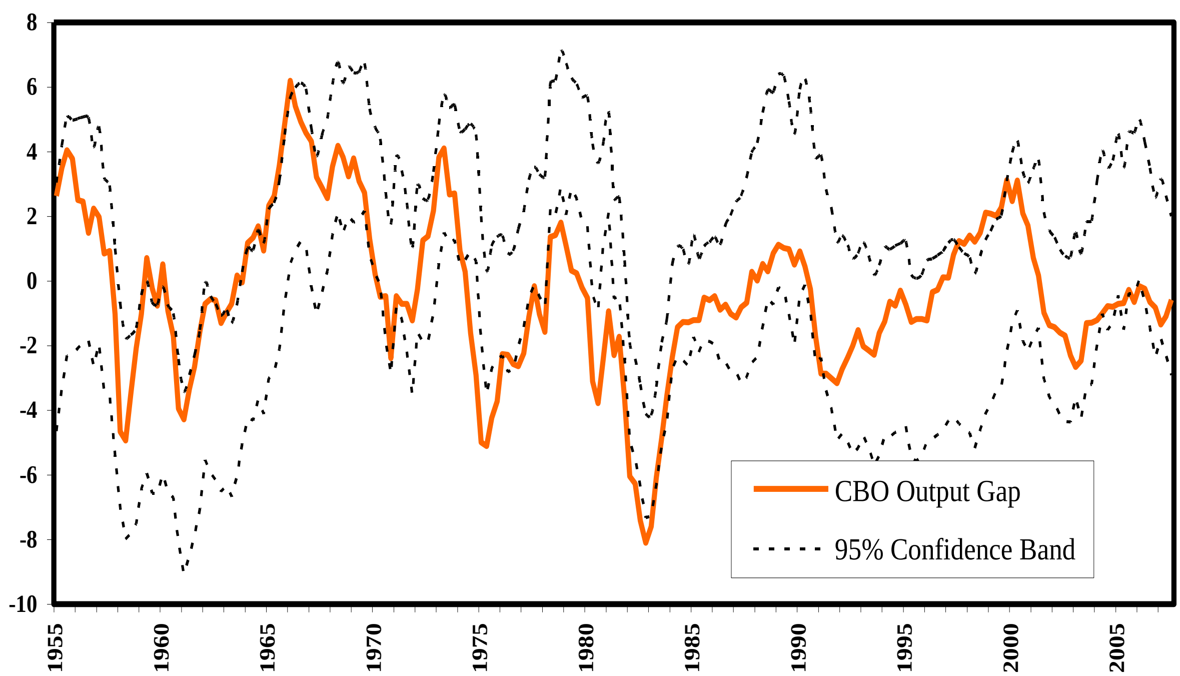
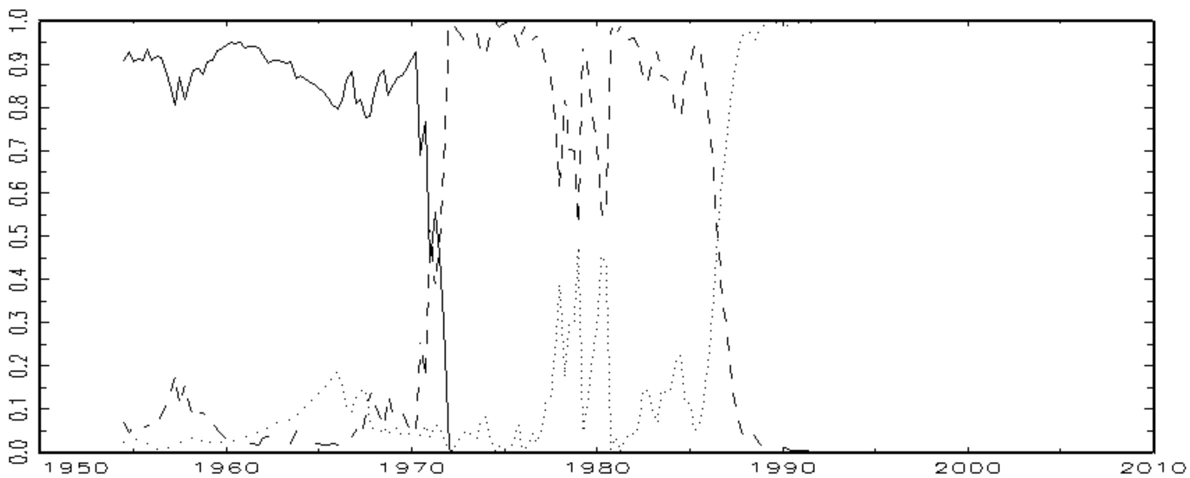
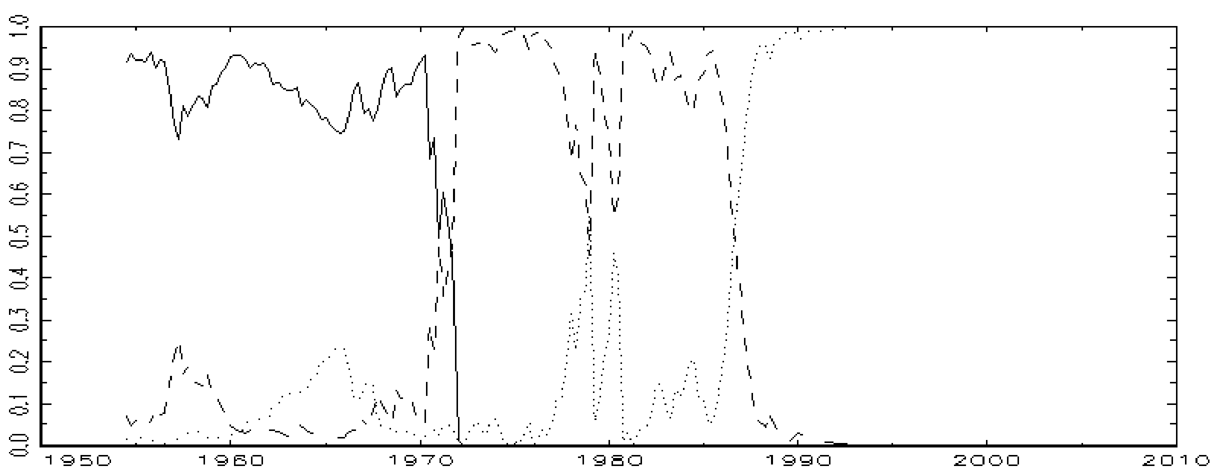


Figure 4a. Filtered probabilities for the baseline UC model



Note: The solid line represents the filtered probability of being in the first regime, the dashed line represents the filtered probability of being in the second regime, and the dotted line represents the filtered probability of being in the third regime. The filtered probabilities are obtained from estimation of the baseline model.

Figure 4b. Filtered probabilities for the bivariate UC model



Note: The solid line represents the filtered probability of being in the first regime, the dashed line represents the filtered probability of being in the second regime, and the dotted line represents the filtered probability of being in the third regime. The filtered probabilities are obtained from estimation of the bivariate UC model.