

Voluntary Quality Disclosure under Price-Signaling Competition

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ABSTRACT

We analyze an oligopolistic competition with differentiated products and qualities. The quality of a product is not known to consumers. Each firm can make an imperfect disclosure of its product quality before engaging in price-signaling competition. There are two regimes for separating equilibrium in our model depending on the parameters. Our analysis reveals that, in one of the separating regimes, price signaling leads to intense price competition between the firms under which not only the high-quality firm, but also the low-quality firm chooses to disclose its product quality to soften the price competition.

JEL Classification: D82, L13, L15, M3

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1 Introduction

Since Akerlof (1970), asymmetric information has long been considered an important factor that causes inefficiencies in markets. It is well understood by now that when consumers cannot observe product qualities, firms can use price as a signaling device for their product quality – the low-quality firms have an incentive to pretend that their products are high-quality, and hence the high-quality firms need to distort their prices to deter imitating behavior by the low-quality firms. Alternatively, to reduce such distortions associated with asymmetric information, firms may choose to subject their products or services to some process that credibly discloses their qualities, if such a disclosure is feasible.

A credible information disclosure would involve using an independent quality evaluation process with public announcement of found quality, or advertisement with potential penalty for misrepresentation in the presence of truth-in-advertising. For instance, some agencies, such as AHAM (Association of Home Appliance Manufacturers) and IAACM (International Association of Air Cleaner Manufacturers), have a “voluntary certification program” that air purifying manufacturers can choose to participate at a cost. These agencies evaluate the participants’ products and make the quality information public. Another example is the disclosure through special interest magazines – a manufacturer may submit its product to such magazine’s review process, so that the magazine’s assessment of the product can be published in an article, and consequently, made available to readers.¹

An interesting observation is that manufacturers with products that are seemingly inferior in quality sometimes chooses to disclose their product qualities. Moreover, such products sometimes are priced higher than other competing products whose disclosed qualities are higher. For example, according to the AHAM ratings, model 30378 produced by Hunter (\$180) outperformed model 201 produced by Blueair (\$299) in all quality categories.² Also, in a recent LCD evaluation by Maximum PC (September 2006), Dell’s 2407WFP (\$850) received a rate of 9 (where 10 is the best), while HP’s LP2465 (\$1000) received a rate of 4. Why would a firm with a relatively inferior product with a higher price disclose information on its product quality? This seemingly puzzling phenomena merits a closer examination.

The objective of this paper is to provide a rational for firms’ choices of quality disclosure

¹For example, for magazines such as Maximum PC or PC World, the manufacturers must submit their products if they want to have their products rated.

²See the website, <http://cadr.org>. Price information is from the internet retailer Amazon. Other websites also provide similar price ranges for these products.

that accommodates the phenomena described above. We look at an oligopolistic industry in which each firm chooses price as well as whether to withhold or advance information about the quality of their products through a disclosure channel. We develop a two-stage game in which two firms compete with products that are differentiated both horizontally (design or brand preferences) and vertically (quality). To illustrate our point in a parsimonious setting, we assume that one firm produces a high-quality product while the product of the other firm is low-quality. In the first stage, the firms choose whether or not to disclose their product qualities simultaneously. A firm’s quality disclosure allows a fraction of consumers to learn about its product quality, while the remaining fraction of the consumers stay uninformed at the end of the first stage. At the second stage, the firms engage in price competition, and may use price to signal their product qualities to the uninformed consumers. To be sure, we want to make it clear that quality disclosure in the first stage, which is our issue of interest, is not a signaling device. On the contrary, it is an instrument that a firm may use to reduce the need of costly distortions to signal in the second stage (when products are finally offered for sale in the market).

We examine two separating equilibrium regimes in our model: High-price signaling (commanding exclusivity) and low-price signaling (commanding large market share).³ First, we find that the high-quality firm has an incentive to disclose information so as to reduce the price distortion. Second and more interestingly, we find that the low-quality firm may also have an incentive to disclose the quality of its product. More specifically, quality disclosure by both high-quality and low-quality firms occurs in a regime where the high-quality firm distorts its price “downward.” In this regime, without quality disclosure the price competition (due to signaling effort) becomes very intense. Thus, by increasing the proportion of the informed consumers in the first stage, the low-quality firm can soften price competition in the second stage. That is, by giving up some uninformed consumers in the first stage (by making them informed), the low-quality firm can reduce the “battleground” for the remaining uninformed consumers in the second stage. This, together with strategic complementarity of prices, makes the low-quality firm better off by unveiling its quality information to the market.

Our result suggests that a firm with relatively lower quality product may have an incentive to voluntarily disclose its quality information, and that such a disclosure takes place with a high price of the product (higher than the price of competing high-quality products).

³See for example, Caminal and Vives (1996) and Hellofs and Jacobson (1999) among others for theoretical analyses and empirical evidences for both cases.

According to our analysis, this can be the case when a high-quality firm can produce at a lower cost. In high-tech industries, for example, producing a higher quality product at a lower cost is not very uncommon. As Gibson, Goldenson and Kost (2006) report, quality improvement and cost reduction are often paired with each other in such industries. In fact, according to their report using Capability Maturity Model Integration standard, companies' quality improvement by 48% are accompanied by cost reduction by 34%.

Literature Review

There is a long literature studying firms' strategies to signal private information to competitors or consumers. A large body of this literature, however, copes with price as a signaling device for a monopolist. Some papers, such as Milgrom and Roberts (1982), Bagwell and Ramey (1988), and Shieh (1993) consider situations in which firms can produce higher quality products at a *lower cost*. They show that a firm with an advantage may distort the price *downward* to signal its cost advantage or higher quality of the product. Other papers, such as Wolinsky (1983), Tellis and Wernerfelt (1987), and Bagwell and Riordan (1991) studied situations in which a higher quality is associated with a *higher cost*. In these papers, a firm may signal the high quality of its product by distorting the price *upward*. Our study encompasses both the cases of high and low costs of producing high-quality products.

There are papers that examine different way to convey signals. Wernerfelt (1994) introduces brand externality as a signaling device. Balachander and Srinivasan (1994) demonstrate that choice of product line also conveys information about a firm's competitive advantage. Lutz (1989) study the usage of warranties to send positive signals to the market. Hellofs and Jacobson (1999) provide empirical evidence that both exclusivity low and high market share can convey quality information. In this regard, we point out that low (high) price signaling strategies in our model are associated with high (low) market share of a signaling firm.

Recently, researchers started paying more attention to signaling in oligopolistic industries. Hertzendorf and Overgaard (2001), and Fluet and Garella (2002) consider price-advertising signals in duopoly, and identify conditions for separating equilibria, and for successful misrepresentation by a low-quality firm. Yehezkel (2008) extends Hertzendorf and Overgaard (2001) by introducing a fraction of informed consumers. This paper is different from ours in two ways. First, in his study, the fraction of informed consumers are exogenous, while in our paper, the fraction of informed consumers are determined endogenously by the firms' disclosure strategies. Second, as in Hertzendorf and Overgaard

(2001), the products are only differentiated vertically in his paper. In our paper, the firms' products are differentiated both vertically and horizontally, raising situations in which the low-quality firm discloses its quality information if the disclosure cost is not too large.

Lastly, there are studies related to credible disclosure vs. signaling. In this branch of research, the paper by Moraga-Gonzales (2000) is more connected to ours. The author considers a model in which firms could endogenously inform a fraction of the consumers through some form of informative advertising. This feature is similar to credible information disclosure in our paper. However, the paper does not consider the effect of information revelation on equilibrium prices. Therefore, in his model, information disclosure only occurs in a pooling equilibrium, and only by the high-quality firm. We contribute to the literature by identifying that, when there is competition, information disclosure occurs also in a signaling equilibrium, and that both the high-quality and the low-quality firms may have incentives to do so. Fishman and Hagerty (2003) and Daughety and Reinganum (2008) also study both disclosure and signaling in a unified model. In Fishman and Hagerty (2003), disclosure and signaling are complements, whereas in our paper, they are substitutes. In Daughety and Reinganum (2008), unlike in ours, if disclosure takes place there is no need for signaling since disclosure informs all consumers.

The rest of the paper is organized as follows. We present the model in the next section. The game in the second stage (price-signaling competition between firms) is analyzed in Section 3. In Section 4, we analyze the game in the first stage (choice of information disclosure) using the results from Section 3. Some extensions to our model are discussed in Section 5. We gather concluding remarks in Section 6.

2 Model setup

We consider a two-stage game with two single-product firms indexed by $i \in I = \{1, 2\}$. For notational purposes we employ $j \in I$ ($j \neq i$) to designate “the other firm.”

The products are differentiated both horizontally and vertically. In the first stage the firms choose whether to disclose their vertical quality, and in the second stage they choose prices. We focus on competition between firms with different qualities and assume that firm 1 produces “high-quality” products and firm 2 produces “low-quality” products. This perfect negative correlation is employed for expositional purpose and our qualitative results hold as long as there is some negative correlation between the types of the firms. As will be discussed in detail in Section 5, a negative correlation in qualities arises from consumers'

relative preferences over product qualities (relative preferences are known to be a frequent phenomenon). We assume that the firms have better knowledge about the industry than the consumers, and hence they know their competitor's product quality. Consumers cannot observe the quality of a product, but they have a prior belief regarding a firm's product quality that is $Pr(\text{firm } i = \text{high-quality}) = 1/2$.

In the first stage, each firm can credibly disclose information about its product quality at a cost γ . The choice of disclosure is denoted by $x_i \in \{0, 1\}$, with $x_i = 0$ (withhold the information) and $x_i = 1$ (disclose the information at the cost γ). Disclosures by the firm are imperfect – we denote by $\lambda(X)$ the probability that a consumer becomes informed of the firms' product qualities when such information is disclosed, where $X = x_1 + x_2$, with $\lambda(0) = 0$, $\lambda'(X) > 0$, and $\lambda(2) < 1$. The situation we have in mind is that there are numerous disclosure channels, through which firms can make their product qualities public information. We assume that it is prohibitively costly for a firm to disclose its information through all possible channels. For simplicity, we model this situation with that a firm can afford information disclosure through only one channel. Likewise, it is prohibitively costly for a consumer to check all such channels for information. Hence, the probability that a consumer remains uninformed is strictly positive, but as $X (= x_1 + x_2)$ increases, a consumer becomes more likely to be informed.⁴ Throughout this paper, we interpret $\lambda(X)$ as “the fraction of informed consumers” at the end of the first stage.

In the second stage, firm 1 (high-quality) and firm 2 (low-quality) choose p_1 and p_2 respectively, with horizontally and vertically differentiated demand $D_i(p_1, p_2, a_i)$. The parameter $a_i \in \{a^L, a^H\}$, with $a^H = a^L + \Delta_a$ and $\Delta_a > 0$. For the fraction λ of consumers (informed at the end of the first stage), $a_1 = a^H$ and $a_2 = a^L$. For the fraction $1 - \lambda$ of consumers (uninformed at the end of the first stage), a_i are replaced by the expected quality $E(a_i) = \mu_i(p_1, p_2) a^H + (1 - \mu_i(p_1, p_2)) a^L$, where $\mu_i(p_1, p_2)$ is the posterior belief about the probability that firm i is selling a high quality good. Without price-signaling, only the fraction λ of consumers will be able to distinguish the two firms. The remaining $1 - \lambda$ of consumers can only use their priors and the information from the pair of prices to update their posteriors about quality. Demand for firm i 's product is described below:

$$\begin{aligned} D_i(p_1, p_2, a_i) &= E(a_i) - p_i + bp_j \\ &= \lambda a_i + (1 - \lambda) (\mu_i a^H + (1 - \mu_i) a^L) - p_i + bp_j, \end{aligned}$$

where $b < 1$ is the cross-price effect on demand. The own-price effect is normalized to 1.

⁴See Grossman (1981) and Hirshleifer and Teoh (2003) for a similar setup.

Note that $\frac{\partial D_i}{\partial p_i} < 0$, $\frac{\partial D_i}{\partial p_j} > 0$, and $\frac{\partial D_i}{\partial a_i} > 0$ (again, the index $j \in I$ denotes “the other firm”). Expected profits for the firms are then given by the expression:

$$\pi_i(p_1, p_2, \mu_i) = (p_i - c_i)D_i(p_1, p_2, E(a_i)) - \gamma x_i,$$

where c_i is the marginal production cost of firm i . To guarantee a strictly positive demand, we assume that $a^L > c_i$. We assume that these specifications are common knowledge.

We close this section by summarizing the timing of the game. At the first stage, the firm 1 and firm 2 decide whether or not to disclose their quality information (choice of $x_1 \in \{0, 1\}$ and $x_2 \in \{0, 1\}$). Accordingly, the proportion of informed consumers (λ) is realized at the end of the first stage. At the second stage, the firms engage in price-signaling competition. In the subsequent sections, we analyze the game by backward induction.

3 Analysis of the second stage – choice of prices

In the second stage we look at the firms’ pricing strategies. The results in this section will be used in Section 4 where we discuss information disclosure by each firm before price-signaling competition. Recall that $1 - \lambda$ of consumers remain uninformed at the end of the first stage, and thus, if profitable, a firm will price so as to signal its product quality to the uninformed consumers.

The building block of our analysis is as follows. First, assuming that firm 1’s signaling equilibrium price, denoted by p^H , exists (later, we show that p^H exists), we define the consumer belief structure. Second, we derive firm 2’s price in an equilibrium, denoted by p^L . Third, we identify two separating regimes, and construct a set of separating price p^H in each regime. Finally, we show that such set in each separating regime is non-empty, depending on parameters, and select a unique p^H .

We proceed by defining a separating equilibrium applying the Perfect Bayesian Equilibrium framework.

Definition 1 *A separating equilibrium entails $\{p_1, p_2\} = \{p^H, p^L\}$ and $\mu_i(p_1, p_2)$ such that:*

- (i) *If $p_i = p^H$ and $p_j \neq p^H$ then $\mu_i(p_1, p_2) = 1$ and $\mu_j(p_1, p_2) = 0$,*
- (ii) *If $p_i = p^H$ and $p_j = p^H$ then $\mu_i(p_1, p_2) = \mu_j(p_1, p_2) = \frac{1}{2}$,*
- (iii) *If $p_i \neq p^H$ and $p_j \neq p^H$ then $\mu_i(p_1, p_2) = \mu_j(p_1, p_2) = \frac{1}{2}$,*

- (iv) $p^H \in \arg \max_{p_1} \pi_1(p_1, p^L, \mu_1(p_1, p^L))$,
- (v) $p^L \in \arg \max_{p_2} \pi_2(p^H, p_2, \mu_2(p^H, p_2))$,
- (vi) $\mu_1(p^H, p^L) = 1$ and $\mu_2(p^H, p^L) = 0$.

The explanation for the equilibrium as follows. Condition (i) states that, if firm i is pricing according to the high-quality product and firm j is not, then consumers believe that firm i is the high-quality firm and firm j is the low-quality firm. According to condition (ii) and (iii), if both firms are choosing the signaling price ($= p^H$), or both firms choose different prices from the one that the high-quality firm would choose ($\neq p^H$), then consumers cannot tell a firm's product quality and revert to their priors (and believe that both firms have the same probability of having high-quality). Condition (iv) specifies that p^H is the optimal price of a high-quality firm given that firm 2 chooses p^L . Likewise, condition (v) specifies that p^L is firm 2's optimal price given that firm 1 chooses p^H . Finally, condition (vi) says that consumers beliefs are confirmed by the separating outcome – the consumers believe that the product is high-quality when the firm charges p^H and that the the product is low-quality when the firm charges p^L . Together with (i) \sim (iii), conditions (iv) and (v) can be rewritten as follows:

$$\begin{aligned} \pi_1(p^H, p^L, 1) &\geq \pi_1(\hat{p}_1, p^L, \frac{1}{2}) \forall \hat{p}_1 \neq p^H, \\ \pi_2(p^H, p^L, 0) &\geq \pi_2(p^H, p^H, \frac{1}{2}), \end{aligned}$$

The above expressions are standard incentive compatibility conditions in signaling models. The first requires that firm 1 (high-quality) prefers the signaling price p^H to some other price that allow mimicry by firm 2 (low-quality). The second requires that firm 2's profit from being truthfully perceived as the low-quality is higher than its profit from pretending to be the high-quality.

With these definitions, we characterize each firm's optimal price and payoff. As often occurs in signaling games, both separating and pooling equilibria may emerge. Furthermore, it is also possible that the high-quality firm's best response price automatically allows separation without distortion. Since such outcomes are trivial situations, we focus on separating equilibria with price distortions to make our point.

Separating equilibrium

To find the separating equilibrium in the model, first we obtain the strategy of firm 2, the low-quality firm (Lemma 1 below), then we determine the set of candidates for p^H , the

separating equilibrium price for firm 1, the high-quality firm (Lemma 2 below). Then, we show that such set is non-empty, depending on parameters (Lemma 3 below). Lastly, we obtain the separating equilibrium price p^H from the set of candidates (Lemma 4 below).

Lemma 1 *In a separating equilibrium, the strategy of firm 2 is $p^L = p_2(p^H) = \frac{a^L + bp^H + c_2}{2}$.*

Proof. See Appendix A. ■

We proceed by finding candidates for firm 1's separating price in equilibrium (candidates for p^H). Recalling Definition 1, condition (iv) requires that firm 1 wants to separate itself from firm 2 (low-quality). We use the superscript S in π_i^S to denote separating outcomes and the superscript N in π_i^N to denote non-separating outcomes.

In a separating outcome, the profit for firm 1 (the high quality) is given by:

$$\pi_1^S(p^H, p^L, 1) = (p^H - c_1)(a^H - p^H + bp^L) - \gamma x_1.$$

When firm 2 is truthfully recognized as the low-quality firm, its price is as stated in Lemma 1: $p^L = \frac{a^L + bp^H + c_2}{2}$. Thus, the profit for firm 1 in a separating equilibrium is:

$$\pi_1^S(p^H, p^L(p^H), 1) = (p^H - c_1) \left(a^H - p^H + b \frac{a^L + bp^H + c_2}{2} \right) - \gamma x_1. \quad (1)$$

If firm 1 deviates to a price $\hat{p}_1 \neq p^H$ while $p_2 = p^L$, then consumers believe with $\mu_1 = \frac{1}{2}$ that firm 1 is the low-quality. In such a case, its profit would be:

$$\pi_1^N(\hat{p}_1, p^L(p^H), \frac{1}{2}) = (\hat{p}_1 - c_1) \left[\lambda a^H + (1 - \lambda) \left(\frac{1}{2} a^H + \frac{1}{2} a^L \right) - \hat{p}_1 + b \frac{a^L + bp^H + c_2}{2} \right] - \gamma x_1.$$

Firm 1's best outcome in this case is obtained by maximizing the above expression with respect to \hat{p}_1 . Such maximization yields:

$$\pi_1^N(\hat{p}_1(p^L(p^H)), p^L(p^H), \frac{1}{2}) = \left(\frac{(2 + b)a^L - 2c_1 + bc_2 + b^2p^H + (1 + \lambda)\Delta_a}{4} \right)^2 - \gamma x_1. \quad (2)$$

In order to determine the set of candidates for p^H that satisfies condition (iv) in Definition 1, we define $h(p^H)$ to be the difference $\pi_1^S(p^H) - \pi_1^N(p^H)$ using the expressions in (1) and (2). We verify whether $h(p^H)$ is positive or negative:

$$h(p^H) = (p^H - c_1) \left(a^H - p^H + b \frac{a^L + bp^H + c_2}{2} \right) - \left(\frac{(2 + b)a^L - 2c_1 + bc_2 + b^2p^H + (1 + \lambda)\Delta_a}{4} \right)^2.$$

Function $h(p^H)$ is concave and quadratic in p^H with two real roots. Let these roots be \underline{p}_1 and \overline{p}_1 , where $\underline{p}_1 < \overline{p}_1$. These roots satisfy condition (iii) in Definition 1 with equality. Let $\mathcal{H} = \{p^H : h(p^H) \geq 0\} = \{p^H : \underline{p}_1 \leq p^H \leq \overline{p}_1\}$. If $p^H \notin \mathcal{H}$, then Condition (iii) in Definition 1 is violated, and firm 1 would prefer not to separate itself from firm 2.

The next condition (condition (v)) in Definition 1, requires that, with firm 1's signaling price p^H , firm 2 would rather choose $p_2 = p^L$ and be truthfully identified as the low quality than attempt to mimic firm 1 by choosing $p_2 = p^H$.

In a separating outcome, firm 2's profit can be written as:

$$\pi_2^S(p^H, p^L, 0) = (p^L - c_2)(a^L - p^L + bp^H) - \gamma x_2.$$

Again, the optimal price for firm 2 in a separating equilibrium is given by Lemma 1: $p^L = \frac{a^L + bp^H + c_2}{2}$. Substituting for this expression in firm 2's profit function we obtain:

$$\pi_2^S(p^H, p^L(p^H), 0) = \frac{(a^L + bp^H + c_2)^2}{4} - \gamma x_2. \quad (3)$$

If firm 2 mimics firm 1, while firm 1 is playing p^H , then firm 2's profit would be:

$$\pi_2^N(p^H, p^H, \frac{1}{2}) = (p^H - c_2) \left[\lambda a^L + (1 - \lambda) \left(\frac{1}{2} a^H + \frac{1}{2} a^L \right) - p^H + bp^H \right] - \gamma x_2. \quad (4)$$

Using the expressions in (3) and (4), we determine another set of candidates for p^H that satisfy condition (v) in Definition 1. As before, we define $l(p^H)$ to be the difference $\pi_2^S(p^H) - \pi_2^N(p^H)$, and verify whether the expression is positive or negative:

$$l(p^H) = \frac{(a^L + bp^H + c_2)^2}{4} - (p^H - c_2) \left[\lambda a^L + (1 - \lambda) \left(\frac{1}{2} a^H + \frac{1}{2} a^L \right) - (1 - b)p^H \right].$$

Function $l(p^H)$ is convex and quadratic in p^H with two real roots. Let these roots be \underline{p}_2 and \overline{p}_2 , where $\underline{p}_2 < \overline{p}_2$. These roots satisfy condition (v) in Definition 1 with equality. Let $\mathcal{L} = \{p^H : l(p^H) \geq 0\} = \{p^H : p^H \leq \underline{p}_2 \text{ or } p^H \geq \overline{p}_2\}$. If $p^H \notin \mathcal{L}$, then condition (v) in Definition 1 is violated, and firm 2 would prefer to mimic firm 1.

Since both (iv) and (v) in Definition 2 need to be satisfied, our discussion indicates that all candidates for p^H must be in the intersection $\mathcal{H} \cap \mathcal{L}$. Thus we have the next lemma.

Lemma 2 *A separating equilibrium can only be supported by $p^H \in \underline{\mathcal{S}} \equiv \{p : \underline{p}_1 \leq p \leq \underline{p}_2\}$ or $p^H \in \overline{\mathcal{S}} \equiv \{p : \overline{p}_2 \leq p \leq \overline{p}_1\}$.*

Proof. The proof follows directly from the fact that $p^H \in \mathcal{H} = \{p^H : \underline{p}_1 \leq p^H \leq \bar{p}_1\}$ and also $p^H \in \mathcal{L} = \{p^H : p^H \leq \underline{p}_2 \text{ or } p^H \geq \bar{p}_2\}$. ■

Lemma 2 says that the region of potential separating prices comes from two distinct ranges, $\underline{\mathcal{S}}$ and $\bar{\mathcal{S}}$. It can be easily seen that when $p^H \in \bar{\mathcal{S}}$, firm 1 is sending a signal by setting a high price, and when $p^H \in \underline{\mathcal{S}}$, firm 1 is sending a signal by setting a low price. In other words when the high quality firm prices too high or too low it is not profitable for the low quality firm to mimic the high quality firm.

Lemma 3 *If c_1 is sufficiently larger (smaller) than c_2 , then $\bar{\mathcal{S}}$ ($\underline{\mathcal{S}}$) is non-empty.*

Proof. See Appendix B. ■

To focus on separating outcomes, we assume that the parameters satisfy the conditions in Lemma 3. Figure 1 below presents the two sets for p^H candidates: $\underline{\mathcal{S}}$ and $\bar{\mathcal{S}}$. In panel (a) of Figure 1, $\underline{\mathcal{S}} \neq \emptyset$ and $\bar{\mathcal{S}} = \emptyset$, and thus only a low price can signal high quality. As pointed out in Lemma 3, this is the case when the high-quality firm's production cost is significantly lower than the low-quality firm's production cost. Here, the high-quality firm can afford to distort its price downward to signal, while it is too costly for the low-quality firm to match the lower price. In panel (b) on the other hand, $\underline{\mathcal{S}} = \emptyset$ and $\bar{\mathcal{S}} \neq \emptyset$, thus only a high price can signal high quality. This is the case when the high-quality firm's production cost is significantly higher than the low-quality firm's production cost. It is not profitable for firm 2 to match firm 1's upward-distorted price because the own-price has a larger impact on the demand than the cross-price ($b < 1$). That is, firm 2 must cope with a large decrease in quantity demanded to mimic firm 1's high price. Consequently, it is less profitable for firm 2 to mimic firm 1.

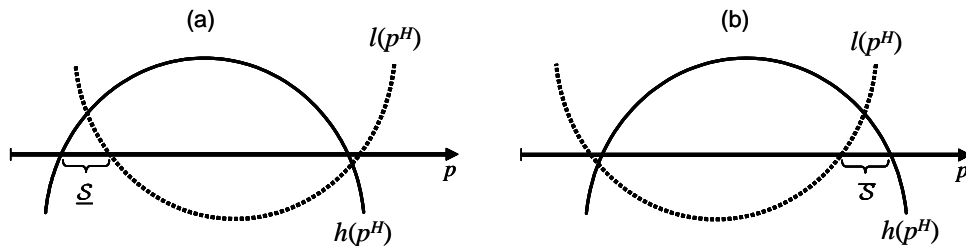


Figure 1. Set of the candidates for p^H

From now on, $p^H \in \bar{\mathcal{S}}$ refers to “a high-price separating equilibrium,” and $p^H \in \underline{\mathcal{S}}$ refers to “a low-price separating equilibrium.” As usual in signaling games, our model yields multiple Perfect Bayesian Equilibria with separation. As will become clear later, however, we note that the firms’ strategy in period 1 is not affected by multiplicity of equilibria in period 2. That is, for any $p^H \in \bar{\mathcal{S}}$ or $p^H \in \underline{\mathcal{S}}$ in period 2, our results for the disclosure strategies in period 1 hold. Therefore, in what follows, we select the following p^H without loss of generality for each regime of $\bar{\mathcal{S}}$ and $\underline{\mathcal{S}}$.

$$\bar{p}^H = \inf(\bar{\mathcal{S}}) \text{ and } \underline{p}^H = \sup(\underline{\mathcal{S}}). \quad (5)$$

The expression for p^H in (5) reflects the idea that firm 1 prefers to deviate its price as little as possible from the one that satisfies the first order condition. Again, the results in the first period hold for any p^H in the sets $\bar{\mathcal{S}}$ or $\underline{\mathcal{S}}$.

We are now ready to determine the firms’ outcomes in these two separating regimes. For convenience, we introduce the following notations for each separating equilibrium:

$$\begin{aligned} \bar{\pi}_1^S &\equiv \pi_1^S(\bar{p}^H, \bar{p}^L, 1) \quad \text{and} \quad \underline{\pi}_1^S \equiv \pi_1^S(\underline{p}^H, \underline{p}^L, 1), \\ \bar{\pi}_2^S &\equiv \pi_2^S(\bar{p}^H, \bar{p}^L, 0) \quad \text{and} \quad \underline{\pi}_2^S \equiv \pi_2^S(\underline{p}^H, \underline{p}^L, 0), \end{aligned}$$

where $\bar{p}^L = \frac{a^L + b\bar{p}^H + c_2}{2}$ and $\underline{p}^L = \frac{a^L + b\underline{p}^H + c_2}{2}$ by Lemma 1.

By solving $l(p^H) = 0$ for p^H , we obtain the exact signaling prices: \bar{p}^H and \underline{p}^H :

$$\begin{aligned} \bar{p}^H &= \frac{(2-b)(a^L + c_2) + (1-\lambda)\Delta_a + \sqrt{(1-\lambda)\Delta_a\{2(2-b)[a^L - (1-b)c_2] + (1-\lambda)\Delta_a\}}}{(2-b)^2}, \\ \underline{p}^H &= \frac{(2-b)(a^L + c_2) + (1-\lambda)\Delta_a - \sqrt{(1-\lambda)\Delta_a\{2(2-b)[a^L - (1-b)c_2] + (1-\lambda)\Delta_a\}}}{(2-b)^2} \end{aligned}$$

Using these expressions for the equilibrium price in the two separating regimes, we obtain the following profit expressions. In a high-price separating equilibrium,

$$\bar{\pi}_1^S = (\bar{p}^H - c_1) \left[a^L + \Delta_a - \bar{p}^H + \frac{b(a^L + b\bar{p}^H + c_2)}{2} \right] \quad \text{and} \quad \bar{\pi}_2^S = \frac{(a^L + b\bar{p}^H - c_2)^2}{4}.$$

Similarly, in a low-price separating equilibrium,

$$\underline{\pi}_1^S = (\underline{p}^H - c_1) \left[a^L + \Delta_a - \underline{p}^H + \frac{b(a^L + b\underline{p}^H + c_2)}{2} \right] \quad \text{and} \quad \underline{\pi}_2^S = \frac{(a^L + b\underline{p}^H - c_2)^2}{4}.$$

In the following section, we proceed to the analysis of the first stage at which each firm decides whether to disclose its quality information.

4 Analysis of the first stage – choice of information disclosure

Using the profit expressions at the second stage, we now analyze the firms' optimal information disclosure strategies at the first stage. Recall that λ is the proportion of informed consumers at the end of the first stage and $\lambda'(X) > 0$, where $X = x_1 + x_2$. Thus, we can see each firm's disclosure strategy simply by looking at the effect of λ on π_i . In other words, the sign of $\frac{\partial \pi_i^S}{\partial \lambda}$ determines $x_i \in \{0, 1\}$, given that γ is small enough. If $\frac{\partial \pi_i^S}{\partial \lambda} > 0$, then firm i will disclose its product quality at the first stage ($x_i = 1$ at the cost γ). If $\frac{\partial \pi_i^S}{\partial \lambda} < 0$, then firm i will withhold its quality information ($x_i = 0$). If γ is too large, then it is clear that $x_i = 0$. Below, we proceed to analyze the two separating equilibrium regimes.⁵

High-price separating equilibrium

In a high-price separating equilibrium, firm 1 (the high-quality) distorts its price upward so that consumers can identify its product quality. The higher the λ at the end of the first stage, the lower the need for firm 1 to distort its prices upward. Thus, firm 1 benefits from disclosing information ($x_1 = 1$) because this increases λ , which in turn reduces the upward distortion in p_1 (\bar{p}^H decreases). However, firm 2 (the low-quality) becomes worse off as λ increases. Again, as λ becomes larger, firm 1's price becomes lower as firm 1 can reduce its upward price distortion. Thus, with $x_2 = 1$, not only more consumers are informed of firm 2's low quality (at the end of the first stage), but also firm 2's price in equilibrium goes down as a response to firm 1's lower price (at the second stage). Together, these effects diminish firm 2's profit. Consequently, firm 2 has an incentive to withhold the information about its product quality ($x_2 = 0$).

The above discussion is summarized in the following proposition.

Proposition 1 *In a high-price separating equilibrium, the high-quality firm discloses information of its true quality ($\frac{\partial \pi_1^S}{\partial \lambda} > 0$) provided that γ is not too large, while the low-quality firm withholds information of its true quality ($\frac{\partial \pi_2^S}{\partial \lambda} < 0$).*

Proof. See Appendix C. ■

⁵In a non-separating equilibrium, firms do not distort their prices, and therefore it is straightforward to verify that only the high-quality firm discloses (certifies) its quality.

Low-price separating equilibrium

In this regime, firm 1 (the high-quality) distorts its price downward (instead of upward) to signal its product quality. That is, for firm 1, the higher the λ at the end of the first stage, the lower the need for downward price distortion. Thus, as in the high-price separating regime, firm 1 benefits from disclosing information (which increases λ). Unlike in the previous regime, however, firm 2 (the low-quality) also benefits from voluntarily disclosing its quality information (contributing to an increase in λ). We present our result in the proposition below.

Proposition 2 *In a low-price separating equilibrium, not only does the high-quality firm disclose information of its true quality ($\frac{\partial \pi_1^S}{\partial \lambda} > 0$), but the low-quality firm also chooses to do so ($\frac{\partial \pi_2^S}{\partial \lambda} > 0$) provided that γ is not too large.*

Proof. See Appendix D. ■

Note that, in the low-price separating equilibrium, the signaling price by firm 1 (\underline{p}^H) is smaller than the low-quality firm's price (p^L). Therefore, information disclosure by the low-quality firm is accompanied by $p^L > \underline{p}^H$. Again, when the high-quality firm can produce at a lower cost, it will aggressively distort its price downward to convince the uninformed consumers at the end of the first stage. This leads to a strategic effect that decreases the payoff of the low-quality firm in two ways. The lower price by the high-quality firm decreases the demand of the low-quality firm, and it also forces the low-quality-firm to respond with a lower price. As a result, the high-quality firm's downward price distortion reduces the low-quality firm's profit.

By disclosing its quality information to the market ($x_2 = 1$), firm 2 increases the proportion of informed consumers, which leads to smaller downward distortion in firm 1's price (\underline{p}^H increases), thus allowing firm 2 to increase its price. These effects together positively contribute to π_2 , and consequently, firm 2 also has an incentive to disclose the information about its product quality.

5 Negative correlation in quality as relative quality preferences

For expositional purpose, we employed perfect negative correlation between the firms' product qualities in our model – when one firm is the high-quality (H), the other firm is the

low-quality (L). This is, the “Nature” chooses either state HL or LH , and the firms’ learn the Nature’s choice while the consumers do not. When the Nature can choose all possible combinations of the state, HL , LH , LL , and HH , *ex ante*, our results still hold, provided that there is some (not necessarily perfect) negative correlation between the firms’ types. In this section, we first discuss a case in which the firms’ type are not perfectly correlated (to make our point, again we focus on the case in which the Nature chose H as firm 1’s type, and L as the firm 2’s type, *ex post*, although all cases are possible from a consumer’s point of view). Then we discuss the circumstances under which some negative correlation between product qualities arises from a consumer’s perspective.

Imperfect correlation between product qualities

With an imperfect negative correlation, all combinations of HL , LH , LL , and HH are possible, but the likelihood of HL or LH is higher than the other cases. In our framework, as long as there is some degree of negative correlation between the firms’ types, our results hold. To see this, suppose $\lambda(x_1, x_2) = \lambda_1(x_1) + \lambda_2(x_2)$, with $0 < \lambda_1(x_1) + \lambda_2(x_2) < 1$.⁶ That is, λ_1 of the consumers are informed by firm 1’s disclosure, and λ_2 of the consumers are informed by firm 2’s disclosure. With perfect negative correlation (as in our model), λ_i of the consumers automatically learn firm j ’s quality information. This is not the case when the firm’s types are not perfectly correlated. However, with some negative correlation, λ_i of the consumers learn that it is “more likely” that firm j ’s quality is the opposite to firm i ’s. One can model that, for λ_i of the consumers (the consumers who learned firm i ’s quality), firm j ’s quality parameter $\theta_j \equiv \rho a_i + (1 - \rho)(a^L + a^H - a_i)$, where ρ represents the correlation parameter with $\rho \in [0(\text{perfect negative}), 1(\text{perfect positive})]$. With $\rho = 1/2$, there is no correlation. With this specification, if the state of nature is such that firm 1 is type H and firm 2 is type L , the demand functions for firm 1 and firm 2 are respectively:

$$\begin{aligned} D_1 &= \lambda_1 a^H + \lambda_2 \theta_1 + (1 - \lambda) (\mu_i a^H + (1 - \mu_i) a^L) - p_i + b p_j, \\ D_2 &= \lambda_2 a^L + \lambda_1 \theta_2 + (1 - \lambda) (\mu_j a^H + (1 - \mu_j) a^L) - p_j + b p_i, \end{aligned}$$

where $\theta_1 \equiv \rho a^L + (1 - \rho) a^H$ and $\theta_2 \equiv \rho a^H + (1 - \rho) a^L$. Notice that these demand functions are essentially the same as the one in our model, except the second term $\lambda_2 \theta_1$ ($\lambda_1 \theta_2$) for firm 1 (2). Therefore, for any $\rho \in [0, 1/2)$, our results hold.

Also, with imperfect negative correlation discussed above, a consumer’s off-the-equilibrium belief can be different from the current structure (in our model, $\mu_1 = \mu_2 = 1/2$ off the equi-

⁶In our model, we let $\lambda(x_1, x_2) = \lambda(x_1 + x_2)$ for simplicity.

librium path since the consumers know that one firm is type H and the other firm is type L). Because consumers know that all states of nature are possible, instead of $\mu_1 = \mu_2 = 1/2$, one way to model can be just $\mu_1 = \mu_2$ off the equilibrium. That is, off the equilibrium path, the uninformed consumers believe that firm 1 and firm 2 are equally likely to be high quality, but not necessarily half and half. For example, suppose the consumers believe that both firms are high quality with certainty ($\mu_1 = \mu_2 = 1$) when $p_1 = p_2 = p^H$, and both are low quality with certatinty ($\mu_1 = \mu_2 = 0$) when $p_1 \neq p^H$ and $p_2 \neq p^H$. With such a belief structure, firm 1 will distort its price even further (compared to the case where $\mu_1 = \mu_2 = 1/2$ off the equilibrium path) to separate itself from firm 2. Thus, firm 2 will have a greater incentive to withhold its quality information in the high-price separating regime. On the other hand, firm 2's incentive to disclose its quality information in the low-price separating regime will be stronger in such a case.

Negative correlations in quality driven by consumers' preference

Here we show that some negative correlation between the firms' types can be implied by the fact that a consumer wants a product of "relatively" better quality. Consider a consumer's utility function in two differentiated products, in which quantity purchased is the consumer's choice variable:

$$U(q_i, q_j) = \sum_{i \in I} [\alpha + \theta_i \delta + f_{\theta_i > \theta_j} \Delta] q_i - \frac{1}{2} \sum_{i \in I} g q_i^2 - \beta q_1 q_2,$$

where α captures some base utility for a product (regardless of quality), θ_i is the quality parameter with $\theta_i \in \{1 \text{ (high-quality)}, 0 \text{ (low-quality)}\}$, δ captures the added absolute utility from a high-quality product, and $\Delta (> 0)$ captures the relative utility of buying the best available product. The function $f_\Omega = \theta_i - \theta_j$ if Ω is true, and $f_\Omega = 0$ otherwise. Also, β is the degree of substitution between the two products, with $0 < \beta < g$. A consumer maximizes her utility for consumption.

$$\max_{q_1, q_2} U(q_1, q_2) + B - \sum_{i \in I} p_i q_i.$$

where B is her budget. By taking derivatives with respect to q_1 and q_2 and solving for the first order condition yields the optimal quantities:

$$q_i = \frac{(g - \beta) a + (g\theta_i - \beta\theta_j) \delta + (gf_{\theta_i > \theta_j} - \beta f_{\theta_j > \theta_i}) \Delta - gp_i + \beta p_j}{g^2 - \beta^2}.$$

Since $f_{\theta_j > \theta_i} = 1 - f_{\theta_i > \theta_j}$, by normalizing $\frac{g}{g^2 - \beta^2} = 1$ and letting $\frac{(g - \beta)a - \beta\Delta}{g^2 - \beta^2} \stackrel{\text{def}}{=} a^L$, $\frac{\beta}{g^2 - \beta^2} \stackrel{\text{def}}{=} b$, and $\frac{\Delta(g + \beta)}{g^2 - \beta^2} \stackrel{\text{def}}{=} \Delta_a$, we can rewrite the demand for firm i as:

$$q_i = a^L + \delta\theta_i - b\delta\theta_j + f_{\theta_i > \theta_j}\Delta_a - p_i + bp_j.$$

This is the demand expression under full information. Incorporating our model assumptions that a consumer remains uninformed at the end of the first stage with probability $1 - \lambda$, the expected demand for firm i is:

$$D_i = a^L + \lambda [\delta\theta_i - b\delta\theta_j + f_{\theta_i > \theta_j}\Delta_a] + (1 - \lambda) [\delta\mu_i - b\delta\mu_j + f_{\mu_i > \mu_j}\Delta_a] - p_i + bp_j.$$

Notice that for the fraction $1 - \lambda$ of consumers, the quality parameter θ_i is replaced with the belief μ_i . This expression for D_i is essentially the same as the one in our model, as long as $\Delta > 0$ (i.e., consumers derive utility from “relative” higher quality). If such relativity does not influence consumer’s utility function ($\Delta = 0$), then $\Delta_a = 0$ (since $\frac{\Delta(1 + \beta)}{1 - \beta^2} = \Delta_a$) and firms quality become fully independent in the way they influence consumer utility. On the other hand, if there is only relative effect and no absolute effect of quality ($\delta = 0$), then the negative correlation between the product qualities becomes “perfect,” followed by the demand:

$$\begin{aligned} D_i &= a^L + \lambda f_{\theta_i > \theta_j}\Delta_a + (1 - \lambda)f_{\mu_i > \mu_j}\Delta_a - p_i + bp_j \\ &= \lambda a_i + (1 - \lambda) [\mu_i a^H + (1 - \mu_i)a^L] - p_i + bp_j, \end{aligned}$$

which is the same as the one in our model.

6 Concluding remarks

In this paper we have studied an oligopoly in which two competing firms have the opportunity to reveal information about their product qualities to a fraction of consumers. Firms use their prices as the signaling device for the remaining uninformed consumers. Two separating equilibrium regimes have been identified. Our analysis suggests that not only a high-quality firm, but also a low-quality firm may prefer to disclose the true quality of its product. The price distortion for signaling by the high-quality firm can either improve the payoff of the low-quality firm (as in the case in which high price signals high-quality) or diminish the payoff of the low-quality firm (as in the case in which low price signals

high-quality). In the latter case, the low-quality firm has an incentive to advance the true quality of its product to the market, so as to relax price competition.

The results of our model have significant practical relevance. In numerous industries, disclosing quality information through some trusted mechanism is a possibility that cannot be ignored. For instance, a large number of different products and services, such as hotels, motor oil, desktop computers, glass, home-networking equipment, to name a few, have the possibility of being assessed and certified by an expert body that reports on the products intrinsic characteristics. Managers responsible for these type of products have to carefully consider whether the strategy of information disclosure is worth for their products. As our model suggests, the answer is dependent not only on the product quality in itself, but also on the competitive landscape that ensues as a result of choice of disclosure. Information disclosure is a way to effectively widen the vertical quality difference between the goods and thus soften competition.

Our result hinges on the following assumptions. First, we adopted Bertrand competition with differentiated products – the products are differentiated not only vertically, but also horizontally. For example, in Hertzendorf and Overgaard (2001), the products are differentiated only vertically, and thus, the quantity demanded for the low-quality firm is zero if the low-quality firm’s price is higher than its competitor’s. In such setting, our results do not hold. Second, as discussed in Section 5, our result requires some degree of (but not necessarily perfect) negative correlation. Third, we considered price as the only signaling instrument. As mentioned in introduction, different signaling devices could be introduced to the model. For instance, dissipative advertising could be considered. Such advertisement can affect the price distortions. However, unless this additional instrument completely removes the price distortion, i.e. some amount of price distortion still remains, our qualitative results still hold.

Appendix

A. Proof of Lemma 1

In a separating equilibrium, firm 2 has no incentive to distort its price to mimic firm 1, and hence it chooses p_2 to maximize: $\pi_2^S = (p_2 - c_2) (a^L - p_2 + bp^H) - \gamma x_2$. The first order condition with respect to p_2 with a simple rearrangement gives: $p_2(p^H) = \frac{a^L + bp^H + c_2}{2}$. ■

B. Proof of Lemma 3

We prove Lemma 3 by showing that, first, $\bar{p}_1 - \bar{p}_2 > 0$ when $c_1 - c_2$ is large enough, and then $\underline{p}_2 - \underline{p}_1 > 0$ when $c_2 - c_1$ is large enough. Since $h(p^H)$ is a quadratic function with two real roots, solving $h(p^H) = 0$ for p^H gives \bar{p}_1 and \underline{p}_1 , where $\bar{p}_1 > \underline{p}_1$. Similarly, $l(p^H) = 0$ gives \bar{p}_2 and \underline{p}_2 , where $\bar{p}_2 > \underline{p}_2$. Using the values for \bar{p}_1 and \bar{p}_2 from $h(p^H) = 0$ and $l(p^H) = 0$ respectively, we have the following expression:

$$\begin{aligned} \bar{p}_1 - \bar{p}_2 &= \frac{2(c_1 - c_2)(4 - b^2) + 2[2(1 + \lambda + 2\lambda b - b) - b^2] \Delta_a}{(4 - b^2)^2} \\ &+ \frac{\sqrt{8(1 - \lambda)\Delta_a \{(4 - b^2)[(2 + b)a^L - (2 - b^2)c_1 + bc_2] - b^2(1 + \lambda) - 2(3 + \lambda)\Delta_a\}}}{(4 - b^2)^2} \\ &- \frac{\sqrt{(1 - \lambda)\Delta_a \{2(2 - b)[a^L - (1 - b)c_2] + (1 - \lambda)\Delta_a\}}}{(4 - b^2)^2}. \end{aligned}$$

We let

$$\bar{X} = \frac{2(c_1 - c_2)(4 - b^2) + 2[2(1 + \lambda + 2\lambda b - b) - b^2] \Delta_a}{(4 - b^2)^2}$$

captures the first term outside the square roots and

$$\begin{aligned} \bar{Y} &= \frac{\sqrt{8(1 - \lambda)\Delta_a \{(4 - b^2)[(2 + b)a^L - (2 - b^2)c_1 + bc_2] - b^2(1 + \lambda) - 2(3 + \lambda)\Delta_a\}}}{(4 - b^2)^2} \\ &- \frac{\sqrt{(1 - \lambda)\Delta_a \{2(2 - b)[a^L - (1 - b)c_2] + (1 - \lambda)\Delta_a\}}}{(4 - b^2)^2} \end{aligned}$$

captures the second and the third term with the square roots. To check whether $\bar{p}_1 - \bar{p}_2$ could be positive, we first inspect \bar{Y} . The term \bar{Y} is positive when: $\delta - \eta \geq 0$, where $\delta \equiv 8(1 - \lambda)\Delta_a \{(4 - b^2)[(2 + b)a^L - (2 - b^2)c_1 + bc_2] - b^2(1 + \lambda) - 2(3 + \lambda)\Delta_a\}$ and $\eta \equiv (1 - \lambda)\Delta_a \{2(2 - b)[a^L - (1 - b)c_1] + (1 - \lambda)\Delta_a\}$. These are the expressions inside the square root terms of \bar{Y} . By solving this expression with equality ($\delta - \eta = 0$) for c_1 , we find that the two square root terms cancel out when:

$$c_1 = \frac{(7 + 4b)a^L + (1 + 3b)}{8 - 4b^2} c_2 + \frac{\{46 + 18\lambda - b[1 - \lambda + 8b(1 + \lambda)]\} \Delta_a}{8(8 - 6b^2 + b^2)}.$$

This value for c_1 necessarily implies that $c_1 > c_2$, since the first term is greater than c_2 , and the second term is positive. However, if $c_1 > c_2$, then, it can be easily checked that $\bar{X} > 0$. Therefore, when $c_1 - c_2 (> 0)$ is large enough, we have $\bar{X} > 0$, and $\bar{Y} = 0$, which imply that $\bar{p}_1 - \bar{p}_2 > 0$.

Similarly, using the values for \underline{p}_1 and \underline{p}_2 from $h(p^H) = 0$ and $l(p^H) = 0$ respectively, we have the following expression:

$$\begin{aligned}
\underline{p}_2 - \underline{p}_1 &= -\frac{2(c_1 - c_2)(4 - b^2) + 2[2(1 + \lambda + 2\lambda b - b) - b^2] \Delta_a}{(4 - b^2)^2} \\
&+ \frac{\sqrt{8(1 - \lambda)\Delta_a \{(4 - b^2)[(2 + b)a^L - (2 - b^2)c_1 + bc_2] - b^2(1 + \lambda) - 2(3 + \lambda)\Delta_a\}}}{(4 - b^2)^2} \\
&- \frac{\sqrt{(1 - \lambda)\Delta_a \{2(2 - b)[a^L - (1 - b)c_2] + (1 - \lambda)\Delta_a\}}}{(4 - b^2)^2}.
\end{aligned}$$

Again, we let

$$\underline{X} = -\frac{2(c_1 - c_2)(4 - b^2) + 2[2(1 + \lambda + 2\lambda b - b) - b^2] \Delta_a}{(4 - b^2)^2}$$

captures the first term outside the square roots and

$$\begin{aligned}
\underline{Y} &= \frac{\sqrt{8(1 - \lambda)\Delta_a \{(4 - b^2)[(2 + b)a^L - (2 - b^2)c_1 + bc_2] - b^2(1 + \lambda) - 2(3 + \lambda)\Delta_a\}}}{(4 - b^2)^2} \\
&- \frac{\sqrt{(1 - \lambda)\Delta_a \{2(2 - b)[a^L - (1 - b)c_2] + (1 - \lambda)\Delta_a\}}}{(4 - b^2)^2}
\end{aligned}$$

captures the second and the third term with the square roots. To see that $\underline{p}_2 - \underline{p}_1 = \underline{X} + \underline{Y} > 0$ when $c_2 - c_1 (> 0)$ is large enough, we take the extreme values: $c_1 = 0$ (the minimum possible c_1) and $c_2 = a^L$ (the maximum possible c_2). With these parametric values, we have:

$$\begin{aligned}
\underline{X} &= \frac{2a^L(4 - b^2) + 2[2(1 + \lambda + 2\lambda b - b) - b^2] \Delta_a}{(4 - b^2)^2}, \\
\underline{Y} &= \frac{\sqrt{8(1 - \lambda)\Delta_a \{(4 - b^2)[(2 + b)a^L + ba^L] - b^2(1 + \lambda) - 2(3 + \lambda)\Delta_a\}}}{(4 - b^2)^2} \\
&- \frac{\sqrt{(1 - \lambda)\Delta_a \{2(2 - b)[a^L - (1 - b)a^L] + (1 - \lambda)\Delta_a\}}}{(4 - b^2)^2}.
\end{aligned}$$

It can be easily seen that $\underline{X} > 0$. For \underline{Y} , again, we look at the expressions inside the square root terms of \underline{Y} . Let $\omega \equiv 8(1 - \lambda)\Delta_a \{(4 - b^2)[(2 + b)a^L + ba^L] - b^2(1 + \lambda) - 2(3 + \lambda)\Delta_a\}$ and $\tau \equiv (1 - \lambda)\Delta_a \{2(2 - b)[a^L - (1 - b)a^L] + (1 - \lambda)\Delta_a\}$. From $\omega - \tau$, we have the expression: $(1 - \lambda)[\{46 + 18\lambda - b[1 - \lambda + 8b(1 + \lambda)]\} \Delta_a + 2(8 + 7b)(4 - b^2)a^L] \Delta_a > 0$ ($\therefore \underline{Y} > 0$), and thus $\underline{X} + \underline{Y} > 0$. ■

C. Proof of Proposition 1

We prove Proposition 1 by showing that $\frac{\partial \bar{\pi}_1^S}{\partial \lambda} > 0$ (which implies that when γ is small enough, $x_1 = 1$) and $\frac{\partial \bar{\pi}_1^S}{\partial \lambda} < 0$ (implying that $x_2 = 0$). Since $\frac{\partial \bar{\pi}_i^S}{\partial \lambda} = \frac{\partial \bar{\pi}_i^S}{\partial \bar{p}^H} \times \frac{\partial \bar{p}^H}{\partial \lambda}$, we first

determine the sign of $\frac{\partial \pi_1^S}{\partial \bar{p}^H}$. In this regime, firm 1 (high-quality) is setting a price different from the first order condition so as to guarantee that consumers can distinguish between firms. Let p_1^{FOC} denotes the undistorted First Order Condition price for firm 1. We know that $\frac{\partial \pi_1}{\partial p_1} = 0$ when $p_1 = p_1^{FOC}$. Because profits are a strictly concave function in own prices, it must be the case that $\frac{\partial \pi_1^S}{\partial \bar{p}^H} < 0$ when $\bar{p}^H > p_1^{FOC}$. Since, in this case, the firm is distorting its price upward, $\frac{\partial \pi_1^S}{\partial \bar{p}^H} < 0$.

Next we determine the sign of $\frac{\partial \bar{p}^H}{\partial \lambda}$:

$$\frac{\partial \bar{p}^H}{\partial \lambda} = \frac{\Delta_a \left\{ -1 - \frac{\underbrace{(2-b)[a^L - (1-b)c_2] + (1-\lambda)\Delta_a}_{(A1)}}{\underbrace{\sqrt{2\Delta_a(1-\lambda)(2-b)[a^L - (1-b)c_2] + [(1-\lambda)\Delta_a]^2}}_{(A2)}} \right\}}{(2-b)^2}.$$

Notice that expressions (A1) and (A2) in the RHS of the above equation are positive, thus the entire expression in the curly brackets is negative, which implies that $\frac{\partial \bar{p}^H}{\partial \lambda} < 0$. Therefore $\frac{\partial \pi_1^S}{\partial \lambda} = \frac{\partial \pi_1^S}{\partial \bar{p}^H} \times \frac{\partial \bar{p}^H}{\partial \lambda} > 0$, meaning that firm 1's profit increases with λ .

Next we analyze the effect of quality disclosure on the profit of firm 2. By the chain rule, $\frac{\partial \pi_2^S}{\partial \lambda} = \frac{\partial \pi_2^S}{\partial \bar{p}^H} \times \frac{\partial \bar{p}^H}{\partial \lambda}$.

$$\frac{\partial \pi_2^S(\bar{p}^H, p^L(\bar{p}^H), 0)}{\partial \bar{p}^H} = \frac{2b(a^L + b\bar{p}^H - c_2)}{4} > 0,$$

since $a^L > c_2$. Furthermore, as seen above, $\frac{\partial \bar{p}^H}{\partial \lambda} < 0$. Thus, $\frac{\partial \pi_2^S}{\partial \lambda} = \frac{\partial \pi_2^S}{\partial \bar{p}^H} \times \frac{\partial \bar{p}^H}{\partial \lambda} < 0$. ■

D. Proof of Proposition 2

As in the previous proof, we show that $\frac{\partial \pi_1^S}{\partial \lambda} > 0$ and $\frac{\partial \pi_2^S}{\partial \lambda} > 0$ (implying that when γ is small enough, $x_1 = x_2 = 1$). Again, $\frac{\partial \pi_i^S}{\partial \lambda} = \frac{\partial \pi_i^S}{\partial \underline{p}^H} \times \frac{\partial \underline{p}^H}{\partial \lambda}$, and we begin the proof by signing the effect of disclosure on the profit of firm 1. We first determine the sign of $\frac{\partial \pi_1^S}{\partial \underline{p}^H}$. In this regime, the firm 1 is playing a price different from the first order condition so as to guarantee that consumers can distinguish between firms. Let p_1^{FOC} denotes the undistorted First Order Condition price for firm 1. When $p_1 = p_1^{FOC}$, we have $\frac{\partial \pi_1}{\partial p_1} = 0$. Because profits are an strictly concave function in own-prices, it must be the case that $\frac{\partial \pi_1^S}{\partial \underline{p}^H} > 0$ when $\underline{p}^H < p_1^{FOC}$. In this regime, the firm is distorting its price downward, and thus $\frac{\partial \pi_1^S}{\partial \underline{p}^H} > 0$.

Next we determine the sign of $\frac{\partial p^H}{\partial \lambda}$:

$$\frac{\partial p^H}{\partial \lambda} = \frac{\Delta_a \left\{ -1 + \frac{\underbrace{(2-b)[a^L - (1-b)c_2] + (1-\lambda)\Delta_a}_{(B1)}}{\underbrace{\sqrt{2\Delta_a(1-\lambda)(2-b)[a^L - (1-b)c_2] + [(1-\lambda)\Delta_a]^2}}_{(B2)}} \right\}}{(-2+b)^2}.$$

We let $W = \{(2-b)[a^L - (1-b)c_2] + (1-\lambda)\Delta_a\}^2$, the expression in the term (B1), and let $Z = 2\Delta_a(1-\lambda)(2-b)[a^L - (1-b)c_2] + [(1-\lambda)\Delta_a]^2$, the expression inside the square root in (B2). It is straightforward to show that $W - Z = \{(2-b)[a^L - (1-b)c_2]\}^2 > 0$. Hence $\frac{(B1)}{(B2)} > 1$, which implies that $\frac{\partial p^H}{\partial \lambda} > 0$. Therefore $\frac{\partial \pi_2^S}{\partial \lambda} = \frac{\partial \pi_1^S}{\partial p^H} \times \frac{\partial p^H}{\partial \lambda} > 0$.

Next we analyze the effect of quality disclosure on the profit of firm 2. Again by the chain rule, $\frac{\partial \pi_2^S}{\partial \lambda} = \frac{\partial \pi_2^S}{\partial p^H} \times \frac{\partial p^H}{\partial \lambda}$.

$$\frac{\partial \pi_2^S(p^H, p^L(p^H), 0)}{\partial p^H} = \frac{2b(a^L + bp^H - c_2)}{4} > 0,$$

since $a^L > c_2$. Furthermore, as seen above, $\frac{\partial p^H}{\partial \lambda} > 0$. Thus, $\frac{\partial \pi_2^S}{\partial \lambda} = \frac{\partial \pi_2^S}{\partial p^H} \times \frac{\partial p^H}{\partial \lambda} > 0$. ■

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