Modeling Key Item Effects

Kirthi Kalyanam, Sharad Borle, and Peter Boatwright*

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*Authors are listed in reverse alphabetical order. The corresponding author is Peter Boatwright, pbhb@andrew.cmu.edu, 412-268-4219, Tepper School of Business – Carnegie Mellon University, 5000 Forbes Ave, Pittsburgh, PA 15213. Fax: 412-268-7345. Kirthi Kalyanam is J.C. Penney Research Professor at Leavey School of Business, Santa Clara University, kkalyanam@scu.edu. Sharad Borle is Assistant Professor at Jones Graduate School of Management, Rice University, sborle@rice.edu.

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Abstract

Retailers operate in an increasingly cost efficient environment. The rapid adoption of inventory systems has highlighted the bottom-line benefits of sparse assortments, and retail purchase environments like that of Wal-Mart have shifted consumers' cost and assortment expectations. While traditional retailers continue to maintain broader assortments than can be found at discount competitors, it is often the case that only a few key items in a category are responsible for the majority of category sales and profits. Given the cost of carrying large assortments of slow selling items, a reasonable approach to managing inventory would be to evaluate the marginal profitability of each individual item and to pare back the assortment so that every item would be independently profitable. Even so, retail buyers typically believe that items in an assortment cannot be evaluated independently. In particular, retailers and manufacturers believe that the mere presence of certain items increases the sales volume of the whole assortment. Despite this common belief that individual items affect sales of an assortment of related items, relatively little is known about the presence of individual key items to sales of a collection of items. This paper provides an empirical study of the role of every individual item in an assortment, estimating demand for each individual item as well as impact of the presence of each item on other individual items and on category sales. Using a unique database from a large national U.S. retailer that includes information on item specific out-of-stocks, we use the natural variation in an apparel category to study the sales impact of the absence of each individual item. Our primary focus in this paper is to identify items that influence category demand, which are items that retailers consider to be key items. The model we develop is a system of equations, accounting for the inter-relationship between sales, demand, and out-of-stocks. In a hierarchical Bayes framework, we estimate a joint model of sales and stockouts, an approach in the spirit of that used by Manchanda, Rossi, and Chintagunta (2005) in a different context. Our model accounts for store level heterogeneity and allows us to evaluate three effects of out of stocks: lost sales, substitution of other individual items, and the impact on the sales of the entire category.

Our results show many items affect category sales over and above their own sales volume contribution. Deconstructing the role of individual items into the three effects of lost own sales, substitution to other items, and the category impact, we find that the category impact has the largest magnitude in the category we studied, men's work T-shirts. The best selling item, Navy XL (extra large size), accounts for 9.5% of category sales. When Navy XL is stocked out 5% of a time period, category sales volume decreases by nearly 12.8%, a figure that is greater than this item's share of the category. Its own sales loss as a percent of category is 1.4%, and the gain in sales due to substitution to other items is 0.4% of category sales. This result in turn suggests that the sales contribution of key items are above and beyond their individual sales, so management of category inventory should account for this effect of key items on category sales. Interestingly, the disproportionate impact of individual items on category sales is not restricted to top selling items. Using a simulation based on our model, we find that almost every single individual item has a similar disproportionate effect on category sales, a result that is counter to the extant literature on assortment reductions, where even large scale cuts in product assortments have at most a modest impact on category sales. Because the extant literature has focused on grocery products, which are more frequently purchased and less discretionary than apparel, a comparison of our results to those of the extant literature supports recent findings that more frequently purchased categories are less adversely affected by reductions in assortment. At least for apparel, it seems that variety is indeed the price of entry in retailing. We also find that the assortment appears to gain attractiveness when certain items are out of stock. For example, net category sales volume increases when the "medium light blue" item is out of stock, a result that is consistent with the discussion in the literature concerning category clutter. We also confirm the importance of using a simultaneous equations framework for this analysis. At the same time that individual item stockouts reduce sales of those items, the stockouts themselves occur in periods of unusually high sales. In fact, category sales and the prevalence of stockouts are positively correlated, so a naïve model would conclude that stockouts increase sales. Finally, we discuss our implications and strategies for category management and inventory models.

Key Words: Apparel Retailing, retail assortment, out-of-stocks, key items, hierarchical Bayes, COM-Poisson

1. INTRODUCTION

> "When you think about it, a key item really is a key performance indicator, because those are the items that drive your company's business."[1]
>
> Tony Bruno
VP of Planning and Allocation
The Bombay Company

> "With that simple merchandise adjustment…to operating a key class / key item business came a change in the culture of the company and a resurrection of the brand"[2]
>
> Robert J. DiNicola
Chairman
Zale Corporation

> "In terms of the overall number of cell phones sold worldwide, it (the Razr) was a very small fraction …. What it really did do though was get Motorola in the minds of a lot of people around the world for being cool again. "[3]
>
> Edward J. Zander
Chairman & CEO
Motorola

For many retailers, such as those in the apparel industry, the items in the store not only serve as individual profit centers but also craft the visual presentation of the store. While retailers might exhibit a broad assortment of items, it is often the case that key items drive sales and profits in a product category. As an example, basic colors such as black and navy might be a key items in the work T shirts product category, and popular retailing adages such as 'Navy is the new black' implicitly refer to key items and their dynamic nature. In a product category like cell phones, as the quote above from CEO Ed Zander shows, Motorola's RAZR model could be considered to be a key item. Retailers and manufacturers think that the value of key items

[1] Quoted in Brad Barth (2004), Bombay Facilitates Key Item Planning with Merchandising Tools, *Fairchilds Executive Technology*, Vol 6, No 5.
[2] Robert DiNicola, (2003), The Resurgence of a National Brand: The Zale Turnaround Story, *Retailing Issues Newsletter*, May Business School, Texas A&M University, Vol 15, No 1, January.
[3] Christopher Rhoads, (2005), Motorola's Modernizer, *The Wall Street Journal*, June 23.

extends beyond their own individual sales revenues; for their belief is that the presence of key items in a category lifts the sales volume of the entire category.

Rightly or wrongly, the ability to stock and manage key items seems to be important to retailers and manufacturers. Job descriptions for retail buyers routinely require the ability to identify and manage key items[4]. Retailers often designate key items as "never out" items and maintain a high inventory position[5]. Annual reports of retailers such as Federated Department Stores[6] and The Zale Corporation highlight their key item merchandising strategies. Software vendors tout the ability to manage key items in their assortment planning tools.[7] Manufacturers also emphasize key item strategies so that they can meet the needs of their retail partners.[8] Retailing textbooks (Levy and Weitz 2004, pp. 420) advise readers to take into account the impact of an item on the overall assortment but do not provide any guidance on the magnitude of these effects or how they should be identified.

In spite of this widespread practical interest there has been very little research investigating key item effects. At the same time, categories such as apparel or other infrequently purchased and discretionary products that provide the context for retailers' discussions of key items have not received much modeling attention. The most basic question is whether the presence of key items lifts the sales of the entire category over and above own sales effects. If key items do lift the sales of the entire category, then the retailing practice of owning these items

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[4] Job descriptions at several retailers such as Burdines, Macy's and Steinmarts for various positions in the buying organization including Buyers to Regional Merchandise Managers specified the identification and promotion of key items as a required skill.

[5] Robert DiNicola, ibid.

[6] Federated Department Stores Inc, 2003 Annual Report.

[7] For example, *Evant* a software vendor to retailers highlights key item planning capabilities in describing its merchandising solutions, "Evant Merchandise Planning 3.5 Gives Retailers the Power to Plan without Boundaries", Press Release, May 18th 2004. Similarly Retek a leading software vendor to retailers highlights that "one of the strengths of our solution is the ability to identify and capitalize on key item opportunities quickly", "Retek Readies the Home Shopping Network for Prime Time", Retek Case Studies, 2005.

[8] Doris Hajewski (2002), "Childrens Clothing Undergoing Singular Change", Milwaukee Journal Sentinel, June 25.

in depth might be justified. A second issue is whether key items are simply best sellers. If category impact and item sales are highly correlated, then retailers can designate key items based on observed sales. The inter-relationship between key items and other items in the category is also of interest. If a key item can substitute for other items, then the inventory of a key item can be a buffer for other items. A field study of these issues would require either cutting each item in the assortment or an appropriately designed experiment.[9]

The contribution of this research is a model to estimate key item effects using out of stocks in an apparel category. An out of stock of an item is a natural experiment that can (1) result in lost sales, (2) lead to substitution to other items and (3) impact the sales of the entire category due to a temporary change in the available selection. The simultaneous modeling of all three effects for an item at the store level is a key contribution of this research.

The empirical study of out-of-stock data requires careful attention to model specification. In retail sales data, exogenous demand shocks can lead to concurrent stockouts of multiple items. We control for demand shocks and concurrent stockouts using a simultaneous equations approach, estimating the joint distribution of sales and stockouts in a hierarchical Bayes model. See Manchanda, Rossi, and Chintagunta (2005) for a similar modeling approach in a different context. Another characteristic of retail sales data for apparel categories is that unit sales for each item at the store level are sparse. For example, in our data set, 2.4 units of each item were sold per store in each time period. In addition, stockouts occur in any time period with probability less than 0.05, leaving little information at the store level to estimate item and store specific parameters. We therefore utilize a hierarchical model that shares information across stores. Additionally, our model allows for over or under dispersion relative to the common

[9] For example in the study by Boatwright and Nunes (2001) items were cut from the assortment based on business criteria not experimental design.

Poisson sales model. We also model substitution in a flexible manner, without imposing the independence of irrelevant alternatives (IIA) restriction of switching proportional to share.

Using our model, retailers can identify key items, estimate their influence on category demand, and estimate their inter-relationship with other items based on empirical evidence rather than simply guesswork or judgment heuristics. Our model allows retailers to calculate the net impact of the stockout of an item on category sales and to deconstruct that impact into the own sales effect, substitution effect and category demand effect. Store sales associates can use the substitution patterns identified by our model in guiding customers to substitute products during stockouts.

Since our estimates are from store sales data, they are accessible and actionable to retailers in a number of ways. First, many retailers have data warehouses in which they capture store sales data. Second, the use of this data fits existing practices for planning and allocation processes, since systems and software typically operate using store sales data. Third, out of stocks are quite common[10] and retailers are increasingly capturing in-stock positions in their data warehouses. These factors make it more feasible for retailers to implement our model and incorporate the findings into their normal businesses processes.

One of our key findings is that out of stocks of certain items, in addition to affecting own sales, *sharply depress* the sales of the entire category. This provides empirical confirmation of the existence of key item effects. In our data set, Navy-Large is a Key Item. An out of stock of Navy-Large has a large impact on the category through a combination of loss of own sales and a decrease in overall sales of the category. Key items are not necessarily top sellers. The item Tan

[10] For example a recent industry study put the average occurrence of out of stocks as between 8% and 10%. See Thomas W. Gruen, Daniel S. Corsten and Sundar Bharadwaj (2002), "Retail Out of Stocks: A Worldwide Examination of Extent, Causes and Consumer Responses", Grocery Manufacturers Association of America. Our data indicates that even though the average stockout for a particular item can be in the single digit range, the variation across stores can be larger.

Extra Large ranks as one of the top five items in terms of overall category impact but it only ranks a 10 in terms of sales. This is a ‘hidden’ key item.

A simulation analysis based on our model estimates shows that although a few key items have a very high impact on category sales, the out of stocks of many other items also depress category sales. It seems that after all, as speculated upon by others (Hoch et al, 1999), variety is indeed the price of entry in retailing. However, after the fact only a few items are best sellers. A second set of findings is with regard to substitution. While substitution proportional to share is a common assumption in the literature, our results show that substitution is not widespread, is limited to only a few colors and can be asymmetric. Consistent with intuition, there is very little substitution across sizes.

Our research contributes to the existing research on the effects of out of stocks. Anupindi, Dada, and Gupta (1998) developed a model that used information on stockouts to infer item demand and substitution patterns; they found significant differences between demand rates and observed sales of products even for items that were rarely out of stock. We extend their work by estimating not only substitution and lost share but by also estimating which items, if any, affect sales of the entire category. Anupindi et al. (1998) contains a thorough discussion of prior models that incorporate stockouts, which did not measure substitution effects nor quantify the impact of individual items on category sales.

Our paper also contributes to the growing research on how consumers make choices within assortments and respond to changes in assortments. Lattin and Roberts (1992) measured how the structure of a category affects product selection. We consider the opposite directional effect, how the presence of individual products affects purchase in the category. Kim, Allenby, and Rossi (2002) examined consumers’ choices of sets of items from assortments to infer utilities on store assortment, while we use the variation in the in-stock items to assess the sales impact of each item's absence. Borle et al. (2005) assessed the impact of a large-scale reduction in assortment on store sales, finding a decline in store sales. Prior work had found the sales impact of assortment reductions to be small (Drèze, Hoch, and Purk 1994, Broniarczyk, Hoyer, and McAlister 1998; Boatwright and Nunes 2001). These previous studies all measured the impact of permanent and simultaneous elimination of multiple items from frequently purchased grocery categories, using experimental data or household data to examine the impact of changes in assortment on consumer shopping behavior.

Our paper examines these effects in the context of an infrequently purchased and discretionary apparel category. We use store level data with out of stock information, a type of data that is more readily accessible to retailers. Our research also builds on and contributes to the literature on measuring the variety of an assortment. Hoch, Bradlow, and Wansink (1999) used the information structure of a category to provide models of perceived variety within product assortments. We empirically infer the contribution of each item on the attractiveness of an assortment, showing how the temporary absence of even small share products disproportionately affects category sales. We formulate an assortment attractiveness index that changes when individual items are out-of-stock. The assortment attractiveness index allows us to study the differential effects of colors and sizes on the attractiveness of the assortment.

The rest of this paper is organized as follows: in the next section (section 2) we describe the data, in section 3 we introduce the hierarchical demand model and in section 4 we discuss the estimates of the model. In section 5, we deconstruct the net impact of a stockout with a simulation analysis. We conclude with a discussion of our key findings and directions for future research.

2. THE DATA

The data comes from a National retailer based in the United States. The data pertains to a two year time period and consists of quarterly sales and stock out information of "Men's work T-Shirts" category in 196 stores spread across the United States. These are pocket less, round neck short sleeve T-shirts and come in 8 colors and 4 sizes (a total of 32 items)[11]. Table 1(a) gives the average quarterly sales per store for these 32 items.

Table 1(a): Average Quarterly Sales per Store

Color	**Size=M**	**Size=L**	**Size=XL**	**Size=XXL**	
Black	2.09	4.38	4.27	2.15	*3.22*
Grey	1.99	4.88	4.84	2.28	*3.50*
Jade	0.75	1.70	1.90	0.85	*1.30*
Lt. Blue	0.91	2.00	1.85	0.87	*1.41*
Navy	2.59	6.33	6.30	2.93	*4.54*
Royal	0.93	2.22	2.20	1.00	*1.59*
Tan	1.17	2.52	2.46	1.11	*1.82*
White	1.12	2.29	2.17	0.93	*1.63*
	1.44	*3.29*	*3.25*	*1.52*	*2.37*

As seen from the table (Table 1a), on average a store sells only 2.37 units of a particular item in a quarter. Further, viewing sales in terms of colors and sizes (which in turn can be viewed as attributes of the item) we see that 'Navy' color followed by 'Grey' and then 'Black' are the largest sellers, while 'Jade' sells the least. Amongst the sizes, 'Large' sells the most and 'Medium' sells the least. The item 'Navy-Large' is the largest selling item and 'Jade-Medium' is the slowest seller (with sales of a mere 0.75 units per store per quarter). The two interesting facets of Table 1(a) are *firstly* the 'low' sales observed and *secondly* 'significant' sales differences across colors and sizes.

[11] The price and promotions in this category were very infrequent and were not distinguishable from seasonal effects.

Table 1(b) provides stockout information for these 32 color-size combinations. The stockout data available was the number of weeks in a quarter the particular item was out of stock. This is the type of data typically available for planning and replenishing apparel products on a seasonal basis.

A striking feature of Table 1(b) is the large range vis-à-vis the small averages. On average a particular item may have stocked out a small % of weeks but in some stores (during some time periods) the extent of stockouts can be significant. On *average* the color 'Tan' stocked out the most amongst all the colors, an average out of stock of 4.03% of weeks in a quarter as compared to the color 'Grey' which stocked out the least (an *average* of 0.84% of weeks in a quarter). The variation in average % weeks stocked out across sizes is much less with all four sizes stocking out about 2% of the weeks in a quarter. Stockouts when they occur not only induce lost sales (the loss of potential sales of the stocked out item) but also cause changes in demand for other items in the category and induce temporary changes in the selection available to shoppers in the category. In summary, the data provides a

Table 1(b): The Range and Average % of Stockouts**

Color	Size=M	Size=L	Size=XL	Size=XXL
Black	0 to 46.15% *2.06%*	0 to 41.67% *1.60%*	0 to 50.00% *1.51%*	0 to 61.54% *3.19%*
Grey	0 to 30.77% *1.10%*	0 to 61.54% *0.57%*	0 to 53.85% *0.70%*	0 to 53.85% *1.00%*
Jade	0 to 38.46% *1.24%*	0 to 46.15% *1.09%*	0 to 46.15% *1.37%*	0 to 69.23% *2.04%*
Lt. Blue	0 to 76.92% *1.75%*	0 to 53.85% *1.83%*	0 to 76.92% *2.37%*	0 to 61.54% *2.12%*
Navy	0 to 69.23% *3.02%*	0 to 46.15% *2.08%*	0 to 46.15% *2.73%*	0 to 61.54% *4.36%*
Royal	0 to 53.85% *2.01%*	0 to 53.85% *1.89%*	0 to 61.54% *2.02%*	0 to 46.15% *2.31%*
Tan	0 to 61.54% *4.54%*	0 to 69.23% *3.72%*	0 to 61.54% *3.81%*	0 to 61.54% *4.06%*
White	0 to 46.15% *3.21%*	0 to 69.23% *1.78%*	0 to 69.23% *2.01%*	0 to 46.15% *2.39%*

** The first entry in a cell gives the *range* and the second entry the *average.*

nice setting to investigate the impact of retail sales and stockouts in the apparel category. In the next section, we introduce the demand model.

3. THE DEMAND MODEL

3.1 Overview of Model Specification

For ease of exposition, we introduce the demand model in stages. We begin with a 'basic' demand model, which assumes no stockouts (Section 3.1.1). The 'own' effects[12] of a stockout are subsequently introduced (Section 3.1.2). Next, we incorporate the 'cross' item effects[13] of a stockout (Section 3.1.3) and the impact on overall category demand (Section 3.1.4). Finally, we introduce how we control for demand shocks (Section 3.1.5).

3.1.1 The *Basic* Demand Model

Let the demand for a T-shirt with color c (c = 1,2,…,8) size g (g = 1,2,3,4) in store s (s = 1,2,…,196) for time t (t =1,2,…,8[14]) be given by $demand_{scgt}$ units. Let $SALES_{scgt}$ be the sales of t-shirt of color c size g in time t in store s. In the *absence* of stockouts observed sales is equal to demand. We assume demand to be distributed COM-Poisson as follows:

$$demand_{scgt} = SALES_{scgt} \sim ComP(\lambda^0_{scgt}, \nu) \tag{1}$$

wherein the parameters (λ^0_{scgt}, ν) specify the distribution of demand.

The COM-Poisson is a two-parameter extension of the Poisson distribution. The extra parameter ν is helpful in characterizing over/under dispersion in the data relative to the Poisson (Shmueli, Minka, Kadane, Borle & Boatwright 2005; Boatwright, Borle & Kadane 2003; Conway & Maxwell 1961). Figure 1 shows a histogram plot of the quarterly sales of T-shirts (32

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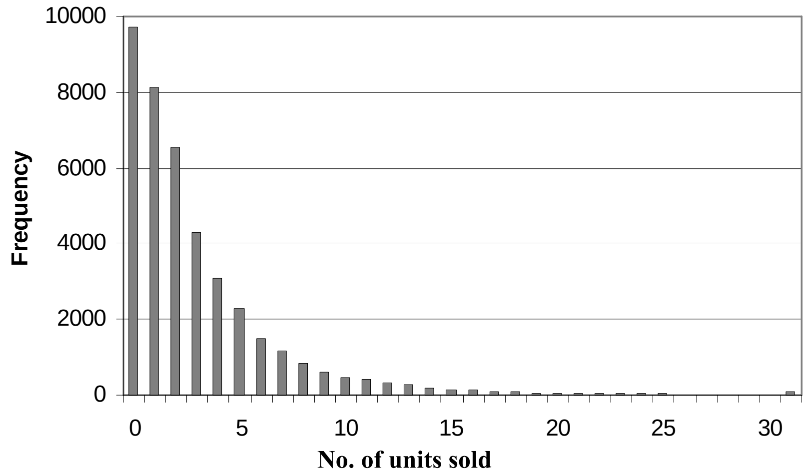
[12] The observed sales for an item being less than the true demand for that item when a stockout for that item occurs
[13] Demand substitution to other items and the impact of item stockout on the overall category sales due to a temporary change in the assortment variety.
[14] Since the data time span is two years

color-size combinations) across the 196 stores. As can be seen, the data is relatively "over-dispersed" and a Poisson may not approximate the data generating process.

Figure 1: Quarterly sales of T-Shirts (32 color/size combinations) across the 196 stores



The parameter λ^0_{scgt} in equations 1 & 2 (given ν) can be interpreted as a measure of central tendency[15]. We specify this parameter as follows,

$$\log \lambda^0_{scgt} \;=\; \beta^{color}_c + \beta^{size}_g + \alpha \log SQFT_s + \gamma_{st} \qquad (2)$$

where β^{color}_c, β^{size}_g are intercepts decomposed into color and size effects[16], and the variable $SQFT_s$ is the square footage of store *s*. The parameter α is the impact of the physical size of the store[17] on demand of various items (the impact being the same for all items in the category). The last term in equation 2, γ_{st}, can be viewed as a store and time period specific demand shock that affects all the items in the assortment. Thus, a 'higher' value for this parameter would imply a 'higher' demand across all items in the assortment in that particular store during that particular

[15] From the p.m.f. of COM-Poison distribution (Boatwright, Borle & Kadane 2003)
[16] For purposes of model identification all coefficients pertaining to the 'Medium' size (g=1) are set to 0.
[17] The store size in terms of square footage is used as a proxy for the 'store volume class'

time period. The parameter captures both time specific effects such as seasonality and exogenous demand shocks. Section 3.1.5 describes this parameter in detail.

3.1.2 The Own Effects of a Stockout

When there are no stockouts, the observed sales reflect the demand for that particular color-size combination in a store in a quarter. Equations 1 and 2 can model demand. However, in the presence of a stockout of color size combination (*c*, *g*), the observed sales $SALES_{scgt}$ is a truncated (and hence biased downwards) representation of the demand. The observed sales need to be adjusted upwards to $SALES^{/}_{scgt}$ so that it is a closer representation of the true demand for that particular color-size combination.

We do so by way of an imputation procedure. We observe the truncated sales $SALES_{scgt}$ whenever there is a stockout. The imputed sales $SALES^{/}_{scgt}$ is a number drawn from the right tail of the demand distribution (equation 1) such that of $SALES^{/}_{scgt} \geq SALES_{scgt}$[18]. The imputation procedure provides an upward adjustment to the observed sales of color size combination (*c*, *g*), in store *s* for quarter *t* when that particular color-size combination is stocked out for some period in that store during that particular time. The demand equation (1) is accordingly modified as follows,

$$SALES^{/}_{scgt} \sim COM\!-\!Poisson\left(\lambda^{0}_{scgt},\ \nu\right) \quad \text{where } SALES^{/}_{scgt} = SALES_{scgt} \text{ when there is no stockout} \tag{3}$$

[18] Performed at the beginning of every MCMC cycle, whenever in any particular store *s*, during any time period *t*, if the color-size combination (*c*, *g*) stocks out. Use the current value of λ^{0}_{scgt} and ν and draw a random number from the truncated COM-Poisson truncated at the observed sales $SALES_{scgt}$ such that the draw $SALES^{/}_{scgt} \geq SALES_{scgt}$

3.1.3 The Cross Item Effects of a Stockout

The demand equation in (3) incorporates the own effects of a stockout, i.e. it provides an upward adjustment to the observed sales when there is a stockout for a particular item in a particular store in a particular time period. However, when stockouts occur it is likely that there are cross item effects. For example, the unmet demand for the stocked-out item may spill over to other items, thus increasing their sales.

We incorporate these effects by modifying demand equation (3) as follows,

$$SALES'_{scgt} \sim COM\text{-}Poisson\left(\lambda'_{scgt},\ \nu\right) \tag{4}$$

where,

$$\lambda'_{scgt} = \left\{\prod_{i=1}^{N_c}\prod_{j=1}^{N_g}\left(\delta_{ic}^{color}\,\delta_{jg}^{size}\right)^{ratio_{sijt}}\right\}\lambda_{scgt}^{0} \tag{5}$$

In the above equations (λ'_{scgt}, ν) are the parameters of the COM-Poisson distribution and δ_{ic}^{color}, $i,c=1,2,\ldots,N_c$, are elements of the N_cxN_c cross impact matrix $\boldsymbol{\delta}^{\mathbf{color}}$, where N_c is the number of colors observed in the data (in our case 8 colors). Similarly δ_{jg}^{size} $j,g=1,2,\ldots,N_g$, are elements of the N_gxN_g cross impact matrix $\boldsymbol{\delta}^{\mathbf{size}}$, where N_g is the number of sizes observed in the data (in our case 4 sizes).

The variable $ratio_{sijt}$ is the proportion of weeks that a t-shirt of color i, size j, in store s, during quarter t was stocked out[19]. It is a measure of the 'extent' of stockout of the particular item in the store in that quarter. Thus, if during a time period, a particular color-size combination never stocked out in a particular store then the value of the corresponding *ratio* variable is zero.

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[19] Thus $ratio_{sijt} = \frac{week_{sijt}}{week_t}$, where $week_{sijt}$ is the number of weeks the t-shirt with color i, size j, in store s during quarter t was stocked out, while $week_t$ is the number of weeks in the quarter t.

The expression $\left\{\prod_{i=1}^{N_c}\prod_{j=1}^{N_g}\left(\delta_{ic}^{color}\delta_{jg}^{size}\right)^{ratio_{sijt}}\right\}$ thus specifies the cross item impact on color size combination (c,g) on account of stockouts in any of the other items. Further, this cross item impact is decomposed across the item attributes viz. color and size with the 8x8 matrix $\boldsymbol{\delta}^{\mathbf{color}}$ specifying the parameters for the demand changes on account of the ‘color’ attribute and the 4x4 matrix $\boldsymbol{\delta}^{\mathbf{size}}$ specifying the parameters for the impact on account of the ‘size’ attribute. By definition $\delta_{ic}^{color}=1$ when i=c, similarly $\delta_{jg}^{size}=1$ when j=g. In line with past empirical work in retail stockouts (Anupindi, Dada & Gupta 1998) and for tractability of the estimation we restrict the cross item impact to be positive, i.e. the elements of the two matrices $\boldsymbol{\delta}^{\mathbf{color}}$ and $\boldsymbol{\delta}^{\mathbf{size}}$ (equation 6) are restricted to be ≥ 1.

3.1.4 Category Impact of a Stockout

The parameter λ_{scgt}^{0} in equation 2 (and 5) is a measure of the ‘core’ demand for color c, size g, in store s, for quarter t. This core demand parameter can be modified to reflect the impact of a stockout on the entire category as follows,

$$\log\lambda_{scgt}^{0}=\beta_c^{color}+\beta_g^{size}+\alpha\log SQFT_s+\underbrace{\left[\sum_{i=1}^{N_c}\sum_{j=1}^{N_g}ratio_{sijt}\left(\theta_i^{color}+\theta_j^{size}\right)\right]}_{\text{AAI}_{st}\text{ (Assortment Attractiveness Index)}}+\gamma_{st}\qquad(6)$$

where, as in equation (2), β_c^{color} and β_g^{size} are the intercepts decomposed into color and size effect and the variable $SQFT_s$ is the square footage of store s. As before, the variable $ratio_{sijt}$ is the proportion of weeks that the color/size combination (i,j) was stocked out in store s quarter t, N_c and N_g are the number of colors and sizes (8 and 4 respectively).

The expression in square brackets (in equation 6) is an attribute based measure of the attractiveness of an assortment. This could be due to the 'variety' of the assortment of T-shirts (Herpen & Pieters 2002). We call it the 'Assortment Attractiveness Index' (AAI) for the assortment in store *s* during time *t*. The θ_i^{color} and θ_j^{size} parameters then are the relative weights of various colors and sizes in the AAI. The magnitudes and signs of these coefficients would be indicative of the importance of that particular attribute (color/size) on the demand for the entire category.[20] When there are no stockouts in a category in a time period, the AAI term is equal to zero.

Thus, equations (4), (5) & (6) specify a model where the demand for an item decomposes into a 'core' demand as specified by the parameter λ_{scgt}^0 and the cross item impact as specified by the expression in curly brackets in equation (5). The core demand in turn is influenced by changes in the attractiveness of the assortment [the AAI term in equation (6)] due to stockouts. These effects (the core demand, the cross item impact and the AAI effect at the category level) are further decomposed into color and size effects.

Thus a stockout of an item can impact the sales of another item in two ways; the *first* is through the cross item impact, where unfulfilled demand of the stocked out item spills over to other items; or more generally, where one item's absence affects demand patterns for other individual items. The *second* is through the impact that the stocked out item has on the attractiveness of the whole assortment—a category level effect. The relative impacts of these two phenomena can critically inform a manager's assortment and inventory planning decisions and hence is of substantive interest.

[20] For the purpose of model identification, all coefficients pertaining to the 'Medium' size (g=1) as well as the first quarter (t = 1) are set to 0.

3.1.5 Controlling for Demand Shocks

A potential concern with the model as specified in equations 4 through 6 is that the stockouts patterns across items in the same time period may not be random, i.e., for example a surge in demand may lead to stockouts of multiple items. Thus, stockouts and demand surges may have a high correlation; this phenomenon if not controlled for may lead to erroneous conclusions from the model. We attempt to control for this by allowing the stockouts to be a function of the store/time specific demand shock parameter γ_{st} (equation 6). We do so by allowing a binomial distribution for $week_{scgt}$ (the number of weeks the color-size combination *c*, *g* was out of stock in store *s* during quarter *t*) as follows,

$$week_{scgt} \sim Binomial\left(weeks_t,\ \frac{\exp(\gamma_{st})}{\exp(\gamma_{st}) + \exp(\omega)} \right) \qquad (7)$$

where $weeks_t$[21] is the number of weeks in quarter *t*, γ_{st} is the store/time specific demand shock parameter (equation 6) and ω is a parameter that along with γ_{st} specifies the binomial probability of number of weeks of stockout given $weeks_t$ weeks in the quarter. Thus, equation 6 allows the stockouts themselves to be a function of demand, partly addressing the concern of 'non-random' stockouts. A hierarchy is also allowed on the store/time specific demand shocks γ_{st} as follows,

$$\gamma_{st} \sim Normal\left(\bar{\gamma},\ \tau^2\right) \qquad (8)$$

where $\bar{\gamma}$ and τ^2 are the mean and variance parameters of the normal distribution.

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[21] Recollect that the variable $ratio_{scgt}$ used in equations 6 & 7 is given by $\frac{week_{scgt}}{week_t}$, where $week_{scgt}$ is the number of weeks the t-shirt with color *c*, size *g*, in store *s* during quarter *t* was stocked out, while $weeks_t$ is the number of weeks in the quarter *t*.

3.2 Model Estimation

A Hierarchical Bayesian framework is used to estimate the demand model as specified by equations (4) through (8). The Bayesian specification is completed by assigning appropriate prior distributions on the parameters to be estimated. The appendix lays out the prior distributions used in the analysis. The model is estimated using a MCMC sampling algorithm[22].

4. THE ESTIMATED COEFFICIENTS

The estimation result is posterior distributions for each of the parameters. These are summarized by their posterior means and standard errors. Tables 2-4 in the following text report these estimates.

4.1 Basic Model Parameters

We first report the estimates of the basic model parameters. The posterior mean (s.e.) of ν (equation 4) is 0.4216 (0.0058); a ν less than 1 indicates that the data exhibits a greater degree of dispersion than the Poisson allows. The posterior mean (s.e.) of α (equation 7) is 0.2951 (0.0010); its positive sign reflects that stores that are physically larger have greater sales volumes in this category, a result that could occur due to category sales being correlated with store square footage. The parameters $\bar{\gamma}$ and τ^2 (equation 8) measure the mean and variance (respectively) in the (log of the) demand shocks[23]. The estimated values for these parameters are 0.1299 (0.0150) and 0.2779 (0.0122) respectively. The posterior means of γ_{st} (equation 6) range from -1.17 to 2.44; this suggests that the spike in category sales due to seasonality and demand shocks is substantial.

[22] details of which can be obtained from the authors on request.

[23] recollect that the γ_{st} in equation 7 can be viewed as store specific, time period specific demand shocks that affect all the items in the store

Table 2: Estimates of the 'Core' Demand Parameters (Equation 6)[24]

Color	β^{color}
Black	-3.1473* (0.01798)
Grey	-2.9264* (0.01886)
Jade	-3.4059* (0.01920)
Lt. Blue	-3.3350* (0.01914)
Navy	-2.9541* (0.01831)
Royal	-3.2680* (0.01939)
Tan	-3.3983* (0.01838)
White	-3.2443* (0.01871)

Size	β^{size}
M	0.0000 0.00000
L	0.4063* (0.00919)
XL	0.4083* (0.00908)
XXL	0.0436* (0.00932)

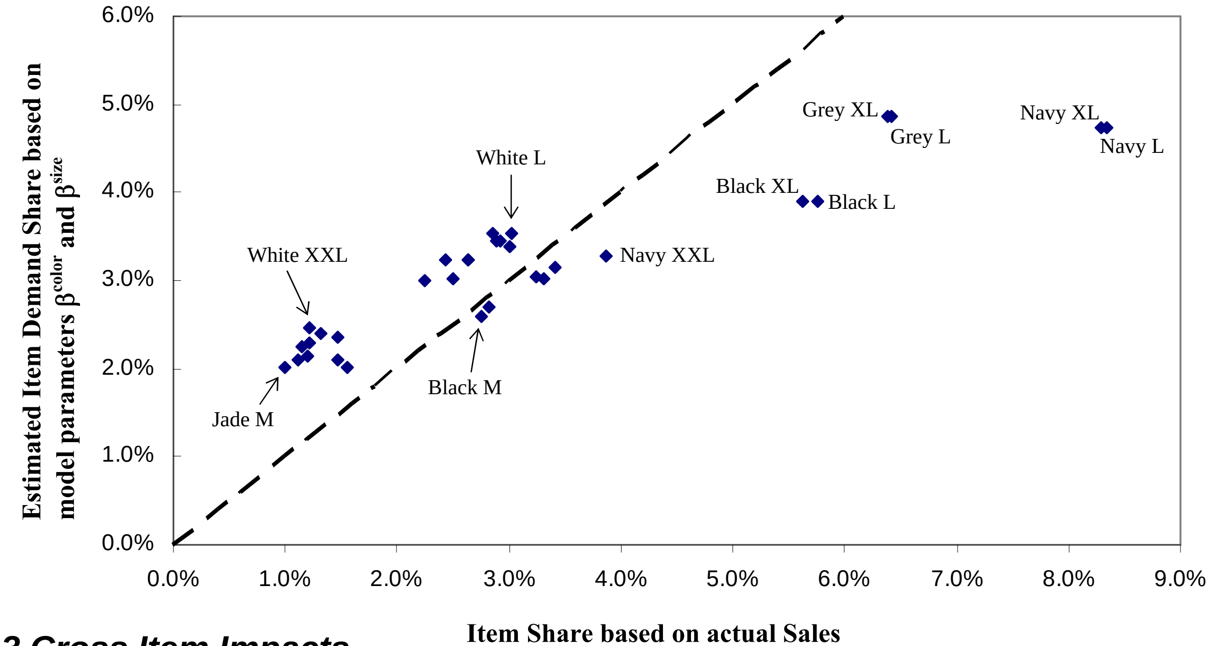
4.2 Core Demand Parameters

In Table 2, we report the estimates of $\boldsymbol{\beta}^{color}$ and $\boldsymbol{\beta}^{size}$ (equation 6). These parameters reflect the relative importance of item attributes. As expected, these estimates correlate with the (log of) item sales reported Table 1a, with correlation of 0.94. In spite of the high correlation, there are some large discrepancies between estimated demand (based on the $\boldsymbol{\beta}^{color}$ & $\boldsymbol{\beta}^{size}$ coefficients) and observed sales which is impacted by stockouts. Figure 2 plots observed sales share plotted versus estimated demand share to illustrate these differences. If observed sales and demand share were comparable, all points would fall on the diagonal (the dashed line). Instead,

[24] The 'significant' estimates have been marked with an asterisk, where estimates are deemed 'significant' when the 95% posterior interval does not contain 0.

some points are far from the diagonal. For instance, *Navy L* & *Navy XL* had a sales share of around 8.5% whereas the estimated demand share for these items is less than 5%.

Figure 2: Item Category Sales Share Vs Item Category Demand Share



4.3 Cross Item Impacts

Tables 3a and 3b contain the parameter estimates which specify the cross item demand impact of one item when another stocks out. This impact is further segregated into *color* and *size* attributes. The interpretation of these tables is as follows: in Table 3a, when the row color stocks out, the parameters in the table reveal the impact on the column colors. For instance, when jade stocks out, the demand for black and for navy both increase. Note that all of the parameters in this table are greater than 1.0 by assumption; the shaded entries are those that reflect "significantly" positive coefficients.[25]

[25] Due to our assumptions, the entire mass of the posterior distribution exceeds 1.0, so a formal test of the hypothesis that a parameter equals 1 is not meaningful. Even so, given the familiarity of classical hypothesis tests based on asymptotic normality, we here report results in this classical format to reflect the precision of the posterior point estimates.

Table 3(a): Estimates of the Cross Item Impact for the *Color* attribute[26]
(The δ_{ic}^{color} parameters in Equation 5)

	Black	Grey	Jade	Lt. Blue	Navy	Royal	Tan	White
Black	1.0000 0.00000	1.1623 (0.11066)	1.1847 (0.14117)	1.1961 (0.15707)	1.0959 (0.07482)	1.2697 (0.16639)	1.1016 (0.08994)	1.2551 (0.15706)
Grey	1.1360 (0.10842)	1.0000 0.00000	1.0988 (0.06449)	1.0860 (0.06132)	1.0397 (0.03184)	1.0548 (0.04220)	1.0629 (0.04972)	1.0666 (0.04837)
Jade	1.1319* (0.04921)	1.0319 (0.02461)	1.0000 0.00000	1.0801 (0.05010)	1.0854* (0.04074)	1.1152 (0.05830)	1.0266 (0.02367)	1.0304 (0.02447)
Lt. Blue	1.0861* (0.03774)	1.0428 (0.02557)	1.0955* (0.04682)	1.0000 0.00000	1.0487 (0.02800)	1.0834 (0.04272)	1.0115 (0.01038)	1.0212 (0.01708)
Navy	1.0031 (0.00287)	1.0949 (0.04910)	1.1273* (0.06828)	1.1093 (0.06782)	1.0000 0.00000	1.2091* (0.08266)	1.1625* (0.05577)	1.1925* (0.08119)
Royal	1.1593* (0.04187)	1.0435 (0.02685)	1.1101* (0.05280)	1.1529 (0.05491)	1.0414 (0.02531)	1.0000 0.00000	1.0129 (0.01125)	1.0213 (0.01714)
Tan	1.0034 (0.00293)	1.1546* (0.03183)	1.1445* (0.04964)	1.1917 (0.05027)	1.0099 (0.00830)	1.1076* (0.04321)	1.0000 0.00000	1.1820* (0.04436)
White	1.2424* (0.04006)	1.0184 (0.01424)	1.0237 (0.01997)	1.0253 (0.02018)	1.0246 (0.01800)	1.0314 (0.02392)	1.0106 (0.01001)	1.0000 0.00000

NOTE: The (*row*, *column*) entry indicates the parameter $\delta_{row,column}^{color}$. The cell entry is impact on the *column* color when the *row* color stocks out.

Table 3(b): Estimates of the Cross Item Impact for the *Size* attribute[27]
(The δ_{jg}^{size} parameters in Equation 5)

	M	L	XL	XXL
M	1.0000 0.00000	1.0134 (0.00929)	1.0019 (0.00177)	1.0034 (0.00307)
L	1.0590* (0.02276)	1.0000 0.00000	1.0028 (0.00272)	1.0079 (0.00675)
XL	1.0067 (0.00583)	1.0042 (0.00400)	1.0000 0.00000	1.0193 (0.01288)
XXL	1.0038 (0.00375)	1.0028 (0.00263)	1.0081 (0.00673)	1.0000 0.00000

NOTE: The (*row*, *column*) entry indicates the parameter $\delta_{row,column}^{size}$ for the impact on the *column* size when the *row* size stocks out.

[26] For visual clarity, the 'significant' estimates have also been shaded, where we define estimates as 'significant' when the posterior mean is more than two standard deviation above 1.0
[27] For visual clarity, the 'large' estimates have also been shaded, where we define estimates as 'large' when the posterior mean is more than two standard errors above 1.0

As one would expect, the magnitudes of cross item impacts between sizes are smaller in general than those between colors, since body sizes are inflexible in the short term. The only "significant" parameter is that which indicates that sales of *medium* items increase when *large* items stock out, although the effect size is very small. Mass produced clothes are not exact in terms of their fit, probably by design so there is some flexibility in accommodating different body sizes. We would expect some substitution around adjacent sizes (M to L and vice versa) but little across non-adjacent ones (example M to XL).

As for cross color demand impacts, Table 3a reveals that not all colors are equal. *Jade* and *navy* are in some ways symmetric in that when *jade* stocks out; demand for *navy* increases (so does the demand for *black*). When *navy* stocks out, demand for *jade* increases (so does the demand for *royal*, *tan* and *white)*. The colors *black* and *grey* are notable in that they have no significant impact on any single color. The colors *tan* (*navy*) are notable in that their stockouts affect five (four) of the remaining seven colors. The colors *navy* and *tan* are also notable in that they are only affected by the stockout of one color. *Navy* demand is affected by *jade*, and *tan* demand is affected by *navy*. At the other extreme, *black* and *jade* are affected by the stockouts of four of the remaining colors.

4.4 Assortment Attractiveness Index Parameters

Table 4 presents the estimates of the $\boldsymbol{\theta}^{\text{color}}$ and $\boldsymbol{\theta}^{\text{size}}$, showing how out of stock items affect demand for the entire assortment. Almost all of these coefficients are significantly negative, showing that stockouts of items with these attributes depress demand for the entire assortment. *Navy's* coefficient is much larger than the others in magnitude, more than twice the magnitude of the next tier consisting of *grey* and t*an*. *Navy's* presence in the assortment is clearly important for sales of the rest of the items. In spite of its low sales share, *tan's* absence is also deleterious to sales of remaining colors. In contrast, stockouts of *lt. blue* (*medium* and *large*) do not harm but actually boost the sales of the assortment, for these coefficients are positive.

Table 4: Estimates of the Category Assortment Attractiveness Index (AAI) Parameters (Equation 6)[28]

Color	θ^{color}
Black	-0.3240* (0.05941)
Grey	-0.4209* (0.06214)
Jade	-0.0754 (0.04953)
Lt. Blue	0.0952* (0.04506)
Navy	-1.0844* (0.05091)
Royal	-0.2570* (0.04893)
Tan	-0.4867* (0.03627)
White	0.0079 (0.03449)

Size	θ^{size}
M	0.0000 0.00000
L	-0.0015 (0.04055)
XL	-0.2541* (0.03258)
XXL	-0.0583* (0.02667)

4.5 Discussion

We propose a model that mines natural variation in store assortments due to out-of-stocks. This natural variation in assortment allows us to study each item's role in the consumer demand for the category. Our model decomposes the impact of an out-of-stock item into three components – lost sales, cross item impact, and category demand impact.

Extant empirical research on stocked out items explores the balance of lost sales and transferred demand and the bias of using sales as a proxy for demand (Anupindi et al 1998). We

[28] The 'significant' estimates have been marked with an asterisk, where estimates are deemed 'significant' when the 95% posterior interval does not contain 0.

also document discrepancies between observed sales and demand, discrepancies which are important to retailers. One key problem with equivocating sales and demand is the self-perpetuation of errors. Consider an inventory analyst who allocates inventory based on sales. She would look at last year's sales (not demand) and allocate inventory based on past sales performance. This makes the best sellers from last year even more available in the current year and the weak sellers from last year (probably due to stockouts) even less available in the current year. This action by the analyst will create a big difference in the core demand parameters and the net impact numbers for the weaker sellers because these items would be more susceptible to stocking out. We see in our scatter plot (Figure 2) a result consistent with this story. The top 2-3 items in terms of sales are also the top 2-3 in terms of being off the diagonal. Confounding due to analyst action is a possible explanation for this observed result.

Extant studies have found a non-trivial relationship between individual items, that stockouts of individual items have significantly increased demand for certain remaining in-stock items (Anupindi et al 1998). Our results confirm that the cross effects are asymmetric, e.g. that *navy*'s effect on tan is much greater than *tan*'s effect on *navy* (Table 3a). One reason for asymmetry would be that the cross effects are correlated to share, in that stockouts of higher selling items would have proportionally greater impacts on sales of remaining items. On average, our results do support the theory that cross effects are correlated to share, as we show in the next subsection (where we provide a simulation exercise). However, the cross effects are not proportional to share, e.g. *Navy XXL* has a much larger impact than its share would suggest.

In addition to measuring these aspects of the impact of out of stocks, we document that the absence of individual items adversely impacts demand for the entire category. Why would the absence of individual items affect demand for the entire category? First, each individual item might contribute to the variety within the assortment (Hoch, Bradlow, and Wansink 1999), and a loss of variety would reduce the desirability of the assortment for consumers with uncertain preferences (Koopmans 1964, Reibstein, Youngblood and Fromkin 1975, Kreps 1979, Kahn and Lehmann 1991) and for those who tend to seek variety (Berlyne 1960, Helson 1964, McAlister and Pessemier 1982, Kahn 1995). Second, retailers work to maintain aesthetically pleasing displays of categories in order to attract consumers; out of stocks may erode the aesthetic value of the category presentation, which is the so called "broken assortment" effect (Smith and Achabal 1997). Both of these reasons support the conclusion that out of stocks would lessen category demand. There is also a theoretical foundation for a result in which some out of stocks would increase category demand. Reasons why purchase amounts would increase after deletion of non-favorite items include the elimination of clutter (Boatwright and Nunes 2001, Borle et al 2005). In addition, Iyengar and Lepper (2000) demonstrated that choosing among too many items is de-motivating and can reduce sales.

The assortment literature has found that grocery retailers carry excess assortment, in that multiple typically low share items can be eliminated from an assortment with no loss in or even an increase in sales (Food Marketing Institute 1993; Boatwright and Nunes 2001, 2004; Drèze, Hoch, and Purk 1994; Broniarczyk, Hoyer, and McAlister, 1998). None of these studies of assortment has comprehensively evaluated every item in an assortment – they examined the impact of a loss of a subset of the items. Our analysis of the category details each and every item's individual role in the category. In the category that we examined, almost every item in the category is important to category demand, in that an out of stock of practically any single item would reduce category sales[29]. This suggests a somewhat different perspective, that

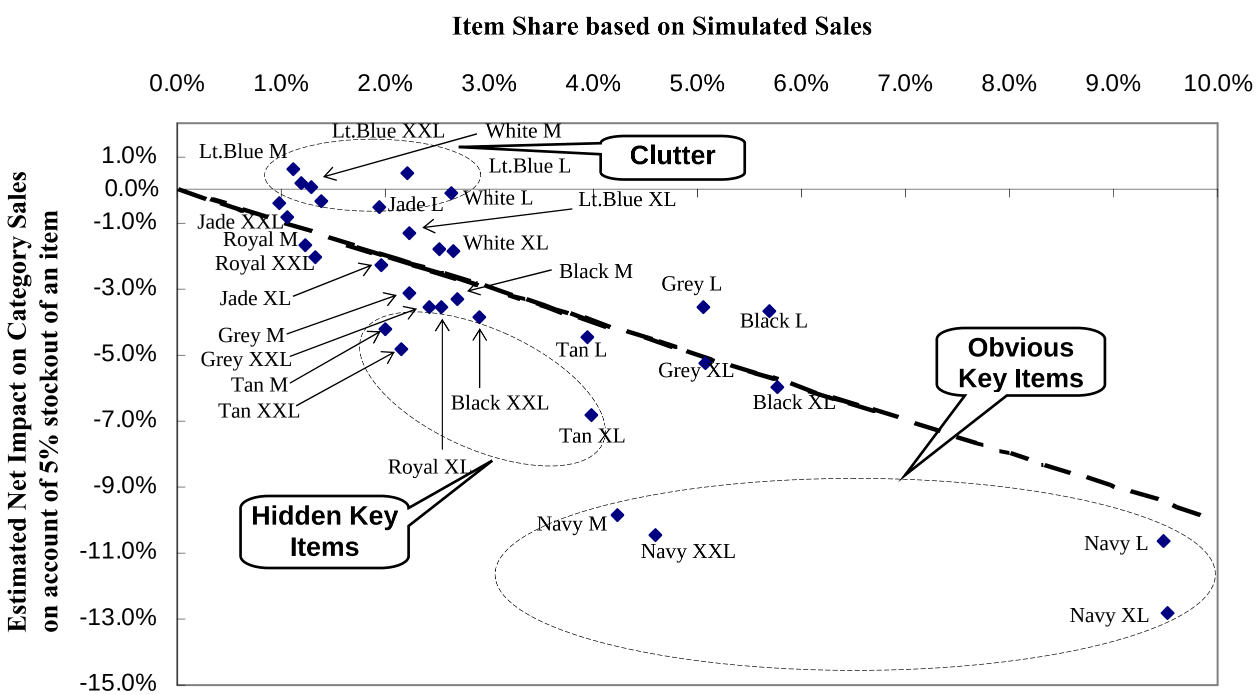
[29] Except for two items, *lt. blue medium* and *lt. blue large*

providing variety with a broad assortment is indeed important in apparel retailing. However, this is simply the price of entry. Consumers concentrate their purchases on a few items.

5. Deconstructing the Net Impact of Stockout: A Simulation Analysis

To assess the magnitudes of the various effects of out of stock items, we conducted a simulation analysis. In the simulation, we assumed that an individual item stocks out at the level of 5%[30] and then used our model to estimate the own sales loss, substitution and category sales impact. Figure 3 shows the results.

Figure 3: Item Share Vs Net Stockout Impact on Category Sales

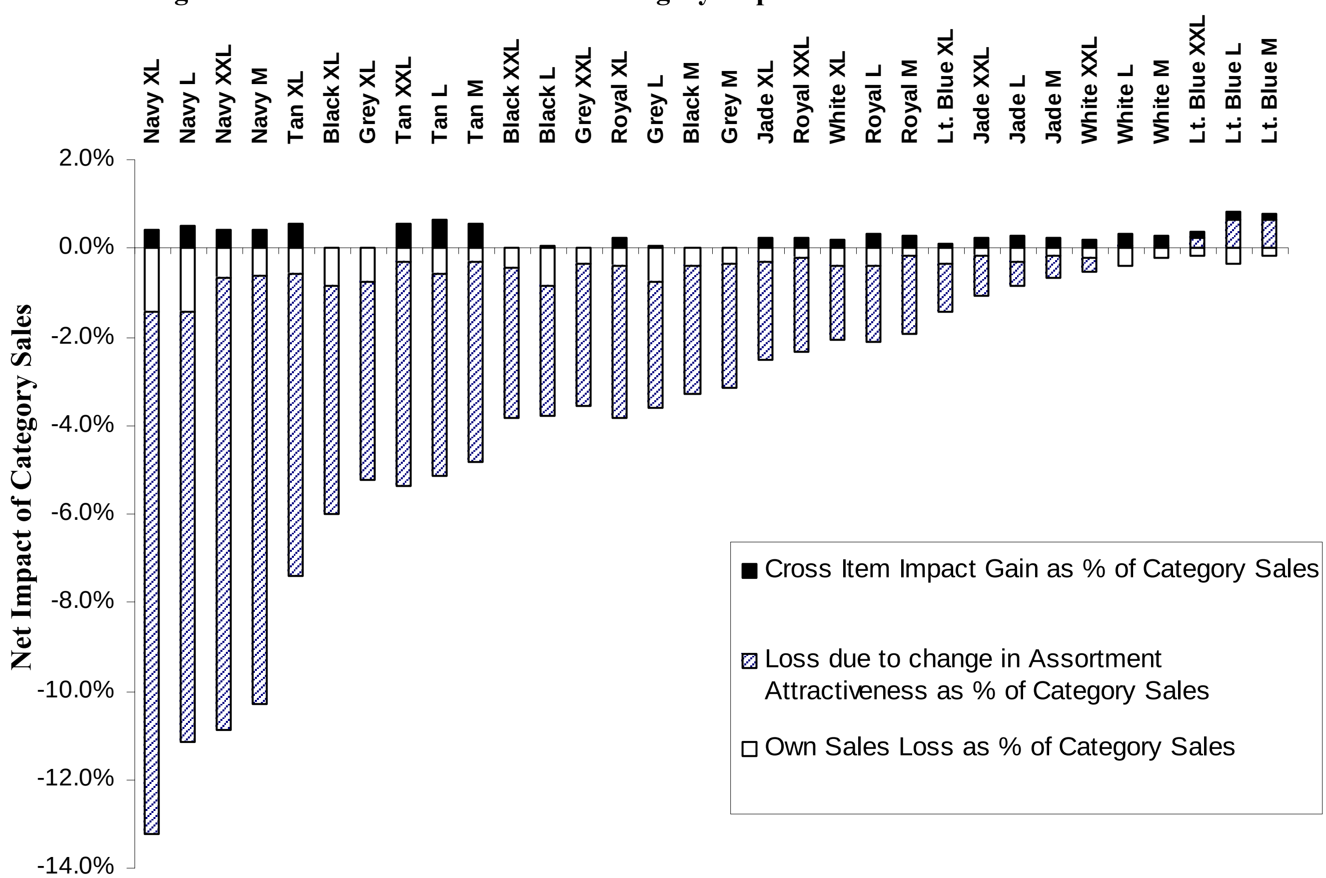


The horizontal axis in Figure 3 is the percent share of an item in the simulated category sales (if there were no stockouts at all) and the vertical axis is the net impact of a 5% stockout of an item on the entire category sales. Taking the example of a particular item say *Navy XL*, in the

[30] That is, it stocks out during 5% weeks in a quarter. In other words the variable $ratio_{scgt}$ in equation 5, 6 is set at 0.05 for all stores, all time periods for that particular item.

simulation we assume that *Navy XL* stocked out 5% (and no other item stocked out), and we use our model to estimate the demand of all items in the category under this condition. So the "◆" corresponding to *Navy XL* shows on the horizontal axis the share of *Navy XL* of category sales (if there are no stockouts) and on the vertical axis the corresponding figure is the net impact of a 5% stockout of *Navy XL* on the entire category sales. *Navy XL*'s own sales account for around 9.5% of total category sales (horizontal axis of Figure 3), but a 5% stockout of *navy XL* (the assumption of the simulation) would cause category sales to decrease by nearly 12.8% (vertical axis of Figure 3).

Figure 4: Deconstruction of the Net Category Impact



This same 12.8% is further deconstructed in the first column of Figure 4. The own sales loss as a percent of the quarter's category sales is 1.4%, the impact on category sales on account of a change in the “assortment attractiveness” (the AAI index) is a negative 11.8%, and the gain due to substitution to other items, the uppermost portion of the column, is about 0.4% of the total category sales. If this last portion of the bar graph, the substitution gain, is subtracted from the other components, the total loss in category sales ends up 12.8%.

Now that we have defined Figures 3 and 4 using *Navy XL* as an example, we next consider the results of all items. On average, the absence of the greatest selling items reduces category sales the most. Figure 3 shows this with a positive correlation between the SKU share of category sales and the net impact on category sales of the stockout of an individual item. *Navy*, in all sizes, is one of the best sellers and has the greatest impact on category sales. We refer to these types of items as ‘obvious key items’. However, the correlation is far from perfect. Note that the *L* size of *Black* and *Grey* have far less impact on category sales than *Navy* in spite of their large share of category sales

Figure 3 also shows that when an individual item stocks out, the loss in sales in the total category often greatly exceeds the sales of the individual item. The diagonal line in the figure shows the point at which the individual item impacts equal to their share of sales. Note that most of the points in the plot are below this diagonal line. This result reveals that almost every single item in this category is integral to the success of the category; that consumers do care about the variety within this assortment, supporting retailers’ insistence on large assortments even when some individual items exhibit quite low sales. This result, that practically every item is integral to the success of the category, is the opposite of results in the extant literature on assortment reductions (Drèze, Hoch, and Purk 1994, Broniarczyk, Hoyer, and McAlister 1998; Boatwright and Nunes 2001, 2004), where even large scale cuts in product assortments have at most a modest impact on category sales. We point out that while the extant literature has focused on grocery items, where categories tend to be frequently purchased and non-discretionary, our study uses an infrequently purchased discretionary apparel category. Our speculation about purchase frequency is consistent with the findings in Borle et al (2005), that more frequently purchased categories are less adversely affected by reductions in assortment.

Figure 4 deconstructs the impact of an individual SKU on category sales into individual components: the loss in own sales, the aggregate gain in sales due to individual cross item effects, and the effect of assortment attractiveness. In general, the loss due to own sales is a trivial portion of the total decline in category sales, in that the middle portions of the bars in Figure 4 are all trivial relative to the total bar heights. Similarly, the cross item effects are also quite small. By far the largest portion of total category loss is due to the change in assortment attractiveness. As an example, the impact of a 5% stockout of *Navy XL* on the assortment attractiveness contributes a 12.8% loss in category sales.

Note also that assortment appears to gain attractiveness when certain items are out of stock. The increased attractiveness actually leads to a gain in category sales. For example, when medium Lt. Blue is out of stock, net category sales increase 0.6%. As noted earlier, this is consistent with the arguments in the literature regarding elimination of clutter.

6. Conclusions

We discussed the conventional wisdom that the presence of some key items in an assortment is critical to sales of that assortment or category. We empirically document this direct impact of individual items on category sales by examining natural variation in the assortment due to stockouts. Our empirical study validates the conventional wisdom of the importance of key items in an assortment and confirms distinct roles of key items in assortments.

To put it another way, we confirm a schizophrenia in retailing, that retailers need assortment to sell the category, but after the fact only a few items are major sellers.

Our results show that best sellers are not necessarily key items. More importantly, we found that key items impact assortment sales in more than one way. Certain items, like *Black* and *Jade* in our data, soak up a portion of sales from out of stock items, sales that might otherwise be lost. Hence it is important that they are in stock. Other items such as *Navy XL* (high sales share) and *Tan M* (low sales shares), should also remain in-stock for a different reason. Their absence has a deleterious effect on sales of all the remaining items. One avenue for future research would be to understand why assortment sales are affected by the absence of key individual items.

We have made some assumptions in framing our model. Note that equation 6 accounts for store heterogeneity only through the variation in square footage of stores, while store sales may vary due to many other factors. We have also evaluated a model that allowed for unobserved store level heterogeneity, finding the results reported here to be robust with respect to store heterogeneity.

We also chose to use a highly flexible model with respect to cross item impact. An alternative approach would be to calculate lost sales of an individual stocked out item and assess what portion went to substitutes and what portion were truly lost. This framework assumes lost sales are less than own sales, while we specifically sought to allow disproportionate effects.

Finally we note the importance of the use of simultaneous equations to assess the effect of stocked out items. At the same time that individual item stockouts reduce sales of those items, the stockouts themselves occur in periods of high sales (unexpected demand shocks). If one were to ignore equation 7, the estimates of the thetas in an equation similar to 6 would be inflated to account for the positive correlation of category sales and stockouts. (The naïve interpretation of such a model would be that stockouts increase sales.)

While not all retailers retain information on out-of-stocks in their data warehouses, our study shows that stockouts provide an empirical context in which to study issues concerning product assortments. Even when average out-of-stocks are quite low, variation in out of stocks can provide the richness needed for empirical research (see Table 1b to see these statistics for our data.)

As one managerial implication of these results, consider the calculation of levels of inventory that stores must retain to avoid out of stocks. Typically the inventory allocations are calculated for each item independently of the remaining items (Fisher, Rajaram and Raman, 2001). At best, such calculations take cross item effects into account (Smith and Agrawal, 2000) with specific restrictions such as switching proportional to share. Our results show, that at least for the category studied here, the impact of key items goes beyond own sales and cross effects on other items and has large implications for the sales of the whole category.

7. **Appendix**

We use a hierarchical Bayes approach using an MCMC sampler (Casella and George, 1992; Gelfand and Smith, 1990) to estimate the model specified by equations 4 through 8. The full conditional distributions used in the estimation can be obtained from the authors on request; the following appendix lays out the prior parameter specifications associated with the model.

A1 Prior Specifications

Table A1: The priors used in the estimation

Parameter	*Priors*
ν	Gamma(2,1)
δ_{ic}^{color}, δ_{jg}^{size} $i,c = 1 \ldots N_c$, $j,g = 1 \ldots N_g$	1.0 + Gamma(1.5,0.1)
β_c^{color}, β_g^{size} $c = 1 \ldots N_c$, $g = 1 \ldots N_g$	Normal(0,100)
α	Normal(0,100)
θ_c^{color}, θ_g^{size} $c = 1 \ldots N_c$, $g = 1 \ldots N_g$	Normal(0,100)
ω	Normal(0,100)
$\bar{\gamma}$	Normal(0,100)
τ^2	Inverse gamma(2.5,2.5)

Table A1 provides the priors used in the estimation. ν is the decay parameter of the COM-Poisson and is defined over the positive real line, values of $\nu > 1$ indicate under-dispersion and values of $\nu < 1$ indicate over-dispersion relative to the Poisson. Looking at the empirical distribution of quarterly sales one might suspect 'over-dispersion' in the data. However, *a priori*, there is little information on the range of values ν can take; very high values of ν though seem unlikely given the dispersion observed in our data. The prior distribution on ν, a gamma(2,1) with a mode at 1 is a reasonable representation of our belief on the values ν can take.

δ_{ic}^{color}, $i,c=1,2,\ldots,N_c$ (equation 5) are parameters of the N_cxN_c cross impact matrix $\boldsymbol{\delta}^{\mathbf{color}}$, where N_c is the number of colors observed in the data (in our case 8 colors). Similarly δ_{jg}^{size} $j,g=1,2,\ldots,N_g$ (equation 5) are parameters of the N_gxN_g cross impact matrix $\boldsymbol{\delta}^{\mathbf{size}}$, where N_g is the number of sizes observed in the data (in our case 4 sizes). The expression $\left\{\prod_{i=1}^{N_c}\prod_{j=1}^{N_g}\left(\delta_{ic}^{color}\delta_{jg}^{size}\right)^{ratio_{sijt}}\right\}$ specifies the cross item impact on color size combination (c,g), on account of stockouts in any of the other items. It multiplicatively impacts the 'core demand' parameter λ_{scgt}^{0} (equation 5). We restrict the cross item impact to be positive, i.e. the parameters δ_{ic}^{color} and δ_{jg}^{size} are restricted to be ≥ 1. Accordingly, we put a prior of 1.0+Gamma(1.5,0.1) on these parameters, which is consistent with our prior beliefs on the range of possible values that they can take.

The parameter α (equation 6) is the impact of the physical size of the store on demand of various items. A prior of Normal (0,100) allows a wide range of prior values for this parameter. The θ_i^{color} and θ_j^{size} parameters (equation 6) are the relative weights of various colors and sizes in the AAI (Assortment Attractive Index). The magnitudes and signs of these coefficients are indicative of the importance of that particular attribute (color/size) on the demand for the entire category. A priori there is little information on either the sign or magnitudes of these parameters and we feel a prior of Normal(0,100) is consistent with this prior belief.

The ω parameter (equation 7) along with γ_{st} (equation 6, 7) specifies the binomial probability of number of weeks of stockout given $weeks_t$ weeks in the quarter. Again, a relatively 'diffuse' prior of Normal(0,100) is in line with our not so sharp prior beliefs on the range of values for this parameter. Similar considerations were followed in setting the normal prior distribution for $\bar{\gamma}$ and the conjugate inverse gamma prior distribution for τ^2 (equation 8).

8. References

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