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MULTI-VENDOR SOURCING IN A RETAIL SUPPLY CHAIN*

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This paper describes a methodology for managing capacity, inventory, and shipments for an assortment of retail products produced by multiple vendors. The vendors differ in lead times, costs, and production flexibility. Product demand is uncertain and fluctuates over time. We develop an optimization model to choose the production commitments that maximize the retailer's expected gross profit, given demand forecasts and vendors' capacity and flexibility constraints. The model has been incorporated into a decision support system, developed in collaboration with supply chain planners at a global retailer of seasonal and fashion merchandise. The software has been tested by two major retailers.
(SUPPLY CHAIN MANAGEMENT; CAPACITY PLANNING; VENDOR MANAGEMENT; SOURCING STRATEGY; STOCHASTIC PROGRAMMING)

1. Introduction

We consider the problem of how to optimally plan and execute the sourcing of seasonal and fashion private-label merchandise carried by department stores and specialty retailers. For a given selling season, the sourcing decisions, typically made by the retail buyer responsible for each merchandise department, include the following components: (1) purchases of raw materials (e.g., fabric) for use by vendors, (2) supply contracts and production commitments with vendors, (3) a weekly plan for sales, shipments, and inventory, and (4) adjustments based on subsequent market information. The goal of this paper is to develop a formal planning methodology for this decision problem that accommodates multiple products and multiple suppliers, and explicitly accounts for demand uncertainty and adjustments to the plan during the season.

The Business Setting

Many of the challenges of this business setting are due to attributes of the demand patterns and the supply base, and how they interact. Demand in this environment typically fluctuates sharply throughout the year. This is exemplified by the data in Figure 1, which illustrates recent sales for a men's casual slacks product. (Since the retailer providing these data aspires to and usually achieves very high fill rates for this product, the difference between sales and demand is insignificant.)

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This type of demand becomes most challenging when production capacity is constrained, which is commonly the case in this industry. Specifically, demand during the peak Fall ("Back to School") and Christmas seasons typically exceeds available manufacturing capacity, while surplus capacity tends to exist during the Spring and Summer. Producing in advance of peak periods improves the ability to meet demand but creates inventory buildup and requires that commitments to production and fabric purchases be made under greater uncertainty.

Sourcing strategies must also reflect the performance capabilities of the supply base. In most cases there are a variety of possible vendors that differ in costs, lead times, and flexibility of production. Vendors with the lowest cost generally offer virtually no flexibility with respect to capacity commitments. These vendors tend to have long lead times for booking capacity (e.g., nine months) and shipment times of several weeks and often require that the total production be allocated relatively evenly throughout the year. More responsive vendors may have shorter lead times and allow greater flexibility vís-a-vís production commitments. Additionally, different vendors may be willing to store limited amounts of finished product prior to delivery for a fee.

Retailers tend to leverage a portfolio of such vendors, resulting in supply chains such as that shown in Figure 2. This enables such strategies as exploiting lower cost production for the more predictable segment of demand, while sourcing the more speculative segment via the more flexible, but more costly, vendors. Operationalizing this in a multi-product, multi-vendor setting is nontrivial and is further complicated by many production and logistical constraints described later. This was our retail collaborator's motivation in chartering this project. In fact, our methodology is unique in its focus on designing contracts with a portfolio of vendors that simultaneously exploits the comparative advantages of each, as opposed to selecting a single most desirable vendor.

Research Contribution

Relative to previous academic research detailed in Section 2, our formulation of the multi-vendor sourcing problem is novel in representing the complex constraints and changing states of information under which different sourcing commitments must be made. We address numerous issues associated with supply chain design and provide insight into a universal question in sourcing: how to balance unit costs versus non-financial attributes such as flexibility. Further, we provide a framework for combining this tradeoff evaluation with risk analysis concepts and methods from decision analysis, as illustrated in Section 4, which appears to be novel among research addressing this class of problem. Overall, our model builds on the key aspects of the literature described in Section 2, incorporating seasonal

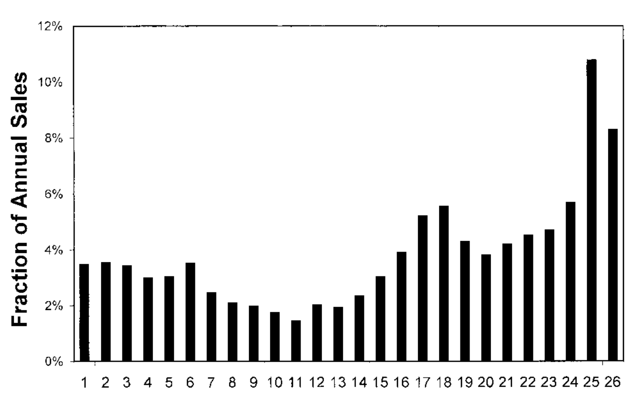

FIGURE 1. Seasonal Patterns in Demand.

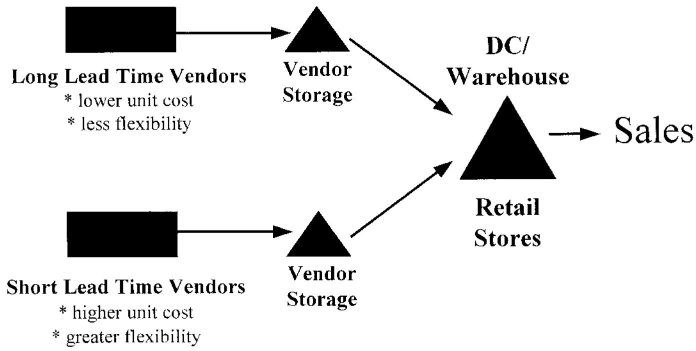

FIGURE 2. Supply Chain Structure.

patterns in demand and detailed production and logistical constraints in a stochastic demand environment with forecast updating. While subsets of these issues have been treated previously, we believe our formulation to be unique in addressing all of them simultaneously.

The formulation described in this paper evolved in close collaboration with retail practitioners, whose involvement occurred at two different levels. A committee of senior executives from different firms regularly reviewed our assumptions and problem framing to ensure the broad applicability of our model to a variety of retail settings. However, the specifics were developed in collaboration with executives and buyers at a particular retailer, who confirmed that our level of detail captures the key complexities faced by retail planners. Their help was especially useful in identifying the cost tradeoffs and constraints most important for sensitivity analysis, leading to variable and constraint modifications that allowed discovery and presentation of the most critical shadow prices. Furthermore, feedback from these buyers and planners was instrumental in the incorporation of our model into a decision support software package with a graphical user interface. Given the depth and breadth of the practitioners' participation, we believe this model to be widely applicable to retail firms that manage the sourcing and production of private label merchandise, and to certain nonretail firms as well.

Organization of This Paper

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 details the mathematical formulation of the optimization model, discussing in depth the assumptions we made to capture the salient features of the particular retail environment. Section 4 outlines an analysis that was conducted in conjunction with our sponsor firm, using representative but disguised data. This will illustrate the types of insights that our framework can deliver. Section 5 summarizes the contributions this research makes to managerial practice, and Section 6 concludes.

2. Literature Review

The use of formal decision models in aggregate production planning has a long tradition and has been the subject of hundreds of academic studies. See Silver and Peterson (1985) for a textbook treatment and some historical background. Nam and Logendran (1992) provide a review of the academic literature, and Buxey (1993, 1995) surveys the usage of such models in practice. The predominant optimization approach is based on linear programming (LP), which allows for non-stationary but deterministic demand, and can handle large numbers of products simultaneously. Forecast uncertainty and information updating are usually dealt with *only in an indirect fashion,* by using a rolling-horizon implementation of a snapshot deterministic solution (the formal term for this is "Open-Loop Feedback Control") (cf. Bertsekas 1976), and also perhaps through the specification of safety stock levels, usually exogenously (e.g., Miller 1979; Gunther 1982; Guerrero, Baker, and Southard 1986; Heath and Jackson 1994).

More direct treatment of demand uncertainty is called for in the retail setting, especially where hard-to-forecast fashion or style goods are involved. This can be provided by newsvendor-style models, but at the expense of sacrificing the dimensionality and detailed constraint structure that can be supported by LP formulations. In this type of approach, the entire selling season for a product is summarized as a small number (possibly one or two) of random variables with known joint probabilities. This allows analytic incorporation of forecast uncertainty into production planning (e.g., Wadsworth 1959; Murray and Silver 1966; Ravindran 1972; Hausman and Peterson 1972; Crowston, Hausman, and Kampe 1973; Hartung 1973; and more recently Fisher and Raman 1996; Eppen and Iyer 1997; Brown and Lee 1998; Donohue 2000), albeit in stylized ways. Various approaches to obtaining the probability distributions of these demand random variables, especially for fashion products, are proposed by Hertz and Schaffir (1960); Riter (1967); Wolfe (1968); Chang and Fyffe (1971), Hausman and Sides (1973), and Riggs (1984).

A classification of the research described above is presented in Table 1. Subsequently, Table 2 provides a more detailed and direct comparison with each of the efforts closest in spirit to our work, as cited in the rightmost column of Table 1.

3. Model Specification

This section outlines the mathematical formulation of the planning problem faced by a retailer leveraging a portfolio of time-phased vendors. Our discussion uses the language of apparel retailing because this is our sponsor firm's primary line of business. However, we believe our underlying methodology to be broadly applicable to other product settings.

TABLE 1

Aggregate Production Planning Approaches for Retail Supply Chain Management, with Representative Examples

	Stochastic Demand	
Deterministic Demand	Small-Dimensioned Problem (few products/constraints/periods)	Large-Dimensioned Problem (many products/constraints/periods)
Classical LP: • Allows for non-stationary demand, and scales well with products, constraints, and periods –Silver and Peterson (1985) –Nam and Logendran (1992) –Buxey (1993, 1995) • Forecast uncertainty and information updating are dealt with only indirectly –Miller (1979) –Guerrero, Baker, and Southard (1986) –Gunther (1992) –Heath and Jackson (1994)	Newsvendor-style Models with Forecast Updating: • Computationally straightforward; structural insights can often be obtained analytically –Wadsworth (1959) –Murray and Silver (1966) –Ravindran (1972) –Hausman and Peterson (1972) –Crowston, Hausman, and –Kampe (1973) –Hartung (1973) –Fisher and Raman (1996) –Eppen and Iyer (1997) –Brown and Lee (1998) –Donohue (2000)	Stochastic Programming: • Computational challenges often require simplifying assumptions, especially regarding representation of forecast updating –Bitran, Haas, and Matsuo (1986) –Eppen, Martin, and Schrage (1989) –Kira, Kusy, and Rakita (1997) Monte Carlo Simulation: • Usually descriptive rather than proscriptive; generality of structural insights is difficult to establish –Nuttle, King, and Hunter (1991) –Hunter, King, and Nuttle (1992, 1996) –King and Hunter (1996)

TABLE 2
Comparison to Most Closely Related Research

	Key Similarities To Our Research	Key Differences
Bitran, Haas, and Matsuo (1986) perform multi-period production planning for families of consumer electronics products, in turn consisting of specific items. Demand occurs in the last period, and estimates of this demand are revised each period. Demands for all items are assumed normal, and the standard deviation of forecast error at each time period is given by an arbitrary, decreasing sequence of numbers that must be provided as data. The updated forecasts at each period also follow a joint normal distribution, with known covariance matrix. The exact problem is a difficult-to-solve, stochastic mixed-integer program, for which the authors develop a deterministic mixed-integer approximation.	• Multi-product planning with forecast updates	• They take production capacity as given, then schedule multi-item production; we consider as decision variables the capacities to be reserved with a variety of vendors at different points in time • They model operations within a single factory at greater detail; our scope spans multiple vendor factories as well as the retailer's distribution center, and includes scheduling of shipments between these • Their representation of demand is more general, but also data-intensive; we pursue a discrete simplification of forecast dynamics to allow exact solution in real time
Eppen, Martin, and Schrage (1989) develop a model to aid in capacity decisions for several lines of cars produced in multiple factories. The sequence of events in each of five years is (1) available capacity is configured (production lines are tooled for specific products), (2) demand occurs, and (3) a production plan is implemented. Demand uncertainty is represented by three "scenarios" for each year, specifying demand and price by product. Scenario probabilities are assigned, assumed independent from year to year. The result is a mixed-integer program that maps out individual sample paths of all possible scenario combinations.	• Scenario approach to modeling demand uncertainty for capacity planning	• Our production decisions are based on imperfect demand signals; theirs assume that all uncertainty has been resolved • Their optimal capacity is selected from a set of predefined possibilities, hence the integer variable structure; ours is chosen from a simplex region defined by constraints that explicitly represent the business relationship between the retailer and each vendor
Kira, Kusy, and Rakita (1997) use a probability structure similar to that of Eppen, Martin, and Schrage (1989), with a single-factory production environment.	• Scenario approach to modeling demand uncertainty in LP context	• Their single-factory production environment is much simpler than ours • Their formulation does not treat capacity planning, nor the nuances of managing a supply chain composed of multiple, independently-managed physical nodes
Nuttle, King, and Hunter (1991) describe software called "The Sourcing Simulator" for planning of apparel sourcing. Its purely descriptive simulation approach allows detailed representation of certain aspects of the setting, especially in the replenishment strategies and consumer behavior. Various studies based on this model (Hunter, King, and Nuttle 1992, 1996; King and Hunter 1996) have validated the importance of the ability to react to improved demand information, a key rationale for the sourcing strategies that we model.	• Similar setting, many similar assumptions • One focus is how vendor replenishment and lead time affect a retailer's performance	• Their scope is confined to a single firm • Their framework assumes single-sourcing (with vendor abstracted as a lead time and reorder frequency), so is silent on how to simultaneously allocate production across a portfolio of time-phased vendors

Timeline of Events and Information Assumptions

Our model addresses the retailer's planning for a specified "selling season." This might correspond, for example, to the Fall season (running from roughly August through January) or the Spring season (February through July). For certain merchandise, some retailers use four or more shorter seasons per year. In some instances a season may be as short as 8 weeks.

In chronological order, the critical time points for the retailer's sequential decision problem are as follows:

$t_0 =$ time at which initial vendor commitments and fabric purchases are made

$t_1 =$ second time at which commitments to vendors are made, for those vendors allowing capacity decisions to be deferred to this time

$t_b =$ beginning of selling season

$t_f =$ end of selling season, when actual demand becomes known.

We assume that our model analysis is performed at some time at or before t_0 for a selling season that spans the horizon (t_b, t_f). The retail planner's information regarding demand evolves continuously over time, shaped by economic forecasts, new fashion and color trends, and observed sales results for similar products. However, for our formulation it is only necessary to define the possible states of information at the specific points in time at which decisions are made.

Our discussions with the retailer's production planning managers indicated that two decision points (at times t_0 and t_1) are adequate for a typical apparel planning decision process. However, the formulation can easily be extended to include more decision points by simply adding more variables to the model.

Evaluation of the expected profit also requires knowledge of the actual demand information at time t_f. To represent the evolving demand information, we define the following random variables:

$X_k \equiv$ a random variable corresponding to the information about the market demand that the retailer observes at time t_k, for $k = 0, 1, f$.

At each time point, X_k has a discrete set of possible values. That is, at time t_f, the actual demand corresponds to one of a discrete set of demand scenarios. We define the following probability distributions to describe the likelihood of observing particular sequences of demand information

$p(\xi_1) \equiv P\{X_1 = \xi_1\}$ for each possible ξ_1 value at time t_1

$p(\xi_f|\xi_1) \equiv P\{X_f = \xi_f|X_1 = \xi_1\}$ for each possible combination of ξ_1 and ξ_f, and

$p(\xi_1, \xi_f) \equiv p(\xi_f|\xi_1)p(\xi_1) =$ the joint probability distribution of X_1 and X_f.

Clearly, this structure can be generalized to characterize information that is revealed in any number of stages, but we will describe only the two-stage case since that corresponds to our particular application.

We treat the final state of information ξ_f as a demand scenario that has a discrete number of possible values. Market "scenarios" are frequently used by retailers in developing marketing plans for alternative contingencies. We model demand uncertainty through discrete scenarios for three reasons. The first reason is analytical tractability. Modeling uncertainty using continuous random variables would obstruct the inclusion of the complexities categorically declared by our corporate collaborators to be essential attributes of their business setting. The second reason is consistency with common managerial practice. Our corporate collaborators indicated that their planning methodology often requires the articulation of "worst case," "most likely," and "best case" scenarios for market uncertainties. However, in the past these scenarios have typically been used only for financial planning, due to a lack of technical know-how for how to translate these into contingency plans for vendor and production management. The third reason is that there is an established precedent in the extant literature for using scenarios to model uncertainty in a variety of contexts. As described in Table 2, Eppen, Martin, and Schrage (1989) and Kira, Kusy, and Rakita (1997) used a scenario approach similar to ours for capacity planning. Smith, Agrawal, and McIntyre (1998) used discrete demand scenarios to obtain optimal inventory and promotional plans for retail chains. Of course, there is a rich tradition in the financial economics literature of modeling uncertainty in the prices of stocks and securities this way (cf. Cox and Rubinstein 1985). More recently, Huchzermeier and Cohen (1996) have used discrete scenarios to study the operations management implications of exchange rate fluctuations.

We extend this scenario concept to include market demand information that is revealed in stages, resulting in the sequential stochastic decision model illustrated in Figure 3. The underlying assumption is that as the selling season gets closer, the sales estimates in the plan will improve for several reasons. For example, there is new sales information for related products. Also, updated sales estimates are at least in part the result of revisions in the merchandise plan, e.g., deciding to feature more or less of a particular type of merchandise, giving it a more or less prominent display and floor space, etc. For changes of this type, there is a good base of experience for the buyers to update their subjective estimates of the demand. This determines the conditional probabilities $p(\xi_f | \xi_1)$. With some assistance from the authors, the retail planners at two major retailers were able to subjectively estimate the required probabilities.

In principle the same method can use early sales results to update the demand probabilities (after the selling season's beginning) and in fact, our formulation approach is compatible with Bayesian updating of the probabilities of the discrete demand levels based on early sales results. However, for this application the vendor deadlines did not permit changes in capacity commitments after the start of the selling season, beyond which only changes at the color, style, or size level are allowed. Since our model is meant to support capacity planning at an aggregate level, this is appropriate for our application. Note also that most papers that consider updating of forecasts in a model of reasonably realistic detail only consider updating prior to the occurrence of any sales. This includes the mathematical-programming–based papers most similar to ours, as described in Table 2. Those models that do accommodate forecast updating based on in-season sales tend to have very simplified inventory analysis that would not scale to the constraint and decision variable complexity in our decision model (e.g., Murray and Silver 1966; Chang and Fyffe 1971; Crowston, Hausman, and Kampe 1973; Hartung 1973; Fisher and Raman 1996).

Decision Variable Definitions

The following indices will be used for variable definitions: j for products, i for vendors, t for the time increment used for production, shipment, and sales decisions (typically in weeks), and q for the time increment used for reservation of capacity (typically in quarters). In this paper, we assume that a product refers to an aggregation of styles, not to an individual SKU (distinguished by style/size/color). Variable names in upper case represent decision variables, while those in lower case or Greek symbols are fixed parameters.

The main basis for classifying vendors (into "short lead time" and "long lead time" types) is the time at which commitments for each product must be made. This is denoted by

$\tau_{ij} \equiv$ time at which a commitment is required by vendor i for product j,

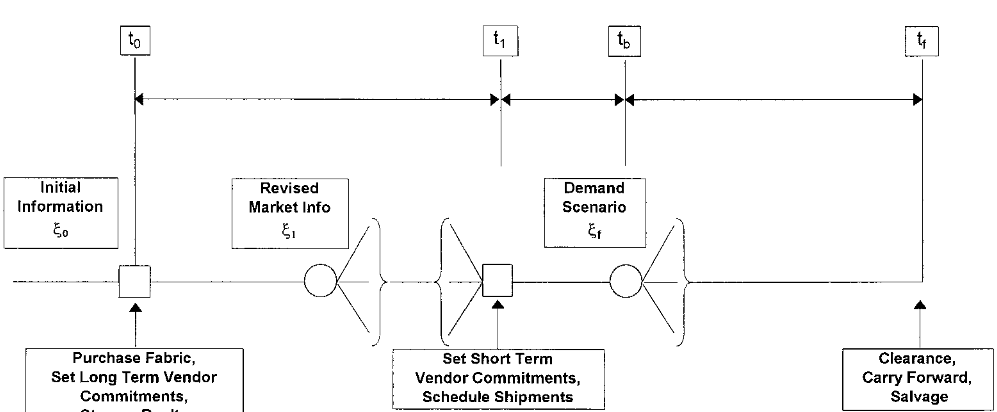

FIGURE 3. Decision Tree for Production Planning.

and the corresponding state of retailer information is

$X_{ij} \equiv$ demand information available at time τ_{ij}, which takes on discrete values ξ_{ij}.

For our implementation, $\tau_{ij} = t_0$ or t_1, since these are the only production commitment time points. It follows that X_{ij} is either X_0 or X_1 for every combination of i and j.

For each possible (ξ_1, ξ_f) combination, the production and inventory variables are defined as follows:

$F_j \equiv$ fabric commitment (in yards) made at time t_0 for product j
$P_{ij}(t|\xi_{ij}) \equiv$ production at vendor i for product j during period t
$Z_i(q|\xi_1) \equiv$ total production by vendor i during quarter q
$Z_j^F(\xi_1) \equiv$ yards of fabric actually used for product j
$M_{ij}(t|\xi_1) \equiv$ beginning inventory of product j stored by vendor i in period t
$S_{ij}(t|\xi_1) \equiv$ shipment quantity of product j from vendor i in period t
$U_j(t|\xi_1, \xi_f) \equiv$ retailer's unit sales of product j in period t
$I_j(t|\xi_1, \xi_f) \equiv$ retailer's beginning inventory of product j in period t

The decision variables depend on the information states in different ways, depending on what information is known when the variable's value is specified. These dependencies determine the dimensionality of the optimization model and variables. We denote this dependence explicitly in our formulation, using the "|" notation. For example, since the production schedule of an item i at a vendor j is fixed at time τ_{ij}, when the state of information is ξ_{ij}, the corresponding vendor production variables are denoted as $P_{ij}(t|\xi_{ij})$. The total production and total fabric usage depend on ξ_1 because they are defined for both short and long lead time production decisions. Similarly, the vendors' inventory and shipment decisions depend on ξ_1 because that is the information available to the vendor when the shipping decisions are made. However, the realized unit sales, and consequently the retailer's on hand inventory, depend on both ξ_1 and ξ_f. This is because the on-hand inventory depends on both the actual demand scenario and all the production decisions, some of which depend on ξ_1. Since the unit sales are affected by the inventory level, this depends on ξ_1 and ξ_f as well. In the optimization program, the information states ξ_1 and ξ_f, are simply treated as additional "subscripts" on variables.

Inventory Balance Equations and Production Constraints

The production, inventory, and shipping variables are related to each other by the following inventory balance equations for the retailer and vendors:

$$I_j(t+1|\xi_1, \xi_f) = I_j(t|\xi_1, \xi_f) + \sum_{i|t \geq \tau_{ij}+l_i} S_{ij}(t-\ell_i|\xi_1) - U_j(t|\xi_1, \xi_f) \qquad \text{for all } j, \xi_1, \xi_f, t, \tag{1}$$

where ℓ_i is the shipping delay for vendor i,

$$M_{ij}(t+1|\xi_1) = M_{ij}(t|\xi_1) + P_{ij}(t|\xi_{ij}) - S_{ij}(t|\xi_1) \qquad \text{for all } i, j, \xi_1, t. \tag{2}$$

When the states of information "subscripts" in one constraint are different for different variables, the variable with fewer subscripts simply keeps the same value for a subset of the equations.

For simplicity, our model considers only the total inventory in the retailer's system, as opposed to inventory levels in individual stores. Once the merchandise reaches the retailer's distribution center (DC), it is usually distributed to the stores and displayed for sale within two to three days. In order to maximize the productivity per square foot, there is generally little storage space in stores, and all store merchandise is placed on display for sale as quickly as possible. Our assumption implies that inventory is generally balanced across the stores, and is appropriate because inventory is rebalanced during the season by allocating replenishments to the stores that most need additional stock. For some merchandise, transshipments are made from one store to another to balance the inventory, but only if the repackaging and shipping costs can be justified. The significant delays in this type of supply chain therefore arise from production commitment lead times, which are usually several months, and shipping times, which may be several weeks for surface shipments.

Constraints on each vendor's storage space can be represented as

$$\sum_j \nu_j M_{ij}(t|\xi_1) \leq w_i(t) \equiv \text{vendor } i\text{'s maximum storage for period } t, \qquad \text{for all } i, t, \xi_1 \tag{3}$$

where $\nu_j \equiv$ storage space required per unit of product j.

A retailer may also specify an upper bound on the amount of inventory contained within its system. This can be specified by

$$\sum_j \nu_j I_j(t|\xi_1, \xi_f) \leq w^R(t) \equiv \text{retailer's maximum storage for period } t, \qquad \text{for all } t, \xi_1, \xi_f \tag{4}$$

This can represent either a physical or budget restriction. In the latter case, ν_j will have a different meaning.

The initial and final inventories may also be required to satisfy constraints of the form:
$I_j(t_b|\xi_1, \xi_f) \geq i_j^0 \equiv$ minimum initial retailer inventory for product j for all ξ_1, ξ_f
$I_j(t_f|\xi_1, \xi_f) \geq i_j^f \equiv$ minimum final retailer inventory for product j for all ξ_1, ξ_f
$M_{ij}(t_b|\xi_1) \geq m_{ij}^0 \equiv$ minimum initial inventory of product j at vendor i for all ξ_1
$M_{ij}(t_f|\xi_1) \leq m_{ij}^f \equiv$ maximum final inventory of product j at vendor i for all ξ_1.

The initial inventory i_j^0 must be sufficient to create an attractive display of merchandise with which to begin the selling season. For continuing, or "basic" products, the minimum final inventory i_j^f may be set to the desired initial inventory for the subsequent season. The vendor's initial inventory m_{ij}^0 can be used to satisfy demand in the current season, while the final inventory m_{ij}^f is available for the subsequent season.

For some aspects of aggregate production planning, managers deem the quarter to be the appropriate increment of time. Using $q(t)$ to denote the quarter corresponding to a time period t, the following relationship tallies the total production of vendor i within a quarter y:

$$\sum_j \sum_{t|q(t)=y} \kappa_j P_{ij}(t|\xi_{ij}) = Z_i(y|\xi_1) \qquad \text{for all } i, y, \xi_{ij} \tag{5}$$

where $\kappa_j \equiv$ production capacity required per unit of product j. This enables us to model quarterly constraints. For instance, to ensure diversification a vendor may be willing to commit only a fraction of its quarterly capacity to a single retailer. On the other hand, less flexible vendors may also insist on a minimum quarterly production commitment from the retailer as a condition for doing business. These can be addressed as follows:

$$\underline{k}_i(q) \leq Z_i(q|\xi_1) \leq \bar{k}_i(q) \qquad \text{for all } i, q, \xi_1 \tag{6}$$

where the bounds do not depend on the demand information. To achieve the economic benefits of level production, certain vendors also permit only limited changes of total production from quarter to quarter, which can be expressed as follows:

$$(1 - \alpha_i) Z_i(q - 1|\xi_1) \leq Z_i(q|\xi_1) \leq (1 + \beta_i) Z_i(q - 1|\xi_1) \qquad \text{for all } i, q, \xi_1 \tag{7}$$

where $0 \leq \alpha_i \leq 1$ and $\beta_i \geq 0$. In general, vendors that allow later commitments also typically allow greater quarter-to-quarter flexibility (larger α_i and β_i parameters).

Production is also constrained by the fabric procurement decision as follows:

$$\sum_i \sum_t \kappa_j^F P_{ij}(t|\xi_{ij}) = Z_j^F(\xi_1) \leq F_j \qquad \text{for all } j, \xi_1 \tag{8}$$

where $\kappa_j^F \equiv$ yards of fabric required per unit of product j.

Modeling Product Demand

The demand pattern for each product over time is an input to the model that is conditional on the demand scenario ξ_f, denoted as follows:

$d_j(t|\xi_f) \equiv$ actual demand for product j in period t.

To specify these values, we used a forecasting model form that has been applied successfully to retail sales forecasting. Econometric marketing studies have found that multiplicatively separable models of the form

$$\begin{pmatrix} \text{Period } t \text{ demand} \\ \text{for product } j \end{pmatrix} = \begin{pmatrix} \text{Total season demand} \\ \text{for product } j \end{pmatrix} \cdot \begin{pmatrix} \text{Seasonality} \\ \text{effect at } t \end{pmatrix} \cdot \begin{pmatrix} \text{Marketing} \\ \text{effects at } t \end{pmatrix}$$

fit observed retail sales data well (Achabal, McIntyre, and Smith 1990; Kalyanam 1996). Thus we let

$$d_j(t|\xi_f) = b_j(\xi_f) \cdot f_j(t) \cdot \rho_j(t) \tag{9}$$

where

$b_j(\xi_f) \equiv$ full-season demand for product j under demand scenario ξ_f

$f_j(t) \equiv$ fraction of total demand for product j that occurs in period t

$\rho_j(t) \equiv$ marketing effects for product j during period t, including price/advertising effects.

This approach greatly reduces the model dimensionality by confining the effect of information updating to the full-season demand, which is a scalar. The full set of relative seasonality factors $f_j(t)$, such as shown in Figure 1, generally do not require updating. Similar representations of demand have been used by Chang and Fyffe (1971), Crowston, Hausman, and Kampe (1973), and Hartung (1973). The specification of demand parameters and price variations due to any retail promotional strategies is exogenous to the optimization model, and hence does not affect the linearity structure.

Calculating Unit Sales

Units sales volume in period t is bounded by the period's demand, so

$$U_j(t|\xi_1, \xi_f) \leq d_j(t|\xi_f) \qquad \text{for all } j, t, \xi_1, \xi_f. \tag{10}$$

While traditional inventory models assume that lost sales occur only when inventory is fully exhausted, in retail marketing environments the *amount* of on-hand inventory can influence sales. For apparel, for example, sales rates can deteriorate as inventory drops because the remaining inventory consists of increasingly broken assortments with incomplete selections of sizes and colors (Smith and Achabal 1998). Low inventory also increases the likelihood that some stores are inadequately stocked, i.e., the inventory is not "balanced." While the relationship between inventory level and sales is not necessarily linear (Smith and Achabal 1998), a linear approximation is reasonable within the range of values of the inventory level that is expected in practice, and it lends considerable analytical tractability to our formulation. Therefore, we allow unit sales to depend upon the beginning inventory according to the following constraints:

$$U_j(t|\xi_1, \xi_f) \leq \eta_j I_j(t|\xi_1, \xi_f) \qquad \text{for all } j, t, \xi_1, \xi_f \tag{11}$$

where $\eta_j \equiv$ maximum fraction of the beginning inventory that can be sold in one period. Because of (1) and the production capacity constraints in (6) and (7), it is also possible that neither (10) or (11) will be binding for a given t.

Constraints (10) and (11) assume that the unfilled demand is lost (to competitors, for example), which is much more common than backordering for most retail merchandise. Backordering, which is actually more straightforward to model, can easily be accommodated within our formulation by modifying the inventory balance equations.

The Objective Function

The objective function will be defined in terms of the following economic parameters:

$\pi_j(t) \equiv$ average selling price for product j in period t

$c_{ij} \equiv$ unit procurement + shipping cost ("landed cost") for product j purchased from vendor i

$r_j \equiv$ residual value per unit of product j at the end of the selling season

$c_j^F \equiv$ cost per yard of fabric for product j

$r_j^F \equiv$ residual value per yard of fabric for product j at the end of the selling season

$h_j \equiv$ retailer's unit holding cost per period for product j

$v_{ij} \equiv$ vendor i's unit storage charge per period for product j

The average selling price $\pi_j(t)$ may vary by time period to allow periodic price markdowns during the season. The value of r_j has different interpretations for basic and fashion items. For a basic item, it corresponds to the unit value of this product in the next selling season (i.e., the avoided replacement cost minus any holding cost). For fashion items it describes a "salvage value." At the season's end, any remaining fashion items may be sold through outlet stores or in bulk to discounters, resulting in markdowns to prices possibly below the original cost.

The expected revenue and cost for each product, denoted as R_j and C_j, respectively, are as follows:

$$R_j = \sum_{t,\xi_1,\xi_f} p(\xi_1, \xi_f)\{\pi_j(t|\xi_f)U_j(t|\xi_1, \xi_f) + r_j I_j(t_f|\xi_1, \xi_f) + r_j^F[F_j - Z_j^F(\xi_1)]\} \tag{12}$$

$$C_j = \sum_{i,t,\xi_1} p(\xi_1)[c_{ij}P_{ij}(t|\xi_{ij}) + v_{ij}M_{ij}(t|\xi_1)] + \sum_{t,\xi_1,\xi_f} p(\xi_1, \xi_f)h_j I_j(t|\xi_1, \xi_f) + c_j^F F_j \tag{13}$$

where $p(\xi_1, \xi_f)$ and $p(\xi_1)$ are the previously defined joint and marginal probabilities, respectively. The total objective to maximize is then $\Sigma_j \{R_j - C_j\}$. This completes the specification of the model, which is concisely stated in the Appendix for reference.

The choice of fabric commitments, production capacity commitments, and shipping schedules that optimize this objective function correspond to a sequence of decisions under uncertainty, where the demand information changes at each decision point. In general, this can be viewed as a stochastic dynamic programming problem (with linear constraints). Unfortunately, the size of the resulting state space and complexity of the objective make this solution approach impractical. However, as long as the states of information are restricted to a discrete set of values, the equations for R_j and C_j are linear in the decision variables, so that this optimization problem is a linear program. This approach for handling uncertainty within an LP formulation was first suggested by Dantzig (1955). Including decision variables whose values may be chosen after the resolution of the uncertainty leads to what is generally termed as a *stochastic linear program with recourse*. See Hansotia (1980) and Infanger (1994) for discussion of various technical aspects of solving such models and extensive reviews of the literature.

4. Model Application

The optimization model described above was implemented as a PC-based decision support system named the *Sourcing Allocation Manager* (SAM). The user interface screens were programmed in Visual Basic, and the optimization engine is LINGO, supplied to us by LINDO Systems. For test problems with four products, four vendors, a 9-month planning horizon, and 27 distinct sample paths of information realizations, the LP has several thousand decision variables and constraints. It can typically be solved on a 300-MHz Pentium II PC in approximately 3–5 minutes. The software implementation and graphical user interface are discussed in detail in Smith, Agrawal, and Tsay (2000).

This section presents as a case study an analysis that was conducted in conjunction with our sponsor firm, using representative but disguised data. The retailer's goal for this analysis was to gain experience with the model and develop an understanding of the key tradeoffs between vendor capabilities and unit costs. While the conclusions obtained are naturally dependent on the particular parameters assumed, they demonstrate the types of managerial insights that our model can provide.

Recall that our model is best suited to analyze sourcing strategies for fashion and seasonal-basic items whose production lead times are large relative to the length of the selling season. However, our model can also be applied to basic products, often managed on a seasonal basis with specific targets for end-season inventory. Treating ending inventory as a constraint allows finite-horizon planning methods to be applied to basic merchandise. For this reason, and also since data were most readily available for a set of three men's basic Tee shirts, merchandise planners at a major retail chain tested SAM to assist in the sourcing of these products. The products were to be sold during a 6-month season in 1999. They consisted of all sizes and colors for three styles called Pocket Tees, V-Neck Tees, and Crew-Neck Tees, which we will refer to as PT, VT, and CT, respectively. To illustrate how the sourcing strategies would differ for fashion or seasonal-basic items, we compared our model's recommendations for representative products (created in consultation with the retailer) to those for the three basic Tee shirts.

Two vendors, called Pacific Supply and Amazon Apparel for this discussion, were candidates to supply these products. The retailer would develop the production and shipping schedules for each vendor and purchase and deliver the necessary fabric. For a quoted cost per unit, the vendors would cut, stitch, package, and ship the merchandise per retailer specifications.

4.1. *Problem Specification*

VENDOR CHARACTERISTICS. The two vendors' characteristics are summarized in Table 3.

Pacific Supply is a small, flexible vendor that is able to respond more quickly to production requests, but charges a higher price. Amazon Apparel is a large capacity vendor that can offer lower prices due to scale economies and efficiencies that are achieved in part by offering little flexibility. Pacific Supply requires a fairly short production capacity commitment lead time (3 months prior to the beginning of the selling season) for PT and CT, but a longer lead time (6 months prior to the beginning of the selling season) for orders of VT. Amazon Apparel

TABLE 3

Vendor Characteristics

	Pacific Supply	Amazon Apparel
Pocket Tee (PT) Unit Cost	$6.39	$5.96
V-Neck Tee (VT) Unit Cost	$6.39	$5.96
Crew-Neck Tee (CT) Unit Cost	$6.66	$5.96
PT Production Lead Time	3 months	6 months
VT Production Lead Time	6 months	6 months
CT Production Lead Time	3 months	6 months
Vendor-to-Retailer Shipment Time	2 weeks	4 weeks
Allowable Production Increase from Quarter to Quarter	50%	5%
Allowable Production Decrease from Quarter to Quarter	50%	5%
Maximum Production per Quarter	40,000	80,000
Minimum Production per Quarter	20,000	20,000
Previously Committed Production	10,000 in Qtrs. 1, 2	0
Storage Capacity (cartons)	1000	1000
Storage Cost (per carton/week)	$2.00	$2.00

requires a capacity commitment lead time of 6 months for all three products. The differences in the vendors' relative flexibility are also reflected in the allowable percentage changes in production from quarter to quarter: Pacific Supply allows each quarter's total commitment to vary within ±50% of the previous quarter's, while Amazon Apparel allows only a ±5% variation.

An interaction among products is created by virtue of their competing for the same limited production capacity. For instance, using Pacific Supply for the production of VT curtails the amount of short lead time capacity available for PT and CT, and vice versa. The optimization model explicitly accounts for such tradeoffs.

PRODUCT CHARACTERISTICS. The sales forecasts generated by the retailer are shown in Figure 4. These indicate that all three products are expected to have seasonal peaks in the Back-to-School and Christmas seasons. Other relevant product information is presented in Table 4.

The average unit selling price for all three products is uniformly \$15.60 per unit throughout the selling season. Although our model is able to handle arbitrary pricing patterns, this retailer's marketing strategy generally deemphasizes price promotions. The unit costs of the products are not shown since they depend on the production decisions. These costs can be computed by adding the fabric cost (approximately \$2.02 per unit) to each vendor's unit costs from Table 4. Since these are continuing products, their residual values at the end of the season are approximations based on the average future replacement cost minus holding cost. The retailer's holding cost per carton is based on a holding cost rate of 15% per year of the retail price, a convention used by this retailer. The initial and final inventories are targets set by the retailer to achieve an attractive presentation in stores carrying these products.

DEMAND UNCERTAINTY. The input screen shown in Figure 5 was used to solicit the retail planners' beliefs about demand uncertainty. The total season volume from their primary forecasts (cf. Figure 4) was reported in the middle column as the "Most Likely" case. The planners then defined what "Low" and "High" demand would mean for each product in terms of a percentage deviation from the "Most Likely" volume. The percentage range input here for a product codifies beliefs about the difficulty of predicting its demand, and therefore the

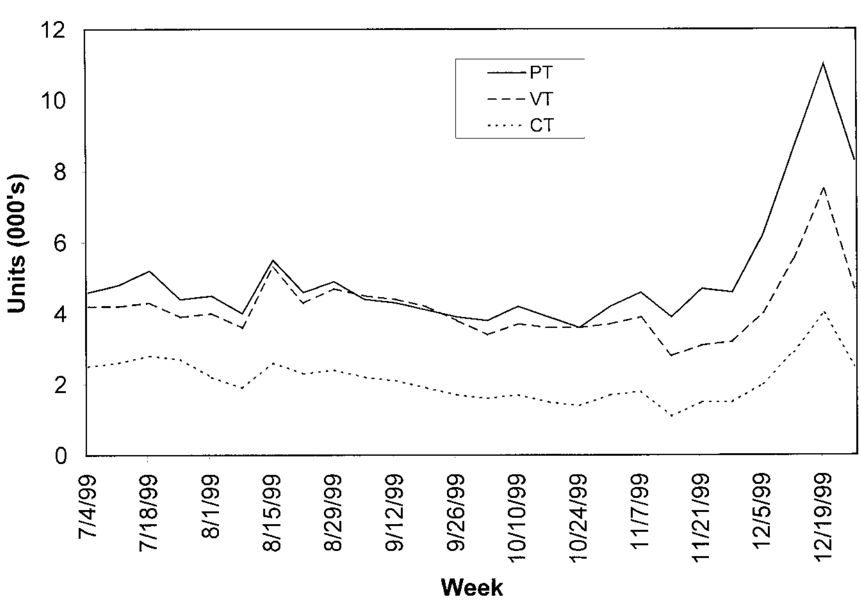

FIGURE 4. "Most Likely" Demand Forecasts for the Selling Season.

TABLE 4
Product Characteristics

	Product		
	PT	VT	CT
Retail Selling Price	$15.60	$15.60	$15.60
Residual Value at End of Season	$7.50	$7.50	$7.50
Units Per Carton	60	60	60
Retailer's Holding Cost (per carton/week)	$2.70	$2.70	$2.70
Initial Inventory Requirement	76,000	54,000	40,000
End of Season Inventory Requirement	72,000	60,000	40,000
Fabric Requirement (yards/unit)	0.72	0.72	0.72
Fabric Cost (per yard)	$2.80	$2.80	$2.80
Residual Value of Fabric (per yard)	$1.00	$1.00	$1.00

extent to which the projections about that product's demand might change between t_0 and t_1. The planners also had to attach relative likelihoods for the occurrence of each state of information. Uncertainty about demand still persists after t_1, leading to the 5 possible final demand realizations shown in Figure 6 (XH, extra high; H, high; M, medium; L, low; XL, extra low). Since the three products are just style variations within the same product class, it was assumed that all would jointly experience the same demand scenario. While not required for our model, this was judged by the retail planners to be a reasonable assumption for these products. This also presents the most challenging case from a capacity planning standpoint.

4.2. *The Optimal Sourcing Plan*

Table 5 summarizes SAM's optimal total-season recommendation for the above inputs. A key attribute of the model is the formal specification of a sequential decision strategy that defines how to react to each demand signal. This was considered by the retail planners to be a substantial improvement over their existing methodologies, which handle contingency planning on a more ad hoc basis. In our example the quantity of PT purchased from Pacific Supply is 37,500 units in the Low scenario, and 57,100 units in both the Most Likely and High scenarios. The quantity is the same for the latter two scenarios due to the cost of the

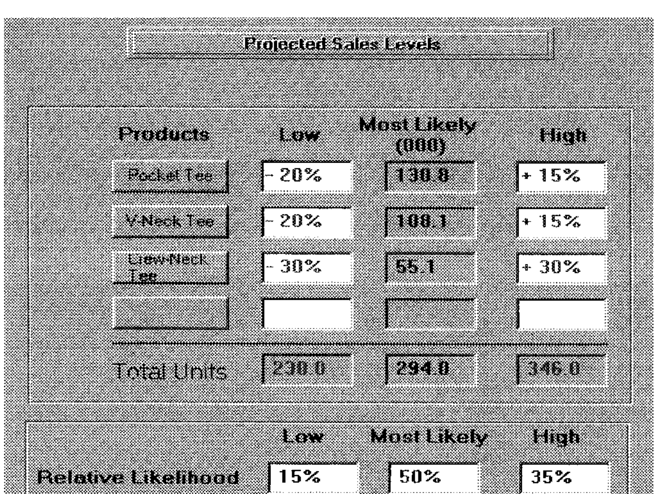

FIGURE 5. Demand Scenario Information.

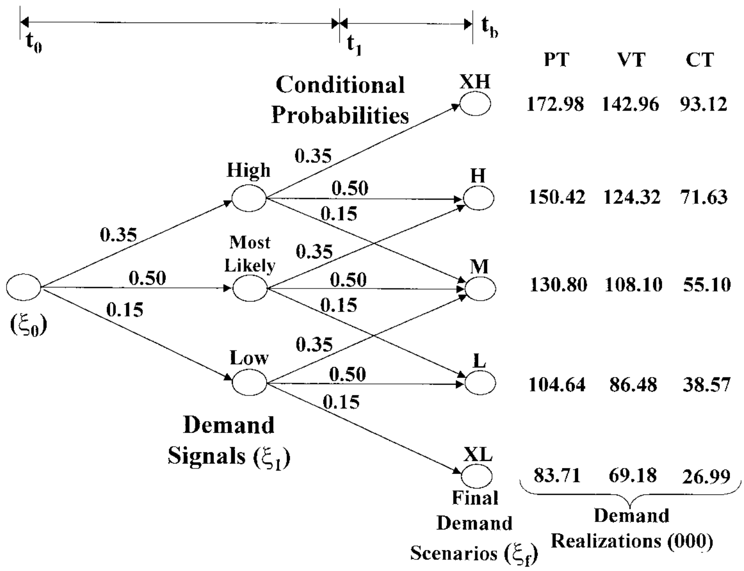

FIGURE 6. Distribution of Total Season Demand.

fabric that must be ordered prior to the placement of the production orders (112,800, 103,600 and 51,600 yards of fabric are ordered for PT, VT, and CT, respectively). To satisfy all demand in the High scenario would require a costly investment in fabric prior to the resolution of demand uncertainty, leaving substantial excess fabric in the Low and Most Likely scenarios.

Note that Amazon Apparel, the less flexible but lower cost vendor, is used to source all three products. Despite higher unit costs, Pacific Supply's flexibility earns a portion of PT and VT production. Interestingly, even though CT is the hardest to predict (i.e., its Low-High range of $\pm 30\%$ in Figure 5 is the widest), its production is entirely allocated to Amazon Apparel since the benefit of Pacific's flexibility on this product does not outweigh the associated cost premium. This sort of cost tradeoff insight can only be obtained through a constrained optimization model.

The weekly production schedules for the PT product that aggregate to the volumes in Table 5 are presented graphically in Figures 7, 8, and 9.

TABLE 5
Total Season Summary of the Optimal Sourcing Plan (All Units in 000s)

	Scenario	Pacific Supply	Amazon Apparel	Total Production	Total Demand	Lost Sales
	Low	37.5	99.5	137.1	104.6	0
Pocket Tees	Most Likely	57.1	99.5	156.7	130.8	0
	High	57.1	99.5	156.7	150.4	7.9
	Low	48.5	95.4	143.9	86.5	0
V-Neck Tees	Most Likely	48.5	95.4	143.9	108.1	0
	High	48.5	95.4	143.9	124.3	6.5
	Low	0	71.6	71.6	38.6	0
Crew-Neck Tees	Most Likely	0	71.6	71.6	55.1	0
	High	0	71.6	71.6	71.6	7.5
	Low	86	266.6	352.6	229.7	0
Total Production	Most Likely	105.6	266.6	372.2	294.0	0
	High	105.6	266.6	372.2	346.4	21.9

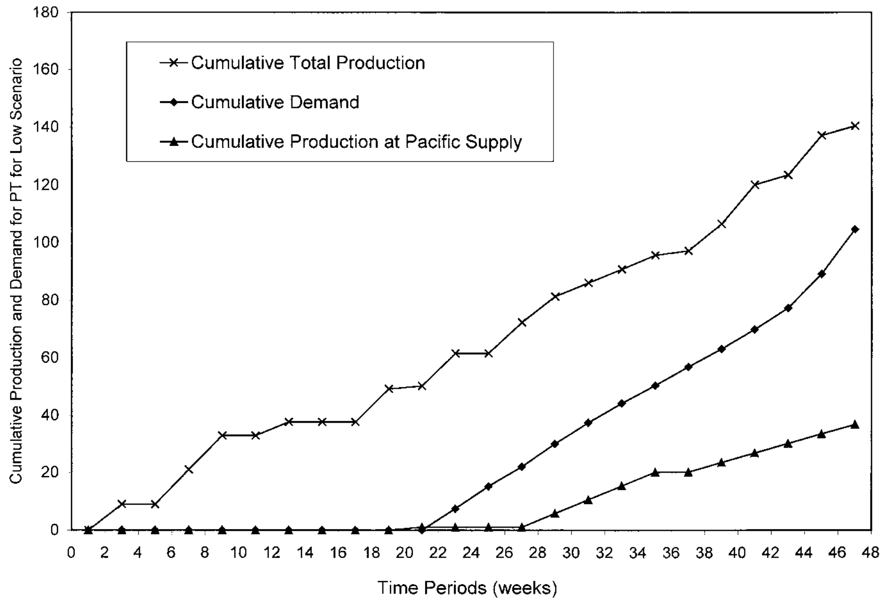

FIGURE 7. Cumulative Demand and Production for PT in “Low” Scenario.

Total production of all products exceeds the forecasted demand in every scenario. This is a consequence of the inventory requirements for merchandise presentation and display, as well as the vendors’ lack of in-process flexibility (as manifested in production lead times and reluctance to adjust production from quarter to quarter). While minimum production requirements could also be a factor in general, this was not the case in this example.

Despite excess production, however, each product still experiences lost sales in the High scenario (cf. Table 5). This is because having excess *over the course of the season* does not guarantee that supply will meet demand in every *individual* period. The divergence is a result

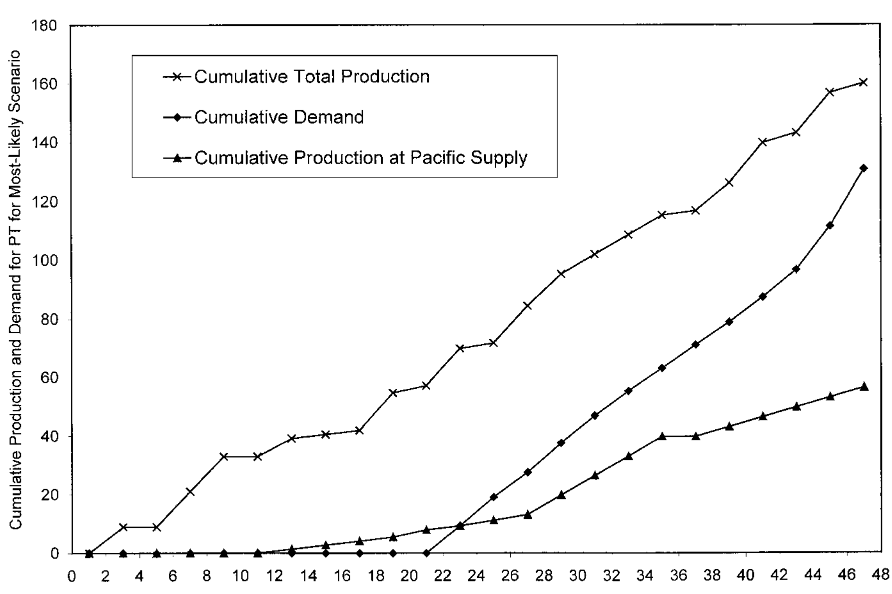

FIGURE 8. Cumulative Demand and Production for PT in “Most Likely” Scenario.

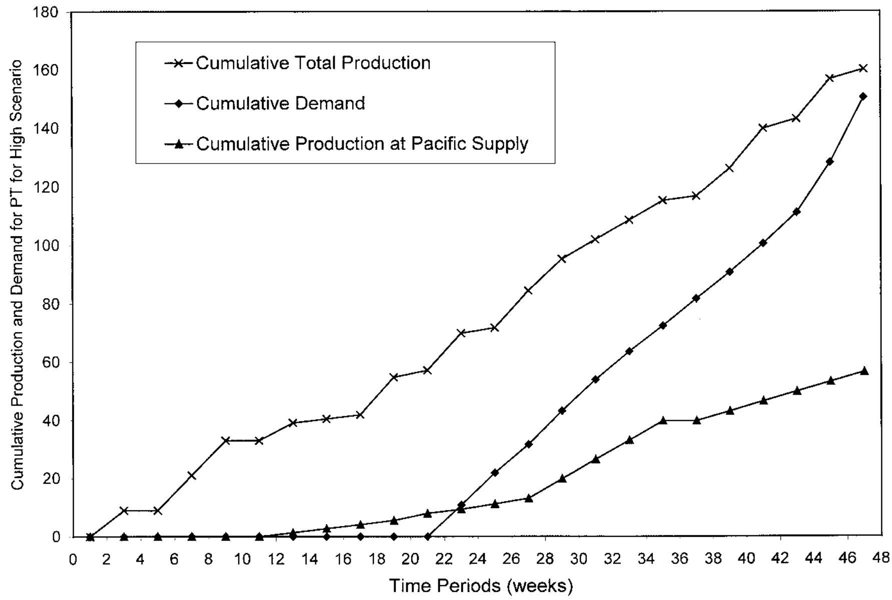

FIGURE 9. Cumulative Demand and Production for PT in "High" Scenario.

of the vendors' production constraints. [Generally, this could also be caused by the sell-through constraints in (11) if $\eta_j < 1$. However, our analysis assumed $\eta_j = 1$ for all j.] Explicit consideration of the intraseason timing of supply and demand is a key distinction of our paper relative to others such as Fisher and Raman (1996).

4.3. *The Value of Optimal Planning*

One of the key strengths of our methodology is its ability to generate sourcing strategies specifying recourse decisions that react to updated information about demand (through orders placed with short lead-time vendors). A natural way to appreciate this is to compare our model's recommendations to actions that sourcing planners might generate using their existing methods. Since the case study described here is a representative abstraction of a problem that this retail firm might face, historical decisions and outcomes were not available. Instead we simulated a "status quo" benchmark consistent with current planning practice, as described below.

In practice, retailer sourcing plans tend to assume that the Most Likely demand scenario will occur with certainty. We used SAM to determine the sourcing behavior that might result from such a deterministic world view, by setting the relative likelihood for the Most Likely scenario to 100%. Since demand information never improves in this case, no gains are realized from postponing sourcing commitment. Thus the model recommends a strategy (production schedule, shipment schedule, and fabric purchase quantities) at time t_0, which is fixed for the entire planning horizon. We then calculated the expected profit associated with this strategy under the stochastic demand environment described in Figure 6, and compared the optimal expected profit associated with the recourse-based plan recommended by SAM.

We found the status quo strategy to have an expected profit of $2,900,850, while SAM suggested a way achieve an expected profit of $3,003,160, a 3.5% improvement. The status quo plan purchases substantially less fabric than the optimal quantities (99,550, 88,000, and 39,660 yards for PT, VT, and CT products, respectively, as compared to 112,800, 103,600, and 51,600 yards), and places smaller total orders with the vendors as well. This is because ignoring the upside demand potential leads to conservatism in the retailer's procurement plans, which incurs substantial lost sales in the higher demand scenarios. This, in fact, is consistent with behavior observed in practice.

Since net profit margins in retailing tend to be very small, this suggests that our methodology can offer meaningful gains. Note also that the status quo strategy is optimal in the assumed deterministic environment, whereas actual sourcing plans developed by planners would likely be suboptimal. Thus, the benefit provided by using SAM is likely to be even higher. (Our findings were similar for the other portfolio types to be defined in the next section. SAM improved expected profit by 3.4, 4.1, and 9.6% in the Seasonal Basic, Fashion, and High-Margin Fashion portfolios, respectively.)

4.4. *An Extended Analysis with Multiple Product Types*

To gain further insights and a broader basis of comparison, we defined three additional product types representative of other parts of this retailer's product line and performed a variety of analyses for all four in parallel. In particular, we considered four separate portfolios, each consisting of three products of a single type. The products are the Tee-shirts called PT, VT, and CT for the Basic type we have analyzed thus far. The model can easily evaluate portfolios comprising arbitrary combinations of types, but we chose this segmentation for expositional clarity. Parameters distinguishing the three new product types, in terms relative to the Basic type, were elicited from discussions with planners at the retailer, and are shown in Table 6.

We began with the demand scenarios that were used for the Basic product, as shown in Figure 5. For Seasonal Basic and Fashion product types the percentage deviations associated with High and Low for the Basic product were doubled across the board to represent the greater difficulty in predicting their demand. At the same time, unsold merchandise for these types were assigned lower residual values to capture their greater cost of obsolescence. Retailers typically compensate for these two factors by increasing the markup, as represented by the High-Margin Fashion product type.

Our model allows any number of lines of analysis, but for illustrative purposes we feature a few that were particularly informative for this retailer. We first present a description of the variability of profit, which can be useful in assessing the financial risk faced by the retailer. We then appeal to shadow prices to evaluate operational changes in production capacity and storage. (These are featured because for the given parameter assumptions, the SAM output suggested that vendor production and storage capacity were the most significant constraints.)

REPORTING AND INTERPRETING THE VARIABILITY OF PROFIT. One of the benefits of building a stochastic model is the availability of distributional information about any system performance metric. This can provide valuable insight about the extent of intrinsic uncertainty to which a decision-maker is exposed. One way to present this information is with a cumulative

TABLE 6
Modified Parameter Values for Alternative Product Types

	Multiplier to Be Applied to Corresponding Parameter for Basic Product		
Product Type	Price	Predictability of Demand	Residual Value
Basic	1	1	1
Seasonal Basic	1	2	0.67
Fashion	1	2	0.4
High-Margin Fashion	1.25	2	0.4

probability plot. Figure 10 does this for the retailer profit associated with the *optimal supply chain strategy* for each of the four portfolio types outlined in Table 6.

As one might expect, the Basic portfolio's profit has the least downside and is the highest on average (expected profit = $3,003,160). High-Margin Fashion (expected profit = $2,999,222) has the greatest uncertainty, displaying significantly greater profit downside than Basic or Seasonal Basic (expected profit = $2,389,783). However, the Fashion portfolio (expected profit = $1,843,000), which in the High scenario could be as profitable as the Seasonal Basic, shows disappointing profitability at all probability levels. This is because the optimal strategy in the High scenario is to purchase no more fabric than what is optimal for the Most Likely scenario, foregoing any opportunity to capitalize on the higher demand. This is because the low salvage value for Fashion forces more conservatism in the production levels. In decision analysis terminology, the Fashion portfolio is stochastically dominated by the others profit-wise.

While designing the product portfolio itself is beyond the scope of our research, the above discussion suggests that our approach can help a retail merchandise manager reconcile his/her product selections with the firm's attitudes toward risk. This can have important implications for how the firm devises various financial and nonfinancial hedging strategies.

SHADOW PRICE ANALYSIS. Important insights from an optimization analysis are often derived from shadow prices and other sensitivity outputs. In vendor sourcing, this information can identify the most critical vendor production and storage constraints, and therefore guide the retailer in negotiating these limits or in identifying alternative vendors with appropriate capabilities. The retailer's storage limits or end-season inventory requirements may also be opportunities for performance improvement.

Because of the multitude of variables associated with the specific time periods and information states, most individual shadow prices in our model are not directly meaningful. However, useful sensitivity information can be obtained by introducing additional variables. For instance, since increases in production and storage capacity would typically be made for the entire horizon rather than by individual periods, it is appropriate to introduce a single variable that increments a given vendor's capacity uniformly in all periods and information states. If this variable is then constrained to be 0, the corresponding shadow price will reveal

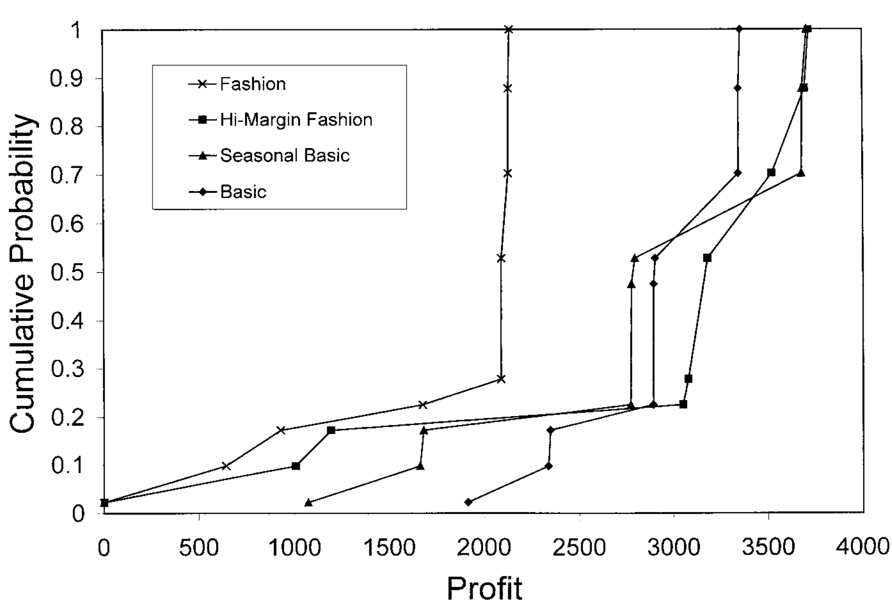

FIGURE 10. Cumulative Distribution of Profits.

the marginal benefit of increasing the vendor's capacities in all periods at once. We add variables for these constraints as follows:

$\widetilde{k}_i \equiv$ increase in quarterly production capacity (000's) for vendor i for all quarters
$\underset{\sim}{k_i} \equiv$ decrease in quarterly minimum production (000's) for vendor i for all quarters
$\underset{\sim}{\omega_i} \equiv$ increase in storage capacity at vendor i (cartons)

The appropriate constraint equations [(6) and (3)] are then replaced with the following:

$$\underline{k}_i(q) - \underset{\sim}{k_i} \le Z_i(q|\xi_1) \le \bar{k}_i(q) + \widetilde{k}_i \qquad \text{for all } i, q, \xi_1 \tag{14}$$

$$\sum_j \nu_j M_{ij}(t|\xi_1) \le w_i(t) + \omega_i \qquad \text{for all } i, t, \xi_1 \tag{15}$$

$$\underset{\sim}{k_i}, \widetilde{k}_i, \omega_i = 0.$$

This enhancement was made for components of the formulation deemed most important by the retail planners: vendor production capacity, vendor flexibility, vendor storage, end-season retail inventory, and product demand. For the given parameter assumptions, the SAM output suggested that vendor production and storage capacity were the most significant constraints. We discuss these in detail below.

THE VALUE OF PRODUCTION CAPACITY. Figure 11 shows the value of additional Pacific Supply production capacity [via the shadow price of the variable $\underset{\sim}{k_i}$ that appears in equation (14)] at different levels of capacity for each of the four product types discussed earlier.

For the Basic portfolio, the value of additional capacity at Pacific Supply is \$1.65 per unit (about 10% of retail price) at the current level (40,000 units/quarter). This is the case even though Pacific Supply's capacity remains underutilized in some periods. In contrast, even though Amazon Apparel is the cheaper source, its capacity will not be fully utilized in any quarter.

The value of additional Pacific capacity is strongly dependent on the product type. The

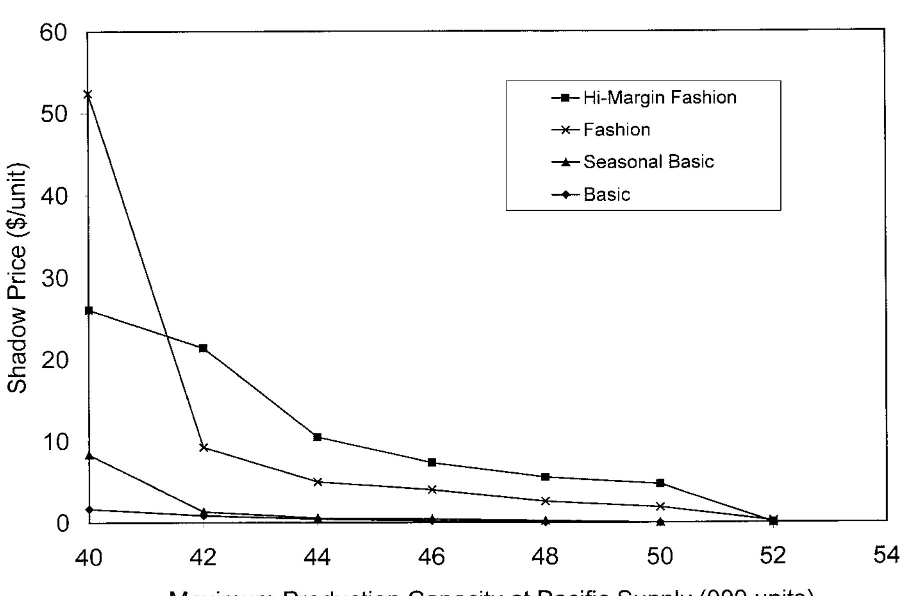

FIGURE 11. Shadow Prices for Maximum Production Capacity at Pacific Supply.

value is 20 to 30 times higher for the Fashion and High-Margin Fashion portfolios than for the Basic portfolio, even though Basics are highly profitable. The greater unpredictability of demand and lower salvage values of the Fashion products impose a premium valuation on Pacific's willingness to delay production commitments until additional market information is obtained.

Note that for all product types the value of additional capacity is initially high, but drops steeply, so that a 10% capacity increase yields most of the benefit. This is an "actionable" insight, in that incremental increases in vendor production capacity can usually be negotiated, unless all the vendor's capacity happens to already be committed to this one particular retailer.

A general lesson from this sensitivity analysis is that not all production capacity is created equal. Capacity cannot be properly valued independently of the nonfinancial conditions on its deployment, such as commitment lead time and allowable flexibility from quarter to quarter. The effect of these conditions is a function of the attributes of the goods that the capacity will be used to produce, in particular the predictability of demand and cost of obsolescence.

THE VALUE OF STORAGE SPACE. Figure 12 displays the value of additional storage space at Amazon Apparel [via the shadow price of the variable $\underset{\sim}{k_i}$ that appears in equation (14)] at different levels of storage space for each of the four product categories. The current level is 1,000 cartons.

For the Basic portfolio, storage at both vendors is fully utilized. However, the shadow price per carton at Amazon Apparel is considerably higher than that at Pacific Supply ($13.20 versus $9.00, respectively). Amazon's lack of storage space forces merchandise to be shipped to the retailer earlier than necessary, creating higher storage costs at the retailer.

The higher demand realizations associated with the High scenario make storage more valuable for Seasonal Basic products than for Basic products. Additional storage has relatively little value for the Fashion portfolio because the optimal production plan responds to the low salvage values with lower production volumes, hence less inventory buildup.

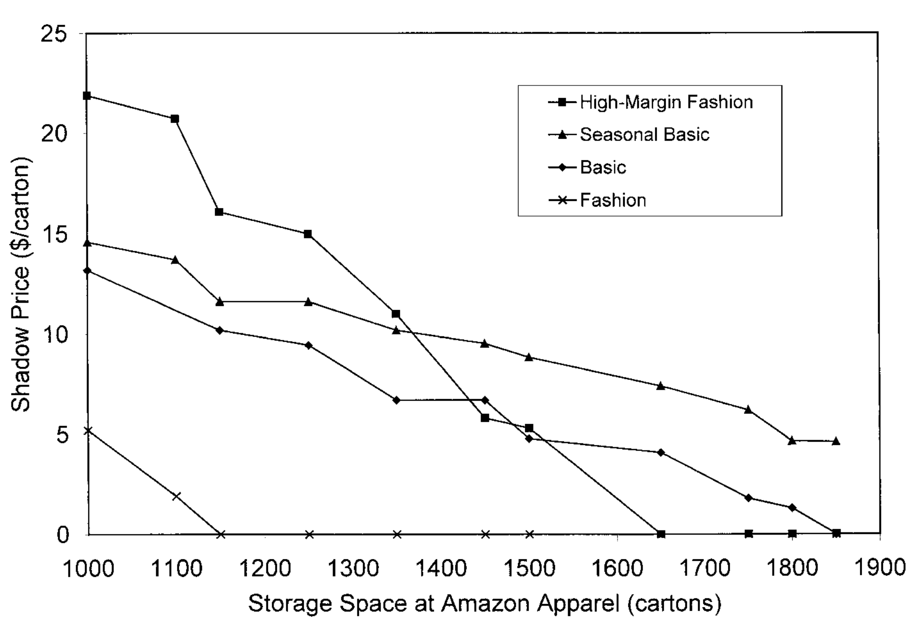

FIGURE 12. Shadow Price for Additional Storage at Amazon Apparel.

The magnitude of the shadow prices for Amazon storage was initially surprising both to us and the retail executives, as the retailer also has access to storage space costing only $0.70 more per carton per week than the vendor's storage. However, due to Amazon's requirement of level production, inventory required for the peak demand must currently be held for many weeks at the retailer. Therefore increasing this vendor's storage is quite beneficial. One executive indicated that more vendor storage can usually be obtained, and thus this too is an "actionable" insight.

Summary of insights from our analysis. The managerial recommendations derived from the retailer's analysis of the Tee shirt sourcing decisions can be divided into two groups, which are outlined below.

1. Sourcing policies for the three products and two vendors
   (a) Amazon Apparel should produce all the Crew Necks because Pacific Supply's cost premium is too high, despite greater flexibility.
   (b) Sufficient fabric should be purchased only for the Most Likely case. The risks of acquiring enough fabric to produce for the High case are too great.
   (c) For these three products, additional vendor production capacity has no significant value.
   (d) There is significant value for increased vendor storage capacity.
2. Sensitivity information by product type for key vendor parameters and constraints
   (a) Additional capacity at Pacific is valuable for both Fashion and Seasonal Basic products, but roughly a 10% increase is sufficient.
   (b) Additional vendor storage is valuable for all products.
   (c) Fashion products must have a significantly higher markup to be attractive relative to Basic and Seasonal Basic products, based on the relative risk and return.

While these recommendations are naturally dependent on the particular parameter assumptions, they exemplify the kinds of managerial insights about sourcing strategies that can be generated by this model.

5. Contributions to Managerial Practice

Our methodology has suggested a number of general insights, which are particularly relevant to practice.

First, we have articulated the value of explicitly incorporating uncertainty of market demand into the analysis of vendor management strategies. The retail practitioners with whom we worked closely during this project revealed that planners rarely formulate contingency strategies for supply chain execution, even though multiple demand scenarios are commonly developed. Such scenarios are typically used only for financial planning, while supply chain planning still commonly assumes that the most likely scenario will occur. Without proactive planning of the type we facilitate, last-minute negotiations and threats against vendors are the prevalent means for coping with market uncertainties. This negatively affects both the short-term and long-term profitability for the retailer and its vendors.

Second, we have shown how the profit risk facing a firm is a function of the nature of its products, the capabilities of its vendor base, and other logistical and environmental constraints. The probability distribution of the stochastic profit could be used to develop appropriate hedging strategies to ensure target financial performance. This would not be possible under any planning methodology based on point estimation of profit.

Third, our analysis suggests and quantifies the value of using a portfolio of vendors with differing delivery capabilities. In practice, while buyers know at an intuitive level that flexibility has value, the inability to quantify this has left them biased toward vendors quoting the lowest unit costs. We have shown that additional vendor flexibility can indeed be worth a reasonable price premium when demand uncertainty exists, and we provide a means to evaluate this tradeoff with a realistic level of detail.

Fourth, we have shed light on a number of factors that impact the inventory turnover realized by retailers. Conventional wisdom suggests that turnover is determined by the replenishment policies adopted at the store level. While this is largely true, we have shown that tension between demand seasonality and the vendors' desire to maintain stable production schedules profoundly affects retailer inventory patterns. Thus, efforts to increase turnover should also consider negotiations with vendors to seek greater production flexibility.

Finally, from an organizational point of view, our methodology provides a vehicle for facilitating cross-functional communication and negotiation. Specifically, in a retail firm the merchandising, sourcing, and finance organizations typically have somewhat conflicting objectives with respect to the inventory management strategy. (In mathematical terms, each group typically perceives a different segment of the overall objective function.) An early insight for us and our corporate sponsors was that our decision support system could serve as a tool for brokering the concerns of these groups by solving the global optimization problem, explicitly quantifying tradeoffs, and, most importantly, defining a common vocabulary for conversation.

6. Conclusion

Estimating the value of adding or dropping a vendor, renegotiating the terms of a supply contract, or improving forecast capability requires the respecification of the production schedule in ways that may differ dramatically from past plans. The complexity of such decisions renders the subjective selection of optimal or even near-optimal plans extremely difficult or impossible. While many retail buyers and merchandise planners rely on extensive databases and query tools for decision support, few have access to computer-based methods for optimal decision making or sensitivity analysis regarding these decisions. Our model and the associated decision support software provide retail planners with the power to identify and evaluate a wide variety of potential supply chain improvements that they are not currently able to consider.

Capturing market uncertainty through discrete scenarios is a familiar mechanism that simplifies the required user inputs and allows the application of linear programming optimization. Because of the many types of production and sales constraints that may apply in a retail business environment, simplicity of use is essential to the practicality of a decision support tool. Tests of our model by buyers and planners within a major retail organization indicates that our framework is compatible with the production commitment decisions they face.

Although our application did not require the approximate optimization methods that have been developed for larger LPs under uncertainty (cf. Infanger 1994), these techniques could allow our formulation to be applied to problems involving much greater numbers of products, vendors, time periods, and states of information. Such methods, coupled with the rapid decline in the cost of computing resources, hold promise for increasing the applicability of our sourcing optimization models.[1]

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Appendix: Complete Statement of the LP Optimization Problem

The complete optimization problem can be stated as follows:

$$\max \sum_j \{R_j - C_j\} \qquad \text{subject to} \tag{A1}$$

$$R_j = \sum_{t,\xi_1,\xi_f} p(\xi_1, \xi_f)\{\pi_j(t|\xi_f)U_j(t|\xi_1, \xi_f) + r_j I_j(t_f|\xi_1, \xi_f) + r_j^F [F_j - Z_j^F(\xi_1)]\} \tag{A2}$$

$$C_j = \sum_{i,t,\xi_1} p(\xi_1)[c_{ij}P_{ij}(t|\xi_{ij}) + v_{ij}M_{ij}(t|\xi_1)] + \sum_{t,\xi_1,\xi_f} p(\xi_1, \xi_f)h_j I_j(t|\xi_1, \xi_f) + c_j^F F_j \tag{A3}$$

$$U_j(t|\xi_1, \xi_f) \le d_j(t|\xi_f) \qquad \text{for all } j, t, \xi_1, \xi_f \tag{A4}$$

$$U_j(t|\xi_1, \xi_f) \le \eta_j I_j(t|\xi_1, \xi_f) \qquad \text{for all } j, t, \xi_1, \xi_f \tag{A5}$$

$$I_j(t+1|\xi_1, \xi_f) = I_j(t|\xi_1, \xi_f) + \sum_i S_{ij}(t - \ell_i|\xi_1) - U_j(t|\xi_1, \xi_f) \qquad \text{for all } j, t, \xi_1, \xi_f \tag{A6}$$

$$M_{ij}(t+1|\xi_1) = M_{ij}(t|\xi_1) + P_{ij}[q(t)|\xi_1] - S_{ij}(t|\xi_1) \qquad \text{for all } i, j, \xi_1, t \tag{A7}$$

$$\sum_j \nu_j M_{ij}(t|\xi_1) \le w_i(t) \qquad \text{for all } i, t, \xi_1 \tag{A8}$$

$$\sum_j \nu_j I_j(t|\xi_1, \xi_f) \le w^R(t) \qquad \text{for all } t, \xi_1, \xi_f \tag{A9}$$

$$\sum_j \sum_{t|q(t)=y} \kappa_j P_{ij}(t|\xi_{ij}) = Z_i(y|\xi_1) \qquad \text{for all } i, y, \xi_{ij} \tag{A10}$$

$$\mathbf{k}_i(q) \le Z_i(q|\xi_1) \le \bar{k}_i(q) \qquad \text{for all } i, q, \xi_1 \tag{A11}$$

$$(1 - \alpha_i) Z_i(q-1|\xi_1) \le Z_i(q|\xi_1) \le (1 + \beta_i) Z_i(q-1|\xi_1) \qquad \text{for all } i, q, \xi_1 \tag{A12}$$

$$\sum_i \sum_t \kappa_j^F P_{ij}(t|\xi_{ij}) = Z_j^F(\xi_1) \le F_j \qquad \text{for all } j, \xi_1 \tag{A13}$$

$$I_j(t_b|\xi_1, \xi_f) \ge i_j^0, \qquad M_{ij}(t_b|\xi_1) \ge m_{ij}^0 \qquad \text{for all } i, j, \xi_1, \xi_f \tag{A14}$$

$$I_j(t_f|\xi_1, \xi_f) \ge i_j^f, \qquad M_{ij}(t_f|\xi_1) \le m_{ij}^f \qquad \text{for all } i, j, \xi_1, \xi_f \tag{A15}$$

$$\text{All variables} \ge 0. \tag{A16}$$

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