# A stochastic decision model for strategic supplier relationship portfolio management

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**Abstract:** Management of a portfolio of strategic supplier relationships is complicated by their dynamism, uncertainty, information vagueness, significant profit impact, long-term orientation, substantial switching cost, and the interdependency among alternatives. To address this challenge, this paper first suggests a three-layer framework for strategic supplier relationship portfolio management. It then proposes a fuzzy binomial tree approximation-based stochastic model to analyse relationship dynamics and the value of a supplier relationship portfolio, taking into account both randomness and fuzziness uncertainty. Furthermore, it develops a decision model for supplier relationship portfolio configuration and adaptive development planning. Numerical examples are provided and some managerial implications are discussed.

**Keywords:** supply chain management; fuzzy sets; stochastic processes; supplier relationship management; portfolio management.

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## 1 Introduction

Today's companies rely heavily on external suppliers for materials, products, services, technology, and innovation. Empirical evidence shows that firms can indeed obtain competitive advantage by managing supplier relationships (Dyer, 1996; Chen et al., 2004; Autry and Golicic, 2010; Chatain, 2011). Obviously, differentiated approaches are needed, since not all suppliers should be dealt with in the same way. This in turn creates a need for some sort of classification (Lilliecreutz and Ydreskog, 2001). Since portfolio models provide differentiated strategic actions for heterogeneous categories of objects or subjects (Turnbull, 1990), purchasing portfolios and portfolios of buyer-supplier relationships have received much attention in recent literature (Bensaou, 1999; Gelderman and Weele, 2005; Gelderman and Semeijn, 2006; Caniëls and Gelderman, 2007). Kraljic (1983) is the first to bring portfolio models into the purchasing arena. The Kraljic portfolio, based on a four-category matrix, has inspired practitioners and researchers to propose variations that suggest a wide range of criteria and dimensions (e.g., Olsen and Ellram, 1997; Bensaou, 1999; Gelderman and Weele, 2005; Gelderman and Semeijin, 2006; Park et al., 2010; Drake et al., 2013). Although extant portfolio models provide a conceptual guide or a qualitative categorisation framework to develop a company's supply strategy (Gelderman and Weele, 2005), they are of little practical use for more detailed analysis (Leek et al., 2006). Furthermore, they work in the fragmented manner of relationship resource allocation, stopping short of quantifying the effect of each dimension upon the total relationship and missing the interdependencies among two or more items in a matrix (Olsen and Ellram, 1997; Zolkiewski and Turnbull, 2000; Wagner and Johnson, 2004).

Conceptual and empirical research on business relationships recognise that transactions or contracting activities between firms occur mostly in the context of established relationships and relationship value is created in an evolutionary fashion (Dwyer et al., 1987; Anderson, 1995; Turnbull et al., 1996; Jap and Anderson, 2007; Lambert and Schwieterman, 2012). In other words, real strategic supplier relationship management takes place at an aggregated level across individual operational transactions in a *long-term* and *dynamic* process. In addition, firms generally possess *flexibility* during the relationship development process to dynamically and adaptively reconfigure and re-optimise their supplier relationship portfolios, e.g., addressing an underperforming relationship by curtailing it or switching to alternatives. Depending on the contract terms and duration, the existence of substitutable alternatives, the comparative relationship states and relationship switching costs, the degree of relationship adaptation flexibility differs across supplier relationships and distinct relationship stages. Furthermore, the valuation of strategic supplier relationships demonstrates substantial, inherent *vagueness and fuzziness*. Due to the complex nature of strategic supplier relationships, information asymmetry between the buyer and the supplier, high aggregation across purchased items, deficiencies in information systems and perceptions of various conditions, many relevant variables at the decision process, even regarding the current state, are not known with certainty or cannot be measured precisely. Usually, supplier relationship valuation has to rely in large part on managerial judgment and estimation to provide input data, which is typically fuzzy in nature. Different from randomness-driven uncertainty, this kind of uncertainty cannot be resolved by the passage of time or experiments, and cannot be modelled appropriately by traditional probability theory and statistics (Zimmermann, 2001). A valuation approach will be more applicable to the extent that it can take into account the fuzziness of managerial judgment. In addition, the fuzziness of estimation is an essential consideration for differentiating relationships. It carries valuable information about and reflects managerial perception of trust, satisfaction, communication, history, the state of collaboration and other issues. For example, an important value driver of long-term or close relationships is that the fuzziness in evaluating relationships decreases significantly with the development of relationships, communication and commitment.

Although a number of studies have discussed purchasing portfolios and portfolios of buyer-supplier relationships, these studies are limited in not sufficiently accounting for the following:

1 dynamism (the long-term evolution of buyer-supplier relationships)

2 uncertainty (vagueness and fuzziness)

3 interdependencies (trade-offs) among portfolio objects.

But these aspects are important since they influence the validity of the portfolio approaches and lead to better purchasing decisions. Furthermore, to the best of our knowledge, extant portfolio approaches predominantly are conceptual, and use case studies or empirical examination. Saen (2010) presents a decision model for selecting appropriate suppliers, considering undesirable outputs[1] and imprecise data. Rezaei and Ortt (2013) propose a fuzzy rule-based system to segment the suppliers of a firm. In the model-based normative research on supplier evaluation, selection, and contracting, attention tends to be focused on the operational metrics of individual transactions, lacking a dynamic and uncertain perspective (Weber et al., 1991; Tsay et al., 1999; Elmaghraby, 2000; De Boer et al., 2001; Cachon, 2003). Vanpoucke et al. (2014) take an initial step in examining the dynamics of buyer-supplier relationships, but the uncertainty and interdependency issues are not addressed. This gap between professional purchasing and academic research motivates the following research questions:

> How should portfolios of supplier relationships be configured under uncertainty? What is the value of a supplier relationship portfolio? How should conflicting goals be traded off in the longitudinal and dynamic relationship process?

To answer these questions, we develop a dynamic, quantitative decision model for supplier relationship portfolio evaluation, configuration, and planning by means of stochastic processes, stochastic dynamic programming, fuzzy sets, and real options thinking. More specifically, we employ a three-layer framework and propose a fuzzy binomial tree approximation-based model to analyse relationship dynamics and the value of the supplier relationship portfolio, taking into account both randomness and fuzziness uncertainty. We focus on analysing supplier relationships at a strategic level, which are characterised by transaction aggregation, high uncertainty, high profit impact, long-term orientation, and substantial switching costs.

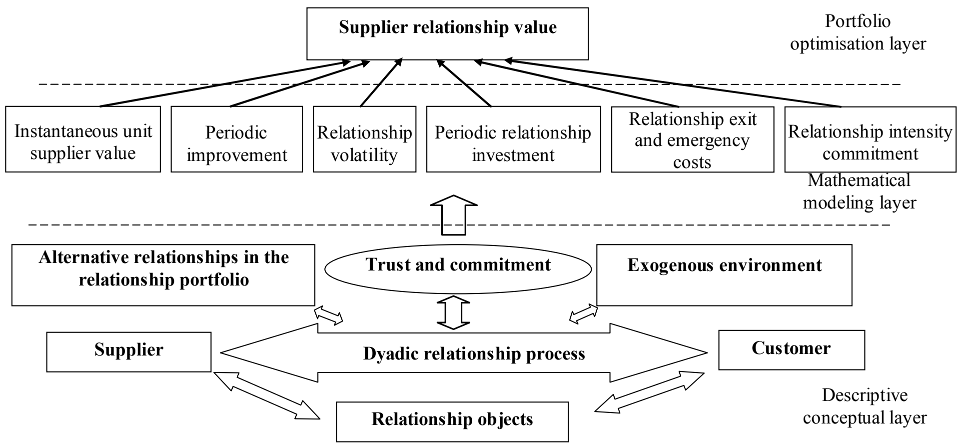
The remainder of this paper is organised as follows. Section 2 outlines a three-layer conceptual framework, defining and distinguishing two relevant value concepts along the way. Section 3 proceeds to model the dynamic stochastic evolution of supplier relationships based on a multidimensional fuzzy binomial tree. An integrated supplier relationship portfolio configuration and planning model is presented in Section 4. Section 5 is dedicated to a detailed numerical study. Section 6 discusses the findings and summarises the managerial insights. Section 7 concludes.

## 2 Conceptual framework

We suggest an integrated three-layer framework to analyse supplier relationship portfolios as depicted in Figure 1. The first layer is a conceptual model that profiles supplier relationships along six dimensions: the feature of relationship objects such as purchased items and services, the organisational characteristics of the supplier and the firm itself, the relationship development process, trust and commitment, alternative relationships in the relationship portfolio and exogenous environment. In this phase, traditional multi-criteria methods are adopted to pare the potential suppliers into a smaller set of qualified suppliers. Costing methods are also used to provide the basis for further analysis in the second layer, i.e., the mathematical modelling layer, which extracts variables in six dimensions to capture various aspects of relationship value creation and facilitate a normative mathematical treatment. At the portfolio optimisation layer, the supplier relationship value as a single aggregated metric derived from the mathematical treatment in the second layer enables relationship portfolio configuration and optimisation. Since the profile analysis at the descriptive conceptual layer has already received much attention in the literature (Weber et al., 1991; De Boer et al., 2001; Park et al., 2010; Lambert and Schwieterman, 2012), this paper concentrates on the mathematical modelling layer and the portfolio optimisation layer.

Value creation is the essential purpose for a customer firm and a supplier firm to engage in a relationship (Walter et al., 2001; Plambeck and Taylor, 2006; Chatain and Zemsky, 2007). We thus introduce a concept of *supplier relationship value* (see Definition 2) that is analogous to the concept of customer relationship value in the literature on relationship marketing (Ravald and Grönroos, 1996) and consistent with value-based management. Business transactions do not occur in a vacuum. They are sequentially correlated ‘episodes’ embedded in an evolving relationship process (Anderson, 1995; Plambeck and Taylor, 2006; Jap and Anderson, 2007). Hence, we first distinguish the periodic supplier value from overall relationship value.

**Figure 1** The three-layer framework




*Definition 1: Instantaneous unit supplier value at period t*, denoted by $V_t$, is the net benefit from the focal supplier relationship for one unit of relationship intensity, relative to a given relationship reference.

To capture the firm's dependence upon a supplier relationship and the interconnectedness of alternatives, instantaneous unit supplier value is compared with a given reference. Like the comparison level for alternatives ($CL_{alt}$) proposed by Kelley and Thibaut (1978), the *relationship reference* represents a nearly substitutable alternative, such as a spot market purchase, performing the activity in-house, or an alternative relationship. Absent such an alternative, the relationship reference is set by the scenario with no supply. In contrast to the order unit at the operational level, the *unit of relationship intensity* at the strategic level is specified by management in terms of transaction volume or monetary transaction value, or a combination of several context-specific dimensions to enable an overview of the relationship. In practice, the instantaneous unit supplier value in a period is obtained by dividing the periodic relationship intensity into the corresponding *aggregated* net benefits derived from the relationship across all relationship objects. These benefits stem from access to technology, resources, markets and information, and manifest as favourable price, high quality, low ordering cost, short lead time, information sharing, better service, and an efficient purchasing system.

*Definition 2: Supplier relationship value* at period *t*, denoted by $R_t$, is the cumulative relative benefits gained minus the total sacrifices made by a supplier relationship from period *t* to the end of the planning horizon, measured relative to a given relationship reference.

Supplier relationship value is the derived relationship metric in the third layer of our framework, whereas instantaneous unit supplier value is one of the determinant state variables in the second layer. The instantaneous unit supplier value evolves over time stochastically. Its periodic increase, termed relationship improvement, directly adds to the relationship value while its volatility results in the uncertainty of the relationship value. Note that relationship improvement can take a negative value, indicating relationship deterioration.

Supplier relationship value comes from three sources. First, it stems from the value of relationship objects, for instance, high quality of exchanged resources, favourable price, and fast time-to-market. This kind of value is mainly represented by instantaneous unit supplier value. Second, the dyadic relationship process and the relationship interaction provide substantial additional benefits such as reduced transaction costs, enhanced trust and commitment, technology transfer, information sharing and collaboration. They affect relationship value through periodic improvement, relationship volatility, commitment to relationship intensity, and periodic relationship investment during the interaction process in the mathematical modelling layer. Third, interdependencies among alternative relationships in the portfolio also contribute to relationship value. For instance, the existence of alternatives enables the firm to promote competition among suppliers, benefit from the different strengths of the various suppliers, diversify supply risk and replace underperforming incumbents (Dwyer et al., 1987; Chatain and Zemsky, 2007). The cost incurred in terminating a relationship is termed *relationship exit cost*. If the entire portfolio is unfavourable, the firm can turn to some emergency sources outside the relationship portfolio. The resulting cost is termed *emergency cost*.

## 3 Modelling supplier relationship dynamics and uncertainty

The mathematical modelling layer in our framework uses the following notation:

| | |
|---|---|
| $i$ | index of a supplier in the supplier relationship portfolio |
| $N$ | number of supplier relationships under consideration |
| $D_t$ | total relationship intensity at period $t$, limited by supply requirement and demand |
| $\lambda_t^i$ | expected periodic relationship improvement of relationship $i$ (can be negative to represent a deterioration) |
| $\sigma^i$ | expected volatility coefficient of relationship $i$, i.e., the standard deviation of $\lambda_t^i$, indicating the risk of the relationship evolution |
| $C_t^i$ | relationship investment in relationship $i$ at period $t$, which is not related to relationship intensity, including relationship overhead cost, investment specific to the supplier, dedicated resources and incentives offered to the supplier at that period |
| $X_t^i$ | relationship exit cost of relationship $i$ at period $t$ |
| $X_t^P$ | emergency cost at period $t$ |
| $q_{t,\min}^i$ | minimum relationship intensity commitment to relationship $i$ at period $t$ |
| $q_{t,\max}^i$ | maximum relationship intensity of relationship $i$ at period $t$, limited by the supplier's capacity |
| $\rho$ | risk-adjusted discount factor |
| $E(.)$ | expectation operator. |

The supplier relationship portfolio is described by $(\vec{Q}_t^P, \vec{S}_t^P)$, where the vector $\vec{Q}_t^P = (q_t^1, \dots, q_t^N)$ denotes the relationship intensity with the respective suppliers at period $t$, and $\vec{S}_t^P = (\vec{s}_t^1, \dots, \vec{s}_t^N)$ represents the state of the relationship portfolio at period $t$. We use the superscript $P$ in a notation to indicate that it concerns a relationship portfolio. The state of relationship $i$ in the portfolio, $\vec{s}_t^i$, in turn is described by a column vector $\vec{s}_t^i = (V_t^i, \sigma^i, \vec{C}^i, \vec{X}^i, \vec{\lambda}^i, \vec{q}_{\min}^i, \vec{q}_{\max}^i)^T$, where $\vec{\lambda}^i = (\lambda_t^i, \lambda_{t+1}^i, ..., \lambda_T^i)^T$ is the projected periodic relationship improvement; $\vec{C}^i = (C_t^i, C_{t+1}^i, ..., C_T^i)^T$ the projected periodic relationship investment; $\vec{X}^i = (X_t^i, X_{t+1}^i, ..., X_T^i)^T$ the projected relationship exit cost; $\vec{q}_{\min}^i = (q_{t,\min}^i, q_{t+1,\min}^i, \cdots, q_{T,\min}^i)^T$ the committed minimum relationship intensity; and $\vec{q}_{\max}^i = (q_{t,\max}^i, q_{t+1,\max}^i, \cdots, q_{T,\max}^i)^T$ the maximum relationship intensity from time $t$ to $T$.

As mentioned before, fuzziness uncertainty cannot be captured by traditional probabilistic treatments. To deal with the vagueness of input data, capture managerial judgement, and reinforce the applicability of our model in practice, we employ fuzzy set theory in modelling supplier relationships. Specifically, we model the instantaneous unit supplier value, periodic improvement, volatility coefficient, relationship exit cost, and periodic investments as fuzzy numbers. Hereafter, we use a tilde (~) to indicate a fuzzy number.

Fuzzy numbers may be represented in various forms. Among them, the triangular form and the trapezoidal form, which accommodates the former as a special case, find the most widespread application as they are easy to understand and implement in practice (Dubois and Prade, 1992; Pedrycz and Gomide, 1998; Lu et al., 2013). For this reason, we adopt the trapezoidal form. For instance, the instantaneous unit supplier value at period $t$ can be represented by a trapezoidal fuzzy number $\tilde{V}_t$ as follows (Kaufmann and Gupta, 1991):

$$\tilde{V}_t = (V_{t1}; V_{t2}; V_{t3}; V_{t4}), t = 1, ..., T. \tag{1}$$

with the membership function

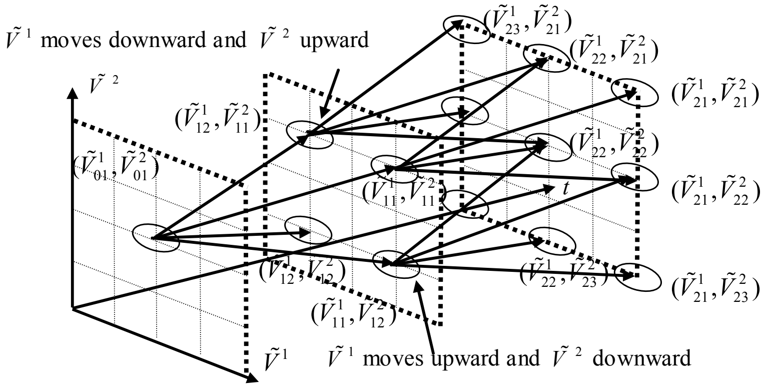
$$\mu_{\tilde{V}_t}(x) = \begin{cases} (x - V_{t1})/(V_{t2} - V_{t1}), & \text{if } V_{t1} \le x < V_{t2} \\ 1 & \text{if } V_{t2} \le x \le V_{t3} \\ (x - V_{t4})/(V_{t3} - V_{t4}), & \text{if } V_{t3} < x \le V_{t4} \\ 0, & \text{otherwise} \end{cases}$$

$V_{t1}$ and $V_{t4}$ are obtained from the pessimistic and optimistic estimates of possible values of instantaneous unit supplier value, respectively. The lower and upper limits of the core, $V_{t2}$ and $V_{t3}$, are obtained from the estimate of its most likely value range.

The dynamics of supplier relationship portfolio are mainly driven by two forces: the uncertain evolution of the instantaneous unit supplier value of each relationship and the adaptive adjustment of relationship intensity $\vec{Q}_t^P$ by the firm in response to relationship state $\vec{S}_t^P$. As argued by Dwyer et al. (1987) and Anderson (1995), and empirically tested by Jap and Anderson (2007), the relational exchange transpires over time; each relationship is the cumulative result of its history as well as the starting point of its anticipated future.

We develop a multidimensional *fuzzy* binomial tree to approximate the stochastic evolution process of the instantaneous unit supplier value of each supplier in the relationship portfolio $\vec{V}_t^P = (\tilde{V}_t^1, \cdots, \tilde{V}_t^N)$. In this binomial tree, the instantaneous unit supplier value of relationship $i$ evolves in dimension $i$ over time. We assume the instantaneous unit supplier values of relationships in the portfolio are not correlated. Figure 2 illustrates an example tree of two supplier relationships with four branches at each step. The instantaneous unit supplier value could evolve from the initial state, $(\tilde{V}_{01}^1, \tilde{V}_{01}^2)$, to four possible states at the next step: $(\tilde{V}_{11}^1, \tilde{V}_{11}^2), (\tilde{V}_{11}^1, \tilde{V}_{12}^2), (\tilde{V}_{12}^1, \tilde{V}_{11}^2)$ and $(\tilde{V}_{12}^1, \tilde{V}_{12}^2)$.

**Figure 2** The multidimensional fuzzy binomial tree with two relationships




We refer to the $j^{\text{th}}$ fuzzy node at time $t$ in dimension $i$ as node ($t$, $i$, $j$), and the instantaneous unit supplier value of supplier $i$ at node ($t$, $i$, $j$) as $\tilde{V}_{tj}^i$, where $t = 0, \ldots, T$, and $j = 1, \ldots, t + 1$. Then, $\tilde{V}_{tj}^i$, $t = 0, \ldots, T$, and $j = 1, \ldots, t + 1$, can either rise upward to $\tilde{V}_{t+1,j}^i (\tilde{V}_{t+1,j}^i = \tilde{V}_{tj}^i + \tilde{u}_t^i)$ with probability $P_t^i$ or descend to $\tilde{V}_{t+1,j+1}^i (\tilde{V}_{t+1,j+1}^i = \tilde{V}_{tj}^i - \tilde{d}_t^i)$ with probability $1 - P_t^i$, where $\tilde{u}_t^i$ is the fuzzy upward and $\tilde{d}_t^i$ the fuzzy downward movement in one step.

The upward and downward movements, $\tilde{u}_t^i$ and $\tilde{d}_t^i$, and the corresponding probability measure in the fuzzy binomial tree, $P_t^i$, should match the expected periodic relationship improvement, $\tilde{\lambda}_t^i$, and the estimated relationship volatility, $\tilde{\sigma}^i$. To find the properties of the fuzzy binomial tree, let us first explore a corresponding traditional binomial tree where all counterpart variables are not fuzzy. The increase of the instantaneous unit supplier value of supplier $i$ and its variance within a marginal time period $\Delta t$ will be (Hull, 2001):

$$\text{Improvement} = \left[P_t^i u_t^i + \left(1 - P_t^i\right) d_t^i\right] = \lambda_t^i \Delta t, \tag{2}$$

$$\text{Variance} = P_t^i \left[u_t^i - \lambda_t^i \Delta t\right]^2 \left(1 - P_t^i\right)\left[d_t^i - \lambda_t^i \Delta t\right]^2 = \left(\sigma^i\right)^2 \Delta t. \tag{3}$$

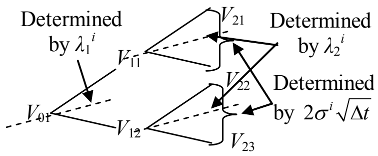
Note that we use notation without a tilde (~) to indicate that it is a counterpart variable in the traditional binomial tree. A third condition should be chosen to determine $P_t^i, u_t^i$ and $d_t^i$. In light of the Girsanov theorem (Neftic, 2000), we can always transform the binomial tree under the unknown original probability measures of the upward and downward movements into a binomial tree under equivalent probability measures with corresponding upward and downward movements while leaving the mean and variance structure intact. To avoid unnecessary fuzzy multiplication operations, we choose equal probabilities for upward and downward movements, that is, $P_t^i = 0.5$, denoted by $P$ hereafter. We obtain the corresponding $u_t^i$ and $d_t^i$ with respect to the equivalent probability measures as follows:

$$u_t^i = \lambda_t^i + \sigma^i \sqrt{\Delta t}, \tag{4}$$

$$d_t^i = \lambda_t^i - \sigma^i \sqrt{\Delta t}. \tag{5}$$

Equations (4) and (5) show $u_t^i$ and $d_t^i$ are determined by $\lambda_t^i$ and $\sigma^i$. Further, they reveal that $\lambda_t^i$ determines the trend of the traditional binomial tree in dimension *i*, whereas $\sigma^i$ determines its spread degree, as illustrated in Figure 3.

**Figure 3** The determinants of the binomial tree in dimension *i*




Analogous to their counterparts in the traditional binomial tree, $\tilde{\lambda}_t^i$ determines the fuzzy trend of our fuzzy binomial tree in dimension *i*, while $\tilde{\sigma}^i$ determines its fuzzy spread.

Then, we obtain $\tilde{u}_t^i$ and $\tilde{d}_t^i$ in the trapezoidal form as

$$\tilde{u}_t^i = \left(\lambda_{t1}^i + \sigma_1^i \sqrt{\Delta t};\, \lambda_{t2}^i + \sigma_2^i \sqrt{\Delta t};\, \lambda_{t3}^i + \sigma_3^i \sqrt{\Delta t};\, \lambda_{t4}^i + \sigma_4^i \sqrt{\Delta t}\right), \tag{6}$$

$$\tilde{d}_t^i = \left(\lambda_{t1}^i - \sigma_4^i \sqrt{\Delta t};\, \lambda_{t2}^i - \sigma_3^i \sqrt{\Delta t};\, \lambda_{t3}^i - \sigma_2^i \sqrt{\Delta t};\, \lambda_{t4}^i - \sigma_1^i \sqrt{\Delta t}\right). \tag{7}$$

To facilitate the computation in our fuzzy binomial tree, we extend the definition of fuzzy numbers to allow the upper bound to be smaller than the lower one and introduce the concept of fuzzy distance.

*Definition 3:* A *fuzzy metric space* is a 2-tuple $(S, \tilde{D})$, where $S$ is a set of fuzzy numbers and the *fuzzy distance* $\tilde{D}$ is a metric on $S$, i.e., a function $\tilde{D}: S \times S \to S$ such that $[\tilde{D}(\tilde{A}, \tilde{B})]^\alpha = [\underline{B}^\alpha - \underline{A}^\alpha; \overline{B}^\alpha - \overline{A}^\alpha]$, where $\tilde{A}, \tilde{B} \in S, [\tilde{D}(\tilde{A}, \tilde{B})]^\alpha, [\tilde{A}]^\alpha = [\underline{A}^\alpha; \overline{A}^\alpha]$ and $[\tilde{B}] = [\underline{B}^\alpha, \overline{B}^\alpha]$ are $\alpha$-cuts.

For two trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$, $\tilde{D}(\tilde{A}, \tilde{B}) = (b_1 - a_1; b_2 - a_2; b_2 - a_3; b_4 - a_4)$. By this definition, the relation $\tilde{B} = \tilde{A} \oplus \tilde{D}(\tilde{A}, \tilde{B})$ holds. Therefore, we obtain the fuzzy distance between two vertically adjacent fuzzy nodes in dimension $i$ in the multidimensional fuzzy binomial tree:

$$\tilde{D}(\tilde{u}_t^{\,i}, \tilde{d}_t^{\,i}) \equiv \left(-\left(\sigma_4^i + \sigma_1^i\right)\sqrt{\Delta t}; -\left(\sigma_3^i + \sigma_2^i\right)\sqrt{\Delta t}; -\left(\sigma_2^i + \sigma_3^i\right)\sqrt{\Delta t}; -\left(\sigma_4^i + \sigma_1^i\right)\sqrt{\Delta t}\right). \quad (8)$$

Interestingly, the fuzzy distance in dimension $i$, $\tilde{D}^i$, is constant over time, independent of $\tilde{\lambda}_t^i$ across distinct periods. Therefore, the fuzzy binomial tree does not overlap and keeps a lattice structure, as show $n$ in Figure 2.

The planning horizon is divided into $T$ review periods, at the beginning of which the firm has the flexibility to adjust its supplier relationships. Consistent with the practice of contractingat the tactical level, which explicitly defines a period of time during which the relationship is inflexible, a review period corresponds to average supply contract duration. We divide each review period further into $M$ steps in the binomial tree. Unlike at each review period, the firm has no flexibility to adjust its supplier relationships at each step within a review period. These $M$ steps within a review period for a given supplier relationship in the binomial tree assume the same upward and downward movements. The upward and downward movements across review periods, however, may be different, depending on the expected periodic improvement, $\tilde{\lambda}_t^i$. Therefore, if the instantaneous unit supplier value of relationship $i$ at period $t$ is at the $ji^{\text{th}}$ node, i.e., $\tilde{V}_t^i = \tilde{V}_{t,j^i}^i$, the conditional probability that it will arrive at the $(j^i + k^i)^{\text{th}}$ node at the next review period $t + 1$, $k^i = 0, \ldots, M$, is

$$P\left\{\tilde{V}_{t+1}^i = \tilde{V}_{t+1,j^i+k^i}^i \,\middle|\, \tilde{V}_t^i = \tilde{V}_{t,j^i}^i\right\} = \frac{M!}{k^i!\left(M-k^i\right)!} \cdot P^{M-k^i} \cdot (1-P)^{k^i}, \quad (9)$$

where

$$\tilde{V}_{t+1,j^i+k^i}^i = \tilde{V}_{t,j^i}^i \oplus M\tilde{u}_t^i \oplus k\tilde{D}^i. \quad (10)$$

The joint conditional probability that the instantaneous unit supplier values in the portfolio evolve from state $(\tilde{V}_{t,j^1}^1, \tilde{V}_{t,j^2}^2, \ldots, \tilde{V}_{t,j^N}^N)$ to $(\tilde{V}_{t+1,j^1+k^1}^1, \tilde{V}_{t+1,j^2+k^2}^2, \ldots, \tilde{V}_{t+1,j^N+k^N}^N)$ is then

$$\begin{aligned} &P\left(\tilde{V}_{t+1}^P = \left(\tilde{V}_{t+1,j^1+k^1}^1, \ldots, \tilde{V}_{t+1,j^N+k^N}^N\right) \middle| \tilde{V}_t^P = \left(\tilde{V}_{t,j^1}^1, \ldots, \tilde{V}_{t,j^N}^N\right)\right) \\ &= \prod_{i=1}^N P\left(\tilde{V}_{t+1}^i = \tilde{V}_{t+1,j^i+k^i}^i \middle| \tilde{V}_t^i = \tilde{V}_{t,j^i}^i\right). \end{aligned} \quad (11)$$

## 4 Supplier relationship portfolio configuration and development planning

Integrated configuration and development planning of the supplier relationship portfolio occurs in our framework's third layer. This deals with how many and which suppliers should be selected and how total relationship intensity $D_t$, which is limited by supply requirements and demand, should be allocated among the selected suppliers over time to maximise the expected value from the supplier relationship portfolio.

The decision variables of relationship portfolio configuration are denoted as follows:

$q_t^i$ relationship intensity with supplier $i$ at period $t$

$q_t^P$ relationship intensity allocated to an emergency source outside the portfolio at period $t$

$I_t^P$ 0–1 indicator variables indicating whether an emergency source is employed at period $t$

$I_t^i$ 0–1 indicator variables for relationship selection, i.e., $I_t^i = 1$ when supplier relationship $i$ is selected at period $t$, otherwise $I_t^i = 0$.

The relationship intensity is subject to the commitment constraint

$$I_t^i q_{t,\min}^i \le q_t^i \le I_t^i q_{t,\max}^i,\ \forall i = 1, ..., N. \tag{12}$$

As demonstrated by the empirical study of Jap and Anderson (2007), transactions in an ongoing relationship are sequentially correlated. We assume that an ongoing relationship can be terminated but an initially unselected relationship cannot be selected later. Hence, the 0–1 indicator variables are subject to

$$I_t^i \le I_{t-1}^i,\ \forall t = 1, ..., T,\ \forall i = 1, ..., N. \tag{13}$$

The *expected supplier relationship portfolio value* is the expected cumulative value of all relationships in the portfolio over the future possible states from period $t$ to the horizon end $T$. The *optimal adaptive supplier relationship portfolio configuration* at any period $t$ is a dynamic adjusted feasible relationship intensity $\vec{Q}_t^P = (q_t^1, ..., q_t^N)$ which maximises the expected portfolio value at period $t$, $E[\tilde{R}_t^P(\vec{S}_t^P)]$, contingent on the revealed portfolio state, $\vec{S}_t^P$. We divide the expected relationship portfolio value into two components: the total instantaneous supplier value at the current period and the expected value gained from the portfolio over future periods. For $t$ = 0, …, $T$–1, the recursive form for the expected portfolio value is provided by the following Bellman equation:

$$\begin{aligned} E\left[\tilde{R}_t^P(\vec{S}_t^P)\right] = \max\Bigg\{ &\sum_{i=1}^{N}\left[I_t{}^i\left(\tilde{V}_t{}^i q_t^i \ominus \tilde{C}_t^i\right) \ominus I_{t-1}^i\left(I_{t-1}^i - I_t^i\right)\tilde{X}_t^i\right] \\ &\ominus I_t^P \tilde{X}_t^P \oplus \rho E\left[\tilde{R}_{t+1}^P\left(\vec{S}_{t+1}^P \middle| \vec{S}_t^P\right)\right]\Bigg\} \end{aligned} \tag{14}$$

such that

$$I_t^i q_{t,\min}^i \le q_t^i \le I_t^i q_{t,\max}^i,\ \forall t = 1, ..., T,\ \forall i = 1, ..., N,$$
$$\sum\nolimits_{i=1}^{N} I_t^i q_t^i + I_t^P q_t^P = D_t,\ \forall t = 1, ..., T,$$
$$I_t^i \le I_{t-1}^i,\ \forall t = 1, .., T,\ \forall i = 1, ..., N,$$
$$q_t^i \ge 0,\ I_t^i \in \{0, 1\},\ \forall t = 1, ..., T,\ \forall i = 1, ..., N.$$

The term $I_t{}^i(\tilde{V}_t{}^i q_t^i \ominus \tilde{C}_t^i)$ in (14) is the value from relationship $i$ at period $t$. The term $I_{t-1}^i(I_{t-1}^i - I_t^i)\tilde{X}_t^i$ is the termination cost if relationship $i$ is an ongoing relationship at period $t$-1 and terminated at period $t$, represented by $I_{t-1}^{i}=1$ and $I_{t}^{i}=0$. $I_{t}^{P}\tilde{X}_{t}^{P}$ is the cost resulting from the emergency source outside the relationship portfolio. $\rho E[\tilde{R}_{t+1}^{P}(\vec{S}_{t+1}^{P}\,|\vec{S}_{t}^{P})]$ is the present value at period $t$ of expected future relationship value.

The first constraint in (14) states that the relationship intensity with supplier $i$ should be no less than the firm's commitment to minimum relationship intensity of relationship $i$ and no greater than the maximum relationship intensity of relationship $i$ at period $t$. The second constraint is the allocation constraint on relationship intensity. For example, the firm's aggregated demand should be allocated among available suppliers and the emergency source.

At the end of the planning horizon (period $T$), we have the boundary condition:

$$\tilde{R}^{P}\left(\vec{S}_{T}^{P},q_{T}\right)=\max\left\{\sum_{i=1}^{N}\left[I_{T}^{i}\left(\tilde{V}_{T}^{i}q_{T}\ominus\tilde{C}_{T}^{i}\right)\ominus I_{T}^{i}(I_{T-1}^{i}-I_{T}^{i})\tilde{X}_{T}^{i}\ominus I_{T}^{P}\tilde{X}_{T}^{P}\right]\right\}. \tag{15}$$

Based on equations (10) and (11), stochastic dynamic programming can solve the relationship portfolio problem (14) from the end of the planning horizon backwards to the initial condition, and obtain the optimal adaptive supplier relationship portfolio configuration and the expected cumulative relationship intensity.

## 5 Numerical study

This section presents a numerical study that illustrate show our modelling framework can be used to assess, configure, and optimise a supplier relationship portfolio under uncertainty. Without loss of generality, we use the same values for fuzzy periodic improvement and periodic relationship investment across review periods in the numerical study. We drop the time index for notational simplicity. To facilitate plotting, we 'defuzzify' fuzzy numbers to ordinary numbers by the defuzzification operator introduced in the Appendix. The model setting used throughout our numerical study is summarised in Table 1.

**Table 1** Parameter assumptions for the numerical study

| | | |
|---|---|---|
| Planning horizon | $T$ | 5 review periods |
| Binomial tree steps in each review period | $M$ | 3 steps |
| Risk-adjusted discount factor | $\rho$ | 0.91 (assumed equal over periods) |
| Fuzzy aversion coefficient | $r_A$ | 0.3 |
| Emergency cost | $\tilde{X}_{t}^{P}$ | (50,000; 52,000; 53,000; 54,000) |
| Total relationship intensity | $D_t$ | 500 unit |

We examine two supplier relationships of a manufacturing company in a purchasing context, as summarised in Table 2. Prospective supplier relationship A is risky but is expected to improve quickly, whereas existing supplier relationship B is less risky but shows less potential for improvement.

**Table 2** Supplier relationships A and B

| *Variables* | | *Supplier A* | *Supplier B* |
|---|---|---|---|
| Instantaneous unit supplier value | $\tilde{V}_0$ | (4; 5; 6; 7) | (13; 14; 15; 16) |
| Periodic improvement | $\tilde{\lambda}$ | (9; 10; 11; 11) | (5; 6; 7; 8) |
| Periodic volatility | $\tilde{\sigma}$ | (9; 10; 11; 12) | (5; 7; 8; 9) |
| Periodic investment | $\tilde{C}$ | (2,000; 2,200; 2,300; 2,500) | (1,000; 1,200; 1,300; 1,500) |
| Relationship exit cost | $\tilde{X}$ | (400; 480; 520; 600) | (1,200; 1,280; 1,320; 1,360) |
| Minimum commitment | $q_{\min}$ | 100 units | 300 units |
| Maximum commitment | $q_{\max}$ | 1,000 units | 1,000 units |

Our model (see Section 4) provides the optimal portfolio configuration and expected relationship value of each relationship over the planning horizon in Table 3, indicating the desirability of building relationships with both suppliers.

**Table 3** Optimal portfolio configuration

| *Supplier* | *Decision* | *Expected cumulative relationship intensity* | *Expected relationship value* |
|---|---|---|---|
| A | Selected | 1,036 | (17,986; 25,656; 32,995; 38,453) |
| B | Selected | 1,464 | (19,857; 30,541; 38,637; 48,831) |

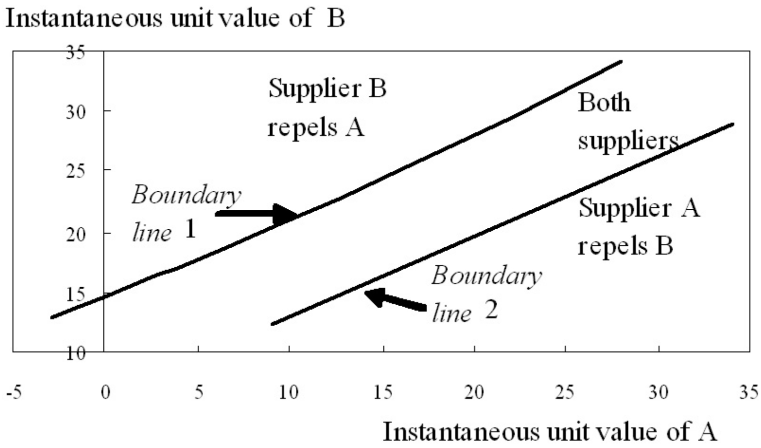
## 5.1 *Competitive effect of relationship portfolios*

Due to the limitations in demand and resources, total relationship intensity with suppliers in a portfolio at each period, $D_t$, is limited. An enhanced relationship with one supplier may result in the impairment of those with alternative suppliers. Consider the previous supplier relationship portfolio. Although both suppliers should be selected at the current period, the development strategy of these two relationships should be dynamically and interdependently adapted to the revealed contingencies over the planning horizon. If one relationship turns out to be more favourable than alternatives in the future, it may force alternatives out of the relationship portfolio. Figure 4 shows the competitive effect by plotting the optimal strategy at period 1. Below boundary line 2, supplier relationship A pushes B out of the relationship portfolio. In contrast, above boundary line 1, supplier relationship B is more valuable than A. In the area between, the two relationships are competitive and should both be maintained.

## 5.2 *Complementarity effect of relationship portfolios*

Besides the competitive effect, relationships in a relationship portfolio may behave as complements. For instance, buying firms might consider retaining a reliable supplier relationship to assure supply alongside a risky one to benefit from its potential improvement. This advantage should be traded off against any cost differential.

**Figure 4** The competitive effect




Consider three example portfolios, which respectively consist of both suppliers, only supplier A, and only supplier B. Table 4 indicates that the portfolio consisting of both suppliers out performs those consisting of only one of them.

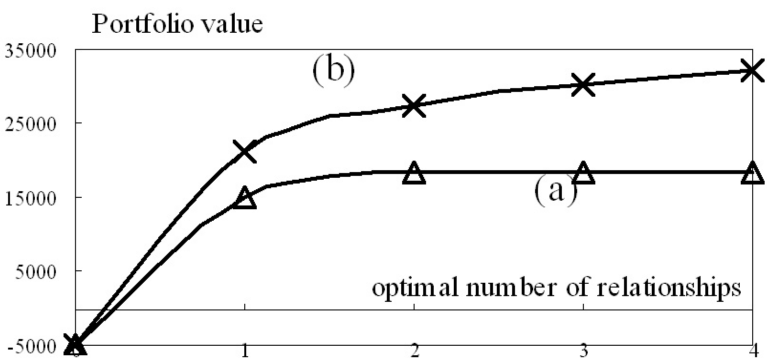
**Table 4** Complementarity effect

| *Cases* | *Supplier A only* | *Supplier B only* | *Both suppliers* |
|---|---|---|---|
| Value | (26,766; 42,479; 57,721; 68,886) | (30,469; 50,121; 65,363; 85,014) | (37,843; 56,197; 71,632; 87,284) |

## *5.3 Optimal size of supplier relationship portfolios*

Determining the number of suppliers and the best way to structure supplier relationships is a crucial aspect of supply chain management (Johnson and Pyke, 2001; Swaminathan and Tayur, 2003). This is closely related to the age-old controversy over sole versus multiple sourcing (Elmaghraby, 2000).

**Figure 5** Optimal number of relationships



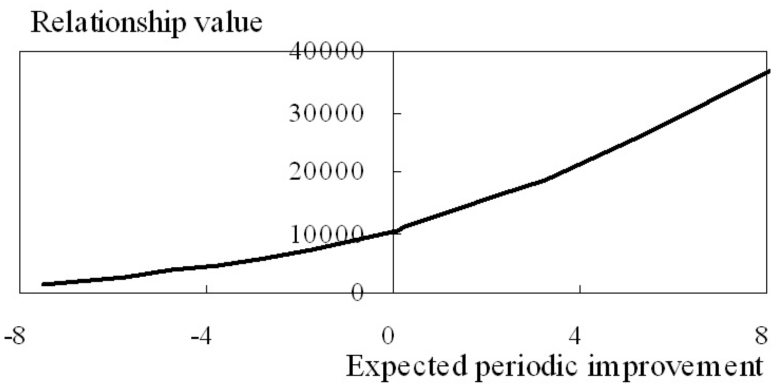
Consider a portfolio of n supplier relationships with the same relationship parameters as relationship A and a review horizon of three periods. The total portfolio value is illustrated by Figure 5(a). Increasing the number of supplier relationships from 0 to 2 increases relationship portfolio value. This is due to the complementarity benefit. However, adopting more supplier relationships will not improve this portfolio. With a smaller number of relationships, buying firms can intensify the selected relationships, concentrate relationship investment and reduce administrative costs. There is a certain threshold, two supplier relationships in our illustration, above which increased relationship investments and reduced relationship value outweigh the benefits gained from relationships with more suppliers.

For low-value, non-strategic commodities, relationship investment $\tilde{C}$ and exit cost $\tilde{X}$ are low. More supplier relationships can be maintained. This is illustrated by (b) in Figure 5 when both relationship investment $\tilde{C}$ and exit cost $\tilde{X}$ decrease to (100; 120; 130; 150) and no minimum relationship intensity $q_{\min}$ is committed. At the other extreme, when relationships require high relationship investment $\tilde{C}$, minimum relationship commitment $q_{\min}$, and exit cost $\tilde{X}$, fewer relationships should be fostered simultaneously.

### *5.4 Make-or-buy decisions and the choice of relationship type*

The make-or-buy problem boils down to whether the relationship value of prospects covers relationship investments and outweighs the value obtainable from the in-house option. In our example case, the portfolio's positive value supports the 'buy' decision.

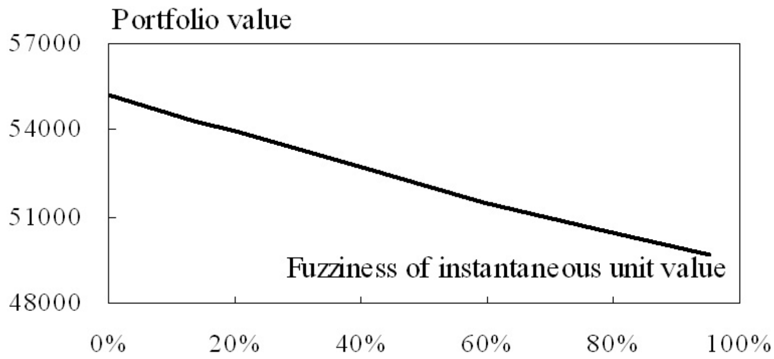
**Figure 6** Impact of the decreased expected periodic improvement of prospective relationship A




Our study reveals that with the increase in relationship investment or the decrease in initial instantaneous unit supplier value, expected relationship improvement and expected relationship duration, relationship value drops and 'make' tends to be appealing. Figure 6 illustrates that the relationship value declines in accordance with the decline in the expected periodic improvement of prospective relationship A. Nevertheless, the effect of relationship improvement on relationship value is not linear. This is because the buyer has the flexibility to adjust its relationship portfolio, which will mitigate the negative effect. Likewise, initial instantaneous unit supplier value and relationship investment each exhibit nonlinear impact. In fact, this property demonstrates the value and the necessity of adaptive relationship portfolio management.

If we measure the fuzziness of a fuzzy number by the ratio of the spread of its core to its defuzzified value, then the fuzziness of instantaneous unit supplier value has a negative effect upon expected relationship value, as suggested in Figure 7. This suggests the value of trust and information. With better information, stronger trust and higher commitment, the customer will encounter fewer opportunistic behaviours by suppliers and thus expect fewer downward movements and more upward movements, which indicate higher periodic improvement. Moreover, better information reduces the fuzziness of estimates of relationship variables. Both increase the supplier relationship value and justify close relationships.

**Figure 7** Impact of fuzziness of the instantaneous unit value of relationship B




## 6 Results and discussion

Proper construction of a supplier portfolio should reflect uncertainty, information vagueness, relationship dynamism, switching costs, and the interdependency among alternatives. Previous sections of this paper presented an integrated modelling framework for supplier portfolio configuration and adaptive development planning, and conducted a numerical study to illustrate how this framework could be used to assess, configure, and optimise a supplier relationship portfolio under uncertainty.

Several points emerge from the numerical study:

1. A supplier relationship portfolio shows both competitive and complementarity effects.
2. Flexibility to adaptively manage supplier relationship portfolios will increase relationship portfolio value, which our model can quantify.
3. Better information, strong trust, and higher commitment contribute to relationship value creation by reducing the fuzziness of relationship parameters and increasing the expected relationship improvement.
4. No sourcing strategy is dominant. Sole and multiple sourcing each have their place.

The same is true of relational and transactional sourcing.

These findings have notable implications. Suppliers can adjust their strategies to reflect the competition effect. A new supplier can increase its competitiveness by decreasing up-front relationship initiation cost, required minimum commitment, and relationship exit cost. Alternatively, a supplier that has a dominant relationship with a customer can require high relationship commitment or increase relationship exit cost to repel rivals and safeguard the relationship.

Besides the competitive effect, relationships in a portfolio may behave as complements. The relationship attributes will determine whether the competitive effect or the complementarity effect will dominate. If the total relationship intensity is expected to be high or if the relationship investment, the relationship exit cost, and the minimum commitment to both relationships are all low, the complementarity effect prevails. Otherwise, the competitive effect dominates. As discussed in Section 5.2, the decrease in the exit cost of relationship B enhances the complementarity effect. Indeed, low exit cost, low relationship commitment, and low relationship investment provide high flexibility to adjust relationship intensity in response to contingencies. This will increase the relationship value since firms need not worry about future adverse relationship evolution.

The results of our study provide actionable insight on determining the number of suppliers and the best way to structure the relationships with these suppliers. For low-value, non-strategic commodities, relationship investment and exit cost are low. More supplier relationships can be maintained. At the other extreme, when relationships require high relationship investment, minimum relationship commitment and exit cost, fewer relationships should be fostered simultaneously.

Our study demonstrates that better information, strong trust, higher commitment, and flexibility to adaptively manage supplier relationship portfolios will increase relationship portfolio value. Our study also reveals that with the increase in relationship investment or the decrease in initial instantaneous unit supplier value, expected relationship improvement, and expected relationship duration, relationship value drops and 'make' tends to be appealing.

Finally, many modern frameworks for supply chain management, such as the Toyota Production System, tend to advocate close and long-term supplier relationships. However, our study suggests that the benefits of intense relationships should be traded off against higher relationship investment and exit cost. Various types of relationships require different degrees of investment, commitment and relationship exit cost, and are therefore appropriate for different situations. For example, when supply security is critical (represented by high emergency cost), collaboration is needed (represented by intense investment). When the relationship objects' life cycle is long (represented by long planning horizon) and demand is high (represented by high total relationship intensity), close relationships are preferable since these may increase relationship value. An arm's length (transactional) relationship is sometimes favourable since this reduces relationship exit cost and enables the buyer to take advantage of flexibility and competitive effects.

## 7 Conclusions

To the best of our knowledge, we are the first to develop a forward-looking, dynamic, fuzzy, stochastic decision model to assess, configure, and optimise a supplier relationship portfolio under uncertainty. This framework can be used to study interdependence effects in a relationship portfolio and trade off conflicting relationship dimensions such as current and future contingent benefits, non-retrievable relationship investment and potential improvement, relationship commitment and adaptation flexibility. By taking a long-term and dynamic perspective, our work enables managers to have a broad view of relationship development and to respond to contingencies adaptively. It reveals and quantifies the value of adaptation flexibility embedded in the longitudinal and dynamic relationship process. It also deals with two sorts of relationship uncertainty, i.e., relationship risk in the sense of randomness and information vagueness in the sense of fuzziness, which are often neglected in the literature. Although at the outset the underlying mathematical formulation seems complex, the fuzzy binomial tree is easy to implement and extend. It can be incorporated into more elaborate decision support systems that analyse supplier relationship portfolios.

Our work provides a basis for further research on dynamic, strategic supplier relationship management. Several areas merit attention. For example, supply contracting and purchasing decision-making that are more operationally oriented should be integrated with strategic supplier relationship management into a consistent framework. Another worth while pursuit would be to analyse interaction, dynamic coordination, incentive compatibility, and risk and profit sharing between the buyer and the supplier in the relationship development process from the point of view of both sides.

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## Notes

1 Desirable and undesirable outputs are terms commonly used in data envelopment analysis (DEA).

## Appendix

### *Fuzzy arithmetic operations*

This Appendix briefly outlines the fuzzy arithmetic operations used in this paper. For more knowledge of fuzzy sets, the reader is referred to Kaufmann and Gupta (1991), Dubois and Prade (1992), Pedrycz and Gomide (1998), and Zimmermann (2001).

According to the representation theorem of fuzzy sets (Pedrycz and Gomide, 1998), all the partial results derived by $\alpha$-cuts can then be merged, reconstructing a solution to the problem in its original formulation. We can derive their fuzzy arithmetic operations for trapezoidal fuzzy numbers from the representation theorem of fuzzy sets and Dubois and Prade (1992) as follows:

Addition: $\tilde{A} \oplus \tilde{B} = [a_1 + b_1; a_2 + b_2; a_3 + b_3; a_4 + b_4]$. (A1)

Subtraction: $\tilde{A} \ominus \tilde{B} = [a_1 - b_4; a_2 - b_3; a_3 - b_2; a_4 - b_1]$. (A2)

Scalar multiplication by an ordinary number $k$:

$$k \otimes \tilde{A} = \begin{matrix} [ka_1; ka_2; ka_3; ka_4], & \text{if } k \geq 0 \\ [ka_4; ka_3; ka_2; ka_1], & \text{if } k < 0 \end{matrix}. \tag{A3}$$

To date, the literature contains there are few mechanisms in the literature for comparing and defuzzifying fuzzy numbers. We propose the following method to fill this gap. Define the defuzzified value of the α-level set $[\tilde{A}]^\alpha = (\underline{A}^\alpha; \overline{A}^\alpha)$ as a real number

$$Def\left([\tilde{A}]^\alpha\right) = \frac{1}{2}\left(\underline{A}^\alpha + \overline{A}^\alpha\right) - \frac{1}{2} r_A \left(\overline{A}^\alpha - \underline{A}^\alpha\right). \tag{A4}$$

where $r_A$ is the fuzziness aversion factor of the decision maker. $r_A$ measures the decision maker's tolerance towards of fuzziness, $0 \leq r_A \leq 1$. At one extreme, $r_A = 0$ indicates that the decision maker is 'fuzziness neutral'. At the other extreme, $r_A = 1$ suggests that the decision maker is totally 'fuzziness averse'.

The defuzzified value of $\tilde{A}$ is defined as the level-weighted average of the arithmetic means of all defuzzified values of the $\alpha$-level set

$$\begin{aligned} Def(\tilde{A}) &= \frac{\int_0^1 \alpha \frac{1}{2}\left[\underline{A}^\alpha + \overline{A}^\alpha - r_A(\overline{A}^\alpha - \underline{A}^\alpha)\right] d\alpha}{\int_0^1 \alpha d\alpha} \\ &= \int_0^1 \alpha \left[\underline{A}^\alpha + \overline{A}^\alpha - r_A(\overline{A}^\alpha - \underline{A}^\alpha)\right] d\alpha. \end{aligned} \tag{A5}$$

It is easy to derive the defuzzified value of a trapezoidal fuzzy number $\tilde{A} = (a_1; a_2; a_3; a_4)$ as

$$Def(\tilde{A}) = \frac{(a_1 + 2a_2 + 2a_3 + a_4)}{6} + \frac{(a_1 + 2a_2 - 2a_3 - a_4)}{6} r_A. \tag{A6}$$

With the defuzzification operator in place, we can compare fuzzy numbers $\tilde{A}$ and $\tilde{B}$ by according to comparing their corresponding defuzzified values.