Ann Oper Res (2017) 257:519–536
DOI 10.1007/s10479-016-2136-1

S.I.: INNOVATIVE SUPPLY CHAIN OPTIMIZATION

Designing structured supply contracts under demand and price uncertainty in an open supply chain

Zhanping Cheng[1] · **Xiaodong Yang**[2] · **Andy A. Tsay**[3]

Published online: 16 February 2016

Abstract This paper develops an integrated model for analyzing and designing structured supply contracts from the perspectives of the buyer, the supplier, and the entire supply chain in an open supply chain. We first present a flexible framework that encapsulates a wide range of contracting types that have been studied previously. We then introduce the concept of relative contract value vis-a-vis a reference alternative, which facilitates addressing explicitly both the demand uncertainty and the price uncertainty within which real supply relationships operate. To guide practitioners in designing optimal supply contracts, we derive closed-form expressions for optimal contract structure, quantity commitment and flexibility, pricing, and sharing policy as well as the conditions to maximize total supply chain profitability associated with a contract. Our research demonstrates that structured contracts consisting of several fixed and/or flexible components are capable of maximizing total supply chain profit and allocating profit between contracting parties arbitrarily.

Keywords Supply chain management · Supply contracts · Price uncertainty · Demand uncertainty · Flexibility

✉ Xiaodong Yang
yangxiaodong@bfsu.edu.cn

Zhanping Cheng
zhpcheng@yahoo.com

Andy A. Tsay
atsay@scu.edu

1 BASF SE, Global Controlling, Speyerer Strasse 2, 67117 Limburgerhof, Germany

2 International Business School, Beijing Foreign Studies University, Beijing 100089, China

3 Leavey School of Business, Santa Clara University, Santa Clara, CA 95053, USA

1 Introduction

Supplier relationship management and supply contracting are emerging as critical ways to improve supply chain performance. Supply contracts, however, are often difficult to design. First, the contracts employed in the real business world are diverse and complex. In many cases, a single contract will combine multiple provisions in order to address a broad range of issues and risks, which enormously complicates contract specification and analysis. Second, although supply contracts can assure supply, safeguard investments in the specific relationship, and reduce uncertainty in cost, they do come with their own hazards. For example, if demand turns out to be weak, committing to buy a fixed quantity would result in significant inventory buildup and write-offs. Third, supply contracting does not occur in an isolated and exclusive supply chain. As argued by Macneil (1980), contracts become irrelevant when the parties have no alternative courses of action. Access to alternatives affects the perception of contract value and the enforcement. Supply contracting requires appropriate pricing, a careful balancing of cost against other priorities, mutually agreeable allocation and sharing of the consequences of uncertainty, and proper design of contract structure.

As a response to these challenges, which are insufficiently addressed in the literature, this paper proposes a flexible framework for analyzing and designing structured contracts from the perspective of the buyer, the supplier, and the supply chain. We provide understanding and general guidelines on how to use and design structured contracts in an open supply chain: how to construct a structured contract, what are the possible values of contract parameters, when the contract can maximize the joint value, and how joint value can be allocated. Our study is unique in four aspects. First, we do not restrict consideration to a given type of supply contracts. Instead, we directly deal with the diversity in contract forms in an integrated manner and provide *a flexible and unified framework* that practitioners can employ and extend to analyze their specific problems in both manufacturing and retailing environments. Thus, practitioners need not go to individual studies under different assumptions and specific contexts in the literature. Second, we do not assume a bilateral monopoly in an isolated and exclusive contracting episode; rather, we allow for dynamically evolving *alternative sources and spot market purchasing*. To achieve this, we introduce a comparative and value-added concept of supply contract value (defined in Sect. 3). Third, we propose a new contract format called a *structured contract*, which may combine multiple clauses specifying quantity commitment and flexibility, price discounts, revenue and cost sharing, return privileges, and capacity reservation terms. We analyze the properties of different structured contracts and provide guidelines and insights regarding how to price, commit to quantity, negotiate for flexibility, and specify a structured contract to deal with demand and price uncertainty. We also develop conditions under which joint supply chain value can be maximized and analyze how this value can be divided. We show that a structured contract consisting of several components provides more powerful mechanisms to achieve supply chain optimality and to divide the joint value between the contract parties. Fourth, we attempt to deal with not only *demand uncertainty* but also *price uncertainty* of each component. To capture the price uncertainty, we introduce a Geometric Brownian process to model the uncertain fluctuation and temporal evolution of the spot market price.

The rest of the paper is organized as follows. Section 2 briefly reviews related literature. Section 3 presents a component-based model to formulate various contract arrangements and structured contracts. Subsequently, we model demand and price uncertainty and introduce the concept of contract value in an open supply chain. Section 4 analyzes the properties of various contract components and discusses how to design optimal contracts. Section 5 concludes. All proofs are relegated to the "Appendix".

2 Related literature

A large body of research has examined replenishment policy under a given supply contract and discussed how to improve supply chain efficiency and achieve channel coordination (see, e.g., Barnes-Schuster et al. 2002; Cachon and Lariviere 2005; Burnetas et al. 2007; Li et al. 2013; Zhao et al. 2014; Roy et al. 2015). Rather than summarize this vast literature, we refer the reader to Tsay et al. (1999), Cachon (2003) and Kleindorfer and Wu (2003) for comprehensive reviews.

Originating from inventory theory, a large branch of supply contract research deals with the uncertainty of demand and emphasizes the value of flexibility in the quantity of purchase (Tsay et al. 1999; Sethi et al. 2004). Less attention has been paid to the *contract price uncertainty*, i.e., the likelihood that the prenegotiated contract price will turn out to be unfavorable relative to alternatives, and the *impact of alternatives and spot market purchasing*.

Li and Kouvelis (1999) and Fotopoulos et al. (2008) develop methodologies to determine the value and the purchase time for time and quantity flexible supply contracts under price uncertainty. One significant difference between their works and ours is that in their setting the contract price rather than the spot market price is stochastic. Further, they assume deterministic demand and do not consider spot market purchasing. Cohen and Agrawal (1999) analyze the choice between long-term fixed-price contracts and short-term contracts which have uncertain price but provide flexibility. Bonser and Wu (2001) link purchasing from a given long-term contract to that from a spot market whose price is stochastic. Martinez-de Albeniz and Simchi-Levi (2005) investigate optimal replenishment strategy under option contracts in the presence of a spot market. Although a so-called portfolio contract is modeled as containing multiple options, the options in a single portfolio contract are assumed to be independent of each other. Li et al. (2009) consider a supply contracting problem in which the buyer faces price and demand uncertainty and compare spot market purchasing with long-term contracting with a single supplier. Like most other studies of supply contracting in the presence of a spot market, these papers focus on designing contracts to maximize the buyer's utility with little attention to the supply chain efficiency, profit split, and structured supply contracts. As explained by Tsay et al. (1999), Pasternack (2002) and De Giovanni and Roselli (2012), contract schemes often influence players' strategies through improvements in their individual profits.

Araman et al. (2003) study optimal replenishment strategy as well as system efficiency under a linear risk-sharing agreement, which resembles a quantity flexibility contract but assumes the price increases linearly with the amount reserved but not purchased. In their work, spot market purchasing serves only as a secondary supply source after the contract quantity is purchased and contract price uncertainty is not explicitly revealed. Wu et al. (2002), Wu and Kleindorfer (2005) and Golovachkina and Bradley (2003) integrate supply contracts with spot market purchasing with a stochastic market price, and investigate both the buyer's and the supplier's perspective. These works, however, are limited to option contracts and do not address structured supply contracts. In addition, to their supplier the spot market is only a secondary channel for liquidating excess inventory and the impact of alternative opportunities is ignored. Thus, contract price uncertainty for the supplier is not explicitly treated.

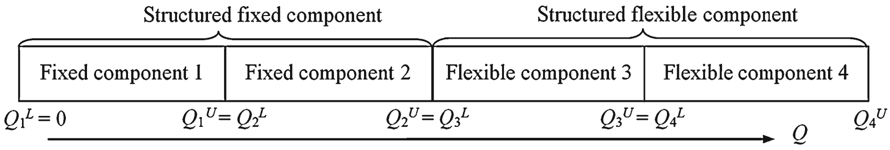

Fig. 1 A structured contract as a combination of fixed and flexible components

3 Modeling the structured supply contract

From an economic point of view, a broad range of supply contracts can be broken into several standard components, which are the smallest contract element containing a specific identifiable feature. Formally, a contract component i at time t is a vector: $(P_{ti}, X_{ti}, Q_{ti}^L, Q_{ti}^U, \theta)$,[1] where P_{ti} is the purchase price which is valid for purchased quantities between Q_{ti}^L and Q_{ti}^U, $Q_{ti}^L < Q_{ti}^U$; X_{ti} the up-front unit cost incurred at the contracting time; and θ the fraction of the total revenue the buyer can retain. Note that X_{ti}, which can also be termed a unit reservation fee, is a parameter usually used for flexible components but not for fixed components. In addition, X_{ti} locks in at the contracting time an obligation to pay $X_{ti}(Q_{ti}^U - Q_{ti}^L)$ at the contract's conclusion, which functions as a sunk cost at the purchasing time, irrespective of the quantity eventually purchased. To simplify notation we will suppress the time index t and the component index i whenever doing so will not cause confusion.

Components are either flexible or fixed. Flexible components provide the buyer a right, instead of an obligation, to purchase up to $Q_{ti} = (Q_{ti}^U - Q_{ti}^L)$. Note that a fixed component in this model cannot be treated simply as a special case of a flexible component, since fixed components push the demand uncertainty and excess inventory to the buyer, whereas flexible components shift demand and price uncertainty and excess inventory to the supplier. A buyer ought to compensate the supplier for the uncertainty reduction achieved by flexible components, perhaps in a proportional manner. The up-front unit cost, X_{ti}, represents such compensation at period t on a unit basis. A structured supply contract is a serial combination of fixed components and/or flexible components indexed by i in the purchase sequence, satisfying $Q_i^U = Q_{i+1}^L$, as depicted in Fig. 1.

This component-based contract representation can formulate a variety of well-known supply contracts, including wholesale contracts (Cachon 2003), quantity flexibility contracts (Tsay et al. 1999), revenue-sharing contracts (Cachon and Lariviere 2005), quantity discount and premium contracts (Tomlin 2003; Burnetas et al. 2007), capacity reservation contracts (Eppen and Iyer 1997), option contracts (Cheng et al. 2011; Barnes-Schuster et al. 2002), return and buyback contracts (Pasternack 1985; Bose and Anand 2007). Consider a structured contract where the buyer reserves a quantity of q_r at a discount price w_1. After demand is revealed, the buyer is allowed to adjust his purchase quantity down to a minimum quantity $q_{\min}$ with a unit penalty cost w_p or up to $q_{\max}$ (total available capacity) at a normal price w_2. Such a contract is a combination of a fixed component $\{P = w_1, Q^L = 0, Q^U = q_{\min}\}$, a flexible component $\{P = w_1 - w_p, X = w_p, Q^L = q_{\min}, Q^U = q_r\}$, and a flexible component $\{P = w_2, X = 0, Q^L = q_r, Q^U = q_{\max}\}$, as shown in Fig. 2.

Similarly, a contract with purchase quantity $q_{\max}$, price w, and a return policy that allows the buyer to return up to q_r of the purchased quantity for credit b per unit is a combination

[1] The notation used in the analysis is summarized in "Appendix 2".

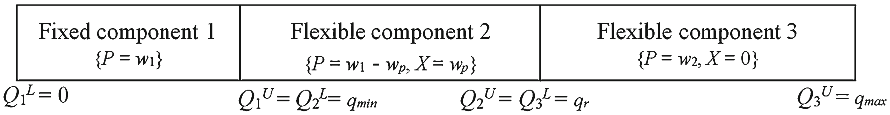

Fig. 2 The example contract as a combination of a fixed component and two flexible components

of a fixed component $\{P = w, Q^L = 0, Q^U = q_{\min} - q_r\}$ and a flexible component $\{P = b, X = w - b, Q^L = q_{\max} - q_r, Q^U = q_{\max}\}$.

In a structured contract, flexible components always follow the fixed components in the quantity order. Flexible components with a quantity flexibility policy should come after those with a return policy since a return policy requires that a quantity as high as its maximum quantity Q^U_{ti} should have been purchased at time t at first, which renders the quantity flexibility below Q^U_{ti} meaningless.

We define *supply contract value*, denoted by V, as the net expected profit accrued over a contract's duration in comparison with a reference alternative. The *reference alternative* represents the best alternative source and provides a benchmark for contract valuation. This paper assumes the spot market is the best alternative source and hence uses the terms "reference alternative" and "spot market" interchangeably. Letting S_t be the price of the reference alternative at period t, the incremental price of a contract component will be $\Delta P = P_{ti} - S_t$. This model also accommodates supply contracts in a retailing setting with an interpretation of S_t as the retail price. In this case, $S_t - P_{ti}$ represents the retailer's unit gross profit margin. Similarly, suppliers have opportunities to sell their products to alternative buyers or on the spot market. Let βS_t be the price that the supplier can achieve outside the focal supply chain, where $0 < \beta \leq 1$. In effect, β represents the supplier's market and bargaining power. We assume the price of the reference alternative S_t is exogenous and evolves over time stochastically, exposing both parties to price uncertainty. Let S_m be the price beneath which suppliers will exit the industry. Denote $S_t^{'} = S_t - S_m$ and assume $S_t^{'}$ follows a Geometric Brownian process with drift parameter a and variance parameter σ (Li and Kouvelis 1999; Hull 2006. Specifically, $\ln[S_t^{'}]$ is normally distributed with mean $\mu_t = (a - \sigma^2/2)t$ and variance $\sigma_t^2 = \sigma^2 t$, where a is the expected appreciation rate of $S_t^{'}$, and σ the volatility coefficient. Alternatively, S_t is lognormally distributed with mean $E[S_t] = (S_0 - S_m)e^{at} + S_m$ and the cumulative distribution function (Hull 2006)

$$F_t(S_t) = N_{0,1}\left(\frac{\ln[(S_t - S_m)/(S_0 - S_m)] - (a - \sigma^2/2)t}{\sigma\sqrt{t}}\right), \tag{1}$$

where $N_{0,1}(\cdot)$ is the standard normal cumulative distribution function.

For manufacturing companies, the demand for components is mainly prescribed by the demand for the higher-level end products. Following Cohen and Agrawal (1999) and Martinez-de Albeniz and Simchi-Levi (2005), we assume purchasing aims to support these demands, and has no price speculation motive. We define demand, D_t, as the quantity to be purchased at the beginning of period t. D_t may have alternative interpretations in different contexts. For instance, in a return contract D_t can be seen as the actual quantity demanded in production during time t. We assume D_t is distributed over $[D_t^a, D_t^b]$ with a truncated normal distribution function $\Phi_t(\cdot)$. Throughout this paper we use a tilde ($\sim$) to distinguish a random variable from a realized value.

4 Designing optimal supply contracts

The sequence of events and the notation in our model are as follows: In the contracting stage, the buyer and the supplier negotiate the contract structure and parameters through multiple offers and counteroffers. In each round, the supplier offers P_i or (P_i, X_i) for each range (Q_i^L, Q_i^U) and the buyer makes his counteroffer $(Q_i^{L'}, Q_i^{U'})$ for each P_i or (P_i, X_i) to optimize his own expected contract value according to anticipated D_t and S. In the purchasing stage, D_t and S_t, $1 \le t \le T$, are revealed. The buyer purchases the fixed quantity committed and optimally allocates the residual demand between flexible components and the spot market to maximize contract value. From the fixed components, the buyer takes any excess inventory, which can be salvaged at hS_t after period t, where $h < \beta$. From the flexible components, the quantity committed but not purchased results in a loss of eS_t for the supplier. This is because the quantity committed but not purchased by the buyer can be salvaged for $(\beta - e)S_t$ after period t. The open supply chain is depicted in Fig. 3.

4.1 Designing optimal fixed components

Fixed components with fixed purchase quantity simplify contract implementation and logistics management, and reduce the variance of orders and production experience by the suppliers. With a fixed component, goods are "pushed" to the buyer, who bears the demand risk in the supply chain (Cachon 2003). Because predetermined quantity enables them to manage capacity more efficiently, suppliers are often willing to attach price discounts to fixed components.

For the buyer, the expected value from a fixed component i in a structured contract, $E[V_{ti}^B]$, is

$$E\left[V_{ti}^B\right] = \int_{Q_{ti}^L}^{Q_{ti}^U} \left[E[\widetilde{S}_t]\left(D_t - Q_{ti}^L\right) + hE[\widetilde{S}_t]\left(Q_{ti}^U - D_t\right) - P_{ti}\left(Q_{ti}^U - Q_{ti}^L\right)\right]\Phi_t'(D_t)dD_t$$
$$+ \left(E[\widetilde{S}_t] - P_{ti}\right)\left(Q_{ti}^U - Q_{ti}^L\right)\int_{Q_{ti}^U}^{\infty}\Phi_t'(D_t)dD_t + \left(hE[\widetilde{S}_t] - P_{ti}\right)\left(Q_{ti}^U - Q_{ti}^L\right)$$
$$\times \int_0^{Q_{ti}^L}\Phi_t'(D_t)dD_t. \tag{2}$$

The first, second, and third terms in Eq. (2) are the buyer's value of purchasing from component i as opposed to the spot market plus the salvage value of excess inventory when $Q_{ti}^L < D_t < Q_{ti}^U$, $D_t > Q_{ti}^U$, and $D_t < Q_{ti}^L$, respectively. Simplifying Eq. (2) yields

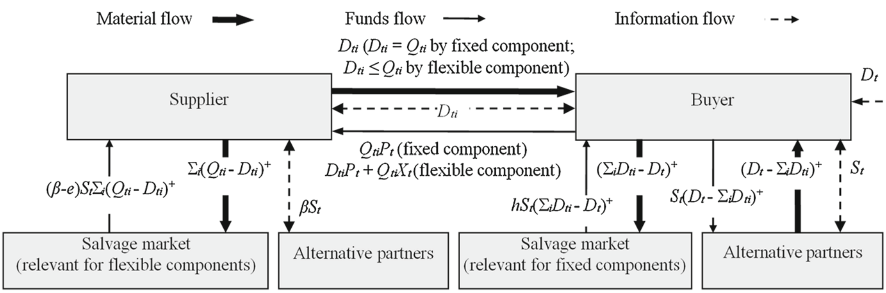

Fig. 3 The open supply chain

$$E\left[V_{ti}^{B}\right]=\left(E[\widetilde{S}_t]-P_{ti}\right)\left(Q_{ti}^{U}-Q_{ti}^{L}\right)-(1-h)E[\widetilde{S}_t]\int_{Q_{ti}^{L}}^{Q_{ti}^{U}}\Phi_t(D_t)dD_t. \tag{3}$$

The buyer's expected marginal value of commitment is then

$$dE\left[V_{ti}^{B}\right]/dQ_{ti}^{U}=\left(E[\widetilde{S}_t]-P_{ti}\right)-(1-h)E[\widetilde{S}_t]\Phi_t\left(Q_{ti}^{U}\right). \tag{4}$$

As long as $(E[\widetilde{S}_t]-P_{ti})-(1-h)E[\widetilde{S}_t]\Phi_t(Q_{ti}^{U})>0$, the buyer is inclined to increase commitment. Given P_{ti}, $P_{ti}<E[\widetilde{S}_t]$, the maximum quantity the buyer is willing to commit, $Q_{ti}^{U}(P_{ti})_{\max}$, will be

$$Q_{ti}^{U}(P_{ti})_{\max}=\Phi_t^{-1}\left(\frac{E[\widetilde{S}_t]-P_{ti}}{(1-h)E[\widetilde{S}_t]}\right). \tag{5}$$

For the supplier, the expected contract component value, $E[V_{ti}^{S}]$, is

$$E\left[V_{ti}^{S}\right]=\left(P_{ti}-\beta E[\widetilde{S}_t]\right)\left(Q_{ti}^{U}-Q_{ti}^{L}\right). \tag{6}$$

From Eqs. (3) and (6), the expected joint value for the entire supply chain, $E[\pi_{ti}^{Fix}]$, is

$$E\left[\pi_{ti}^{Fix}\right]=E\left[V_{ti}^{B}+V_{ti}^{S}\right]=(1-\beta)E[\widetilde{S}_t]\left(Q_{ti}^{U}-Q_{ti}^{L}\right)-(1-h)E[\widetilde{S}_t]\int_{Q_{ti}^{L}}^{Q_{ti}^{U}}\Phi_t(D_t)dD_t. \tag{7}$$

Then, the marginal value of commitment for the supply chain is

$$dE\left[\pi_{ti}^{Fix}\right]\Big/dQ_{ti}^{U}=(1-\beta)E[\widetilde{S}_t]-(1-h)E[\widetilde{S}_t]\Phi_t\left(Q_{ti}^{U}\right). \tag{8}$$

Since Eq. (7) is concave, the global optimal commitment for the supply chain will be

$$Q_t^{U^*}=\Phi_t^{-1}((1-\beta)/(1-h)). \tag{9}$$

$\beta E[\widetilde{S}_t]<P_{ti}<E[\widetilde{S}_t]$ must hold in order to ensure the willingness of both parties to enter the contract. Hence, $Q_t^{U^*}>Q_{ti}^{U}(P_{ti})_{\max}$, indicating that the buyer commits less than the supply chain optimal quantity. Thus we establish for the open supply chain setting a property that is known of wholesale-price-only contracts in "selling-to-the-newsvendor" types of closed supply chains (Cachon 2003):

Proposition 1 *A single fixed component cannot achieve optimality for the supply chain.*

The joint supply chain value of a contract consisting of a single fixed component is maximized at $P_t=\beta E[\widetilde{S}_t]$, which gives the entire supply chain value to the buyer. Therefore, supplier participation will be contingent on the ability to reallocate profit within the supply chain.

Under a revenue-sharing policy which stipulates that the supplier can share a fraction $1-\theta$ of the buyer's total revenue, the buyer's expected contract value will be

$$E\left[V_{ti}^{B}\right]=\left(\theta E[\widetilde{S}_t]-P_{ti}\right)\left(Q_{ti}^{U}-Q_{ti}^{L}\right)-\theta(1-h)E[\widetilde{S}_t]\int_{Q_{ti}^{L}}^{Q_{ti}^{U}}\Phi_t(D_t)dD_t. \tag{10}$$

θ affects value allocation without direct effect on joint supply chain value. Thus, as in the selling-to-the-newsvendor setting (Cachon and Lariviere 2005), in the open supply chain environment we can obtain:

Proposition 2 (1) *In a fixed component, a revenue-sharing policy can induce the buyer to commit up to* D_t^b *when* θ *is chosen appropriately, where* $P_{ti}/E[\widetilde{S}_t] \le \theta \le P_{ti}/(hE[\widetilde{S}_t])$. (2) *If* $\theta = P_{ti}/(\beta E[\widetilde{S}_t])$, *a revenue-sharing policy can achieve the supply chain optimum. Adjusting* θ *and keeping* $P_{ti} = \theta\beta E[\widetilde{S}_t]$ *will allocate joint value arbitrarily.*

Revenue-sharing, however, incurs administrative burden. The supplier may have difficulty monitoring the buyer's revenues and verifying whether they are being appropriately shared. Therefore, revenue-sharing *might not be implementable* in certain situations (Cachon and Lariviere 2005).

Proposition 3 (1) *Decomposing a fixed component with a price* P_t *into a discount scheme with n subcomponents such that* $E[\widetilde{S}_t] \ge P_{t1} > \cdots > P_{tn} = P_t \ge \beta E[\widetilde{S}_t]$ *will transfer a portion of the buyer's value to the supplier.* (2) *The maximum value of the supply chain can be achieved by a contract containing n fixed components with* $P_{tn} = \beta E[\widetilde{S}_t]$. *As* $n \to \infty$, *the supplier can extract the supply chain's entire expected joint contract value,* $E[\widetilde{S}_t]\int_\beta^1 \Phi_t^{-1}((1-x)/(1-h))dx$.

Adjusting n and $P_{t1}, \ldots, P_{t,n-1}$ changes how the joint value will be split. $\sum_{i=1}^n P_{ti}(Q_{ti}^U - Q_{ti}^L)$ is the supplier's total revenue. Figure 4 illustrates how for a fixed supply chain structure the supplier can capture more value via several fixed components, i.e., C_2, C_3, and C_4, with decreasing prices (incremental quantity discount policy) than from a contract with a single fixed component C_1. In contrast, a portion of value will be transferred from the supplier to the buyer through the use of an increasing price scheme (incremental piece-wise quantity premium policy) with components C_5, C_6, and C_7. Thus, as found in the newsvendor environment, a discount scheme can generally be used to achieve supply chain optimality (e.g., Burnetas et al. 2007).

4.2 Designing flexible components

In providing flexibility a supplier relieves the buyer of some of the consequences of uncertainty. This is consistent with the "pull" principle and may improve the supply chain efficiency when the supplier is better positioned to accommodate uncertainty due to an ability to risk-pool (Cachon 2003). Flexible components, however, are difficult to analyze due to their flexibility and interdependence. We examine flexible components by means of marginal analysis. Let us first consider a *single* flexible component with $Q_t^L = 0$. Recall that we drop the component index i throughout this work when treating a single component. Given (X_t, P_t) and Q_t^U, the buyer's expected value is the value expected to be realized at purchasing time t minus the up-front reservation cost:

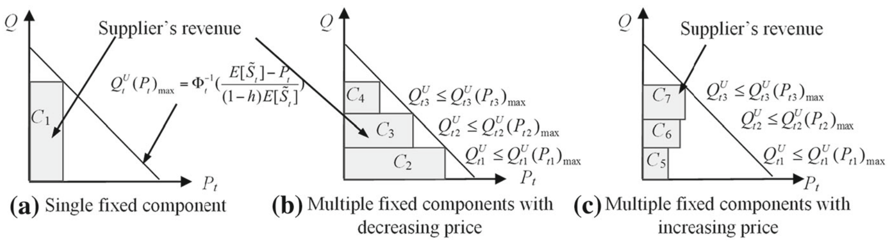

Fig. 4 Fixed components that differ in pricing scheme and structure

$$E\left[V_t^B\right]=\left[\int_0^{Q_t^U} D_t\Phi_t^{'}(D_t)dD_t+\int_{Q_t^U}^{\infty} Q_t^U\Phi_t^{'}(D_t)dD_t\right]\int_{P_t}^{\infty}(S_t-P_t)F_t^{'}(S_t)dS_t-X_tQ_t^U. \tag{11}$$

The buyer's marginal expected value with respect to the reserved quantity Q_t^U is

$$dE\left[V_t^B\right]\Big/dQ_t^U=\left(1-\Phi_t\left(Q_t^U\right)\right)\int_{P_t}^{\infty}(S_t-P_t)F_t^{'}(S_t)dS_t-X_t. \tag{12}$$

As long as $dE[V_t^B]/dQ_t^U \geq 0$, the buyer is willing to reserve more. The maximum quantity up to which the buyer is willing to reserve, $Q_t^U(X_t, P_t)_{\max}$, is

$$Q_t^U(X_t, P_t)_{\max}=\Phi_t^{-1}\left(1-\frac{X_t}{\int_{P_t}^{\infty}(S_t-P_t)F_t^{'}(S_t)dS_t}\right), \tag{13}$$

where $X_t \leq \int_{P_t}^{\infty}(S_t-P_t)F_t^{'}(S_t)dS_t$.

For the supplier, the expected contract value is

$$\begin{aligned}E\left[V_t^S\right]=&\left[\int_0^{Q_t^U} D_t\Phi_t^{'}(D_t)dD_t+\int_{Q_t^U}^{\infty} Q_t^U\Phi_t^{'}(D_t)dD_t\right]\\&\int_{P_t}^{\infty}\left[P_t+(e-\beta)S_t\right]F_t^{'}(S_t)dS_t+\left(X_t-eE[\widetilde{S}_t]\right)Q_t^U.\end{aligned} \tag{14}$$

The supplier's marginal expected value with respect to the commitment is

$$dE\left[V_t^S\right]\Big/dQ_t^U=\left(1-\Phi_t\left(Q_t^U\right)\right)\int_{P_t}^{\infty}\left[P_t+(e-\beta)S_t\right]F_t^{'}(S_t)dS_t+X_t-eE[\widetilde{S}_t]. \tag{15}$$

Summing Eqs. (11) and (14) provides the expected joint value of the contract for the supply chain:

$$E\left[\pi_t^{Flex}\right]=\left(Q_t^U-\int_0^{Q_t^U}\Phi_t(D_t)dD_t\right)(1+e-\beta)\int_{P_t}^{\infty}S_tF_t^{'}(S_t)dS_t-eE[\widetilde{S}_t]Q_t^U, \tag{16}$$

which is a concave function of Q_t^U.

Note that Eqs. (12), (13) and (15) can be extended to a flexible component in a structured contract consisting of *multiple* components since the marginal expected value with respect to Q_{ti}^U of flexible component i is a function of (X_{ti}, P_{ti}) at the point Q_{ti}^U, independent of Q_{ti}^L and the conditions of other flexible components. For any flexible component, however, the ultimate negotiated Q_{ti}^U will be greater than $Q_{t,i-1}^U$ but not greater than the maximum quantity up to which the buyer is willing to reserve, $Q_{ti}^U(X_{ti}, P_{ti})_{\max}$, i.e., $Q_{t,i-1}^U < Q_{ti}^U \leq Q_{ti}^U(X_{ti}, P_{ti})_{\max}$.

The marginal value of Q_{ti}^U in a flexible component i for the supply chain is then:

$$dE\left[\pi_t^{Flex}\right]\Big/dQ_{ti}^U=\left(1-\Phi_t\left(Q_{ti}^U\right)\right)(1+e-\beta)\left(E[\widetilde{S}_t]-\int_0^{P_{ti}}S_tF_t^{'}(S_t)dS_t\right)-eE[\widetilde{S}_t]. \tag{17}$$

Proposition 4 (1) *The maximum quantity the buyer is willing to reserve and the buyer's value of flexible component* i *are decreasing in* X_{ti}, $0 \leq X_{ti} \leq E[\widetilde{S}_t]$. *If* $X_{ti}=0$, *flexible component i is always valuable for the buyer and the maximum quantity up to which the buyer is willing to reserve reaches its maximum D_t^b.*

(2) *The quantity the buyer is willing to reserve and the buyer's value as well as the supply chain's expected value are non-increasing in P_{ti}.*

Different pairs (X_{ti}, P_{ti}) result in different expected joint value and splits of that value. X_{ti} affects the joint value indirectly through the buyer's reserved quantity. Reducing P_{ti} by ΔP_{ti} affects the expected value in an open supply chain in three respects. First, when the realized demand D_t is above Q_{ti}^L and the realized market price S_t is higher than P_{ti}, there is a *value-transfer effect*, i.e., a value of $(\min(D_t, Q_{ti}^U) - Q_{ti}^L)\Delta P_{ti}$ will be transferred from the supplier to the buyer. Second, there is a *purchase-winning effect* . When the realized market price satisfies $P_{ti} - \Delta P_{ti} < S_t < P_{ti}$, the buyer will turn from the spot market to the supply contract at the purchasing stage. Third, there is a *commitment-inducing effect*. The buyer has an incentive to reserve more at the contracting stage.

Proposition 5 *For a single flexible component, the following are optimal for the supply chain:*

$$P_t^* = F_t^{-1}(0), \text{ and } X_t^* = \left(E[\widetilde{S}_t] - P_t^*\right) e/(1 + e - \beta). \tag{18}$$

A single flexible component is not always able to achieve maximum supply chain value.

In specific contexts P_t may be subject to some additional constraints. For instance, the supplier may require that P_t exceed the marginal production cost. The supply chain optimal P_t is always the lowest allowable price.

Partially consistent with the finding of Pasternack (1985) that coordination in a selling-to-the-newsvendor setting occurs with a partial return refund, in our open supply chain environment both no compensation ($X_t = 0$) and complete compensation ($X_t = \int_{P_t}^{\infty}(S_t - P_t)F_t^{'}(S_t)dS_t$) are supply chain suboptimal. When there is no compensation, i.e. $X_t = 0$, the buyer will reserve a quantity greater than or equal to his maximum demand, i.e., $\Phi_t(Q_t^U) = 1$. As indicated by Eq. (16), the marginal supply chain value is $dE[\pi_t^{Flex}]/dQ_t^U = -eE[\widetilde{S}_t]$, implying that the buyer will overcommit. In contrast, complete compensation gives the buyer no incentive to commit the supply chain optimal quantity.

In the selling-to-the-newsvendor setting, a continuum of pairs (X_t, P_t) exists for unlimited return contracts with partial refund (Pasternack 1985) or option contracts (Cheng et al. 2011) to coordinate the system. In the presence of a stochastic spot market, however, the contract price must be set sufficiently low to prevent the spot market from "stealing" the purchase from the contract. This result is also observed in Wu et al. (2002) and Golovachkina and Bradley (2003). But unlike these works, we demonstrate that when the supplier has alternative opportunities, the constraint $P_t = F_t^{-1}(0)$ may make a flexible component fall short of providing the supplier an incentive to achieve the supply chain optimum. This complements the finding of Bose and Anand (2007) that an equilibrium return policy generally makes the supplier worse off in comparison with a price-only contract when the contract price is exogenously set low.

In contrast to a contract containing a single component, a structured contract possesses a more powerful mechanism to share the consequences of uncertainty, transfer value, and provide incentives for both parties to maximize the joint contract value and improve the supply chain efficiency, as demonstrated next.

Proposition 6 *A structured contract consisting of n flexible components with price $P_{ti} = F_t^{-1}(0)$ and increasing X_{ti}, $1 \leq i \leq n$, satisfying*

$$eE[\widetilde{S}_t] - \left(1 - \Phi_t\left(Q_{ti}^U\right)\right)\left(P_{ti} - (\beta - e)E[\widetilde{S}_t]\right) \leq X_{ti} \leq \left(1 - \Phi_t\left(Q_{ti}^U\right)\right)\left(E[\widetilde{S}_t] - P_{ti}\right), \tag{19}$$

can maximize joint supply chain value and split the joint value arbitrarily.

Proposition 7 *For flexible component i in a structured supply contract, $\{P_{ti}, X_{ti}, Q_{ti}^L, Q_{ti}^U\}$, the following relation holds:*

$$\begin{aligned} eE[\widetilde{S}_t] &- \left(1 - \Phi_t\left(Q_{ti}^U\right)\right)\int_{P_{ti}}^{\infty}[P_{ti} - (\beta - e)S_t]F_t^{'}(S_t)dS_t \\ &\leq X_{ti} \leq \left(1 - \Phi_t\left(Q_{ti}^U\right)\right)\int_{P_{ti}}^{\infty}(S_t - P_{ti})F_t^{'}(S_t)dS_t. \end{aligned} \tag{20}$$

The lower and the upper bounds on X_{ti} intersect at the supply chain optimal quantity $Q_t^{U^*}$ for a given P_{ti}.

Consider an illustrative numerical example used throughout this paper: $E[\widetilde{S}_t] = 10, a = 0$, $S_m = 4$, $\sigma = 0.2$, $h = 0.55$, $e = 0.1$, $\theta = 0$, and $\beta = 0.7$. Demand follows a truncated normal distribution over the range [0, 200] with expected value 100 and standard variance 20 . Consider a flexible component with $P_{6i} = 8$ at period 6.

Figure 5 illustrates how the lower and the upper bound of X_{6i} vary with the increase of the commitment. When the commitment is above the supply chain optimal quantity $Q_t^{U^*}$, no unit compensation X_{6i} can motivate both parties to increase commitment.

Proposition 8 *When $X_{ti} > 0$, the buyer's marginal value of Q_{ti}^U in flexible component i and the maximum quantity up to which the buyer is willing to reserve increase in σ_t. There is a threshold P_t^a, where $(S_0 - S_m)\exp(at - \frac{1}{2}\sigma_t^2) + S_m \leq P_t^a < (S_0 - S_m)\exp(at + \frac{1}{2}\sigma_t^2) + S_m$, such that the supply chain's marginal value of Q_{ti}^U increases in σ_t if $P_t > P_t^a$ and decreases in σ_t if $P_t < P_t^a$.*

This proposition implies the buyer tends to reserve a larger quantity from flexible components over time because the future market price becomes more volatile. This also suggests that the supplier would tend to increase contract price or require more compensation for providing flexibility when the uncertainty of market price increases. The supply chain's marginal value may decrease over time due to increased market price volatility when P is low. The

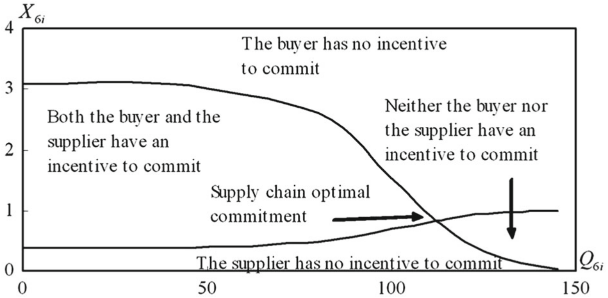

Fig. 5 Bounds on unit compensation

underlying reason is that the buyer's opportunistic behavior, i.e., purchasing from the spot market at a lower price instead of from the contract, will be more sensitive to the increase in market price volatility when P is low.

Proposition 9 *Let P_t be the minimum price the supplier is willing to offer. If $h < \beta - e$ and $(1+e-\beta)\int_0^{P_t} S_t F_t^{'}(S_t)dS_t < (1-\beta)E[\widetilde{S}_t]$, it is supply chain optimal to employ fixed components in a structured contract over the quantity range*

$$0 \le Q_t \le \Phi_t^{-1}\left(\frac{(1+e-\beta)\int_0^{P_t} S_t F_t^{'}(S_t)dS_t}{(1+e-\beta)\int_0^{P_t} S_t F_t^{'}(S_t)dS_t - (e+h-\beta)E[\widetilde{S}_t]}\right), \tag{21}$$

and flexible components over the quantity range

$$Q_t > \Phi_t^{-1}\left(\frac{(1+e-\beta)\int_0^{P_t} S_t F_t^{'}(S_t)dS_t}{(1+e-\beta)\int_0^{P_t} S_t F_t^{'}(S_t)dS_t - (e+h-\beta)E[\widetilde{S}_t]}\right). \tag{22}$$

Otherwise, it is supply chain optimal to employ fixed components.

An optimal mixture of fixed and flexible components is supply chain optimal in some cases in our model, as firm orders plus options (Barnes-Schuster et al. 2002) or minimum commitment plus flexible quantity (Tsay et al. 1999) in the selling-to-the-newsvendor setting. Consider the case of $P_6 = 8$ at period 6 in the previous numerical example.

Figure 6 shows that the marginal supply chain value of an optimal contract consisting of fixed components is greater than that of an optimal contract consisting of flexible components over the range $(0, \widehat{Q}_t)$. Fixed components may be advantageous over flexible components due to the buyer's possible opportunistic behavior. With the increase in Q_t, the benefit of quantity flexibility will increase, which mitigates the adverse effect of the buyer purchasing from the spot market, and makes flexible components more valuable. In other cases, however, the benefit of quantity flexibility might not be sufficient to offset the adverse effect of the buyer's possible opportunistic behavior or the buyer may be better positioned than the supplier to process the excess inventory. In other words, using fixed components improves supply chain efficiency.

This proposition also indicates that fixed components tend to be better for the supply chain with the increase of h, whereas flexible components become appealing with the decrease of e.

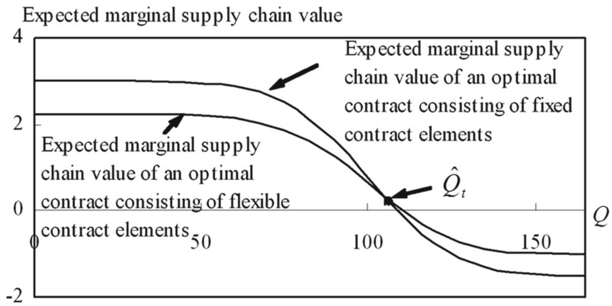

Fig. 6 Expected marginal supply chain value of fixed and flexible components

5 Conclusions

This paper develops an integrated model for understanding and designing more flexible and structured contracts from the perspective of the buyer, the supplier, and the system as a whole in an open supply chain, defined as one in which both parties have possible alternative partners. Our research demonstrates that a wide range of contract types can be viewed as a composite of fixed components and flexible components, where changing the composition alters the allocation between the contract parties of exposure to the uncertainty in price and demand. Our work also suggests that the concept of relative contract value with respect to a reference alternative provides a means to analyze the interaction between a contract and its alternatives and quantify the ramifications of contract price uncertainty. To assist practitioners with supply contract design, we explored different contract structures and analyzed how to configure a structured contract, when the total supply chain profit is maximized, and how this profit is allocated between the parties.

Our paper reveals some key managerial insights: (i) A contract consisting of only a single fixed component cannot maximize joint supply chain value, and a contract consisting of only a single flexible component is not always able to maximize joint supply chain value. (ii) Structured contracts consisting of several fixed and/or flexible components are capable of maximizing total supply chain profit and allocating profit between contract parties arbitrarily. (iii) It is supply chain optimal to partially compensate the supplier for providing flexibility. There exists a range for the unit cost within which both the buyer and the supplier have an incentive to increase contract quantity. Beyond the optimal supply chain quantity, however, it is impossible to provide such an incentive to both parties. (iv) With the increase in market price volatility, the buyer's marginal contract value and reserved quantity for a flexible component tend to increase. The marginal joint supply chain value increases in market price volatility when the contract price is greater than a certain threshold, but decreases otherwise. (v) From the supply chain's perspective, flexible components are not necessarily superior to fixed components in an open supply chain. Depending on the conditions, it may be optimal to adopt fixed components, flexible components, or a combination.

We look forward to future research that addresses some of the limitations of this paper. Our formulation uses the spot market as the reference alternative for each party. In reality, contracts may be written with a range of other partners or supply chains. To address this explicitly would require modeling a system with multiple suppliers and multiple buyers. Our general multi-period framework focuses only on the design of supply contracts for individual periods, but does not consider the ramifications of carrying inventory across periods, such as the price speculation motive. Allowing such possibilities would expand the dimensionality of strategies, and accordingly the space of possible contracts. In these the contract structures could change from period to period, or govern the actions taken across multiple periods collectively. The increase in complexity might call for heuristic approaches that can find reasonable policies when optimal ones are too difficult to obtain.

Appendix 1: Proof of Propositions

Proof of Proposition 2 Equation (10) gives the buyer's maximum commitment

$$Q^U_{ti}(P_{ti})_{\max} = \Phi_t^{-1}\left(\frac{\theta E[\widetilde{S}_t] - P_{ti}}{\theta(1-h)E[\widetilde{S}_t]}\right), \tag{23}$$

which shows that the buyer will commit only $Q_{ti}^U(P_{ti})_{\max} = D_t^a$ if $\theta = P_{ti}/E[\widetilde{S}_t]$ and increase commitment to $Q_{ti}^U(P_{ti})_{\max} = D_t^b$ with the increase of θ to $P_{ti}/(hE[\widetilde{S}_t])$. Equations (23), (9), and (10) indicate that the buyer will commit the supply chain optimal quantity and the buyer's contract value will be $E[V_{ti}^B] = \theta E[\pi_{ti}^{Fix}]$, if $\theta = P_{ti}/\beta E[\widetilde{S}_t]$. □

Proof of Proposition 3 The total commitment is dependent on P_{tn}, irrespective of the price of components 1 through $n-1$. By employing fixed components with decreasing component price P_{ti} , $E[\widetilde{S}_t] \geq P_{t1} > \cdots > P_{tn} \geq \beta E[\widetilde{S}_t]$, $1 \leq i \leq n$, the supplier can induce the buyer to commit up to $Q^U(P_{tn})_{\max}$ and gain from the higher price of components 1 through $n-1$. If $P_{tn} = \beta E[\widetilde{S}_t]$, the supply chain achieves its maximum joint value. As $n \to \infty$, if we keep setting $P_t = E[\widetilde{S}_t] - (1-h)E[\widetilde{S}_t]\Phi_t(Q_t)$ for Q_t over the entire range $[0, Q_{tn}^U]$, the buyer's marginal value will tend to zero. The supplier will obtain the entire supply chain value, i.e.,

$$\sum E\left[V_{ti}^S\right] = \int_{\beta E[\widetilde{S}_t]}^{E[\widetilde{S}_t]} \Phi_t^{-1}\left(\frac{E[\widetilde{S}_t] - P_t}{(1-h)E[\widetilde{S}_t]}\right) dP_t, \tag{24}$$

which can be simplified to $E[\widetilde{S}_t]\int_\beta^1 \Phi_t^{-1}((1-x)/(1-h))dx$. □

Proof of Proposition 5 Proposition 4 demonstrates that the supply chain value is maximized if $P_t^* = F_t^{-1}(0)$. Substituting $P_t^* = F_t^{-1}(0)$ into Eq. (17), we obtain the optimal supply chain commitment $Q_t^{U^*} = \Phi_t^{-1}((1-\beta)/(1+e-\beta))$, which indicates the corresponding $X_t^* = e(E[\widetilde{S}_t] - P_t^*)/(1+e-\beta))$ from Eq. (13). Equation (15) yields the second derivative with respect to the commitment Q_t of the supplier's value of a single flexible component:

$$d^2E\left[V_t^S\right]\Big/dQ_t^2 = -\Phi_t^{'}(Q_t)(P_t - (\beta - e)E[\widetilde{S}_t]). \tag{25}$$

$P_t^* = F_t^{-1}(0)$ indicates that $P_t^* \leq S_m$. From Eq. (15), we know $dE[V_t^S]/dQ_t = 0$ at $Q_t = Q_t^{U^*}$. In some cases the supplier always suffers a loss by a supply chain optimal contract consisting of a single flexible component. For example, if $S_m < (\beta - e)E[\widetilde{S}_t]$, $d^2E[V_t^S]/dQ_t^2 > 0$, i.e., $E[V_t^S]$ is a convex function in Q_t, when $0 < Q_t < Q_t^{U^*}$. Since $E[V_t^S] = 0$ at $Q_t = 0$ and $dE[V_t^S]/dQ_t = 0$ at $Q_t = Q_t^{U^*}$, $E[V_t^S] < 0$ at $Q_t = Q_t^{U^*}$. In other words, the supplier's expected value from a single flexible component is negative. Therefore, the supply chain optimum cannot be achieved. □

Proof of Proposition 6 Proposition 7 gives the upper and the lower bounds on X_{ti} as in Eq. (20). If Eq. (20) holds for component n, the buyer will reserve the supply chain optimal quantity $\Phi_t^{-1}((1-\beta)/(1+e-\beta))$. Adjusting X_{ti} but keeping $P_{ti} = F_t^{-1}(0)$ for $1 \leq i \leq n-1$ changes how the joint value will be split. As $n \to \infty$, at the lower bound, i.e., $X_{ti} = eE[\widetilde{S}_t] - (1 - \Phi_t(Q_{ti}^U))(P_{ti} - (\beta - e)E[\widetilde{S}_t])$, the supplier's marginal value $dE[V_t^S]/dQ_t^U$ is always zero and the buyer obtains the entirety of the supply chain value. Conversely, at the upper bound, i.e., $X_{ti} = (1 - \Phi_t(Q_{ti}^U))(E[\widetilde{S}_t] - P_{ti})$, the supplier obtains the entire supply chain value. □

Proof of Proposition 7 The marginal value for the buyer should be non-negative. Equation (15) indicates $X_{ti} \geq eE[\widetilde{S}_t] - (1 - \Phi_t(Q_{ti}^U))\int_{ti}^{\infty}[P_{ti} - (\beta - e)S_t]F_t^{'}(S_t)dS_t$. Similarly, Eq. (12) suggests $X_{ti} \leq (1 - \Phi_t(Q_{ti}^U))\int_{ti}^{\infty}(S_t - P_{ti})F_t^{'}(S_t)dS_t$. The lower and the upper bounds on X_{ti} intersect at

$$Q_t = \Phi_t^{-1}\left(1 - \frac{eE[\widetilde{S}_t]}{(1+e-\beta)(E[\widetilde{S}_t] - \int_0^{P_{ti}} S_t F_t^{'}(S_t)dS_t)}\right), \tag{26}$$

which is exactly the supply chain optimal quantity $Q_t^{U^*}$ for a given P_{ti}, indicated by Eq. (17). □

Proof of Proposition 8 Integration by parts transforms Eq. (13) into

$$Q_t^U(X_{ti}, P_{ti})_{\max} = \Phi_t^{-1}\left(1 - \frac{X_{ti}}{E[\widetilde{S}_t] - P_{ti} + \int_0^{P_{ti}} F_t(S_t)dS_t}\right). \tag{27}$$

To prove $\int_0^{P_{ti}} F_t(S_t)dS_t$ increases in σ_t, let $S_t^{'} = S_t - S_m$ and $P_{ti}^{'} = P_{ti} - S_m$. Equation (1) gives

$$\frac{d\int_0^{P_{ti}} F_t(S_t)dS_t}{d\sigma_t} = \int_0^{P_{ti}^{'}} N_{0,1}^{'}\left(\frac{\ln\left(S_t^{'}/S_0^{'}\right) - \mu_t}{\sigma_t}\right)\left(1 - \frac{\ln\left(S_t^{'}/S_0^{'}\right) - \mu_t}{\sigma_t^2}\right)dS_t^{'}. \tag{28}$$

Let $\mu_t = (a - \sigma^2/2)t$, $\sigma_t^2 = \sigma^2 t$, and $x = (\ln(S_t^{'}/S_0^{'}) - \mu_t)/\sigma_t$, i.e., $S_t^{'} = S_0^{'}\exp(x\sigma_t + \mu_t)$. Then

$$\begin{aligned}\frac{d\int_0^{P_{ti}} F_t(S_t)dS_t}{d\sigma_t} &= \frac{S_0^{'}}{\sqrt{2\pi}}\int_{-\infty}^{\frac{\ln(P_{ti}^{'}/S_0^{'})-\mu_t}{\sigma_t}} \exp\left(-\frac{x^2}{2}\right)(-x+\sigma_t)\exp(x\sigma_t+\mu_t)dx \\ &= \frac{P_{ti}^{'}}{\sqrt{2\pi}}\exp\left(-\frac{(\ln\left(P_{ti}^{'}/S_0^{'}\right) - \mu_t)^2}{2\sigma_t^2}\right) > 0,\end{aligned} \tag{29}$$

which indicates that the buyer's optimal reserved quantity is an increasing function of σ_t.

From Eq. (12), it is easy to prove that $dE[V_t^B]/dQ_t^U$ increases with σ_t. Whether the supply chain's marginal value increases or decreases with σ_t depends on P_{ti}. We rewrite Eq. (17) as:

$$\begin{aligned}dE\left[\pi_t^{Flex}\right]\Big/dQ_{ti}^U &= \left(1-\Phi_t\left(Q_{ti}^U\right)\right)(1+e-\beta)\left(E[\widetilde{S}_t]-P_{ti}F_t(P_{ti}) + \int_0^{P_{ti}} F_t(S_t)dS_t\right) \\ &\quad - eE[\widetilde{S}_t].\end{aligned} \tag{30}$$

Consider the term $-P_{ti}F_t(P_{ti}) + \int_0^{P_{ti}} F_t(S_t)dS_t$. From Eqs. (1) and (29), we obtain

$$\begin{aligned}\frac{d(-P_{ti}F_t(P_{ti}) + \int_0^{P_{ti}} F_t(S_t)dS_t)}{d\sigma_t} &= \frac{1}{\sqrt{2\pi}}\left[P_{ti}^{'} + \left(P_{ti}^{'}+S_m\right)\left(\frac{\ln\left(P_{ti}^{'}/S_0^{'}\right) - at}{\sigma_t^2} - \frac{1}{2}\right)\right] \\ &\quad \exp\left(-\frac{\left(\ln\left(P_{ti}^{'}/S_0^{'}\right) - \mu_t\right)^2}{2\sigma_t^2}\right).\end{aligned} \tag{31}$$

Obviously, $P_{ti}^{'} + (P_{ti}^{'} + S_m)\left(\frac{\ln(P_{ti}^{'}/S_0^{'})-at}{\sigma_t^2} - \frac{1}{2}\right)$ determines the sign of $\frac{d(dE[\pi_t^{Flex}]/dQ_{ti}^U)}{d\sigma_t}$. We can see that there is a threshold P_{ti}^a for P_{ti} satisfying $\ln\left(\frac{P_{ti}^a - S_m}{S_0 - S_m}\right) = at + \frac{1}{2}\sigma_t^2 - \frac{P_{ti}^a - S_m}{P_{ti}^a}\sigma_t^2$ such that $dE[\pi_t^{Flex}]/dQ_{ti}^U$ increases in σ_t when $P_{ti} > P_{ti}^a$ and decreases in σ_t when $P_{ti} < P_{ti}^a$. Furthermore, we obtain $(S_0 - S_m)\exp(at - \frac{1}{2}\sigma_t^2) + S_m \le P_{ti}^a < (S_0 - S_m)\exp(at + \frac{1}{2}\sigma_t^2) + S_m$. □

Proof of Proposition 9 From Eq. (8), we know the supply chain value of fixed components increases in Q_t when $0 \leq Q_t \leq \Phi_t^{-1}((1-\beta)/(1-h))$. From Eq. (17), we know it is supply chain suboptimal to employ flexible components if $(1+e-\beta)\int_0^{P_t} S_t F_t^{'}(S_t)dS_t > (1-\beta)E[\widetilde{S}_t]$, since the marginal supply chain value is negative and the increase in Q_t only results in more loss.

Consider the range $(1+e-\beta)\int_0^{P_t} S_t F_t^{'}(S_t)dS_t \leq (1-\beta)E[\widetilde{S}_t]$. Let $G(Q_t) = \frac{dE[\pi_t^{Flex}]}{dQ_t} - \frac{dE[\pi_t^{Fix}]}{dQ_t}$. Then

$$G(Q_t) = \Phi_t(Q_t)\left\{(1+e-\beta)\int_0^{P_t} S_t F_t^{'}(S_t)dS_t - (e+h-\beta)E[\widetilde{S}_t]\right\} - (1+e-\beta)\int_0^{P_t} S_t F_t^{'}(S_t)dS_t. \tag{32}$$

If $h > \beta - e$, then $(e+h-\beta)E[\widetilde{S}_t] > 0$. Therefore,

$$(1+e-\beta)\int_0^{P_t} S_t F_t^{'}(S_t)dS_t - (e+h-\beta)E[\widetilde{S}_t] < (1+e-\beta)\int_0^{P_t} S_t F_t^{'}(S_t)dS_t. \tag{33}$$

This indicates that $G(Q_t) < 0$, and in turn that fixed components are supply chain advantageous over flexible components. $h > \beta - e$ suggests that the buyer is better positioned than the supplier to process the excess inventory.

If $h < \beta - e$, then $(e+h-\beta)E[\widetilde{S}_t] < 0$. The marginal supply chain value of optimal flexible components will be greater than that of optimal fixed components, i.e., $G(Q_t) > 0$, when $Q_t > \Phi_t^{-1}\left(\frac{(1+e-\beta)\int_0^{P_t} S_t F_t^{'}(S_t)dS_t}{(1+e-\beta)\int_0^{P_t} S_t F_t^{'}(S_t)dS_t - (e+h-\beta)E[\widetilde{S}_t]}\right)$. Otherwise, it will be less than or equal to that of optimal fixed components. □

Appendix 2: Notation

The notation used in the analysis is summarized as follows.

a	Expected appreciation rate of $S_t^{'}$
b	Return credit per unit in a return contract
C_i	Component i of a contract
D_t	Demand at period t
D_t^a	Minimum possible demand at period t
D_t^b	Maximum possible demand at period t
eS_t	Unit loss at the supplier resulting from the quantity committed but not purchased by the buyer
$F_t(S_t)$	Cumulative distribution function of S_t
hS_t	Salvage value of unit excess inventory at the buyer after period t
P_{ti}	Purchase price in contract component i at period t
$q_{\max}$	Maximum quantity the buyer can purchase
$q_{\min}$	Minimum quantity the buyer must purchase
q_r	Reserved quantity in a back-up or return contract
Q_{ti}^L	Lower purchase quantity breakpoint of contract component i at period t
Q_{ti}^U	Upper purchase quantity breakpoint of contract component i at period t
Q_t^{U*}	Supply chain optimal purchase quantity at period t

S_m	Minimum spot market price, beneath which suppliers will exit the industry
S_t	Price of the reference alternative at period t
V_{ti}^B	The buyer's expected value from contract component i at period t
V_{ti}^S	The supplier's expected value from contract component i at period t
w	Purchase price in a back-up or return contract
w_p	Unit penalty cost in a back-up contract paid by the buyer to the supplier for each unit reserved but not purchased
X_{ti}	Up-front unit cost incurred at the contracting time for contract component i at period t
βS_t	Price at which the supplier can sell her products outside the supply chain
θ	Quota of the total revenue that the buyer can retain
μ_t	Mean of $\ln S_t^{'}$ at period t
π_{ti}^{Fix}	The supply chain's expected value from fixed contract component i at period t
π_{ti}^{Flex}	The supply chain's expected value from flexible contract component i at period t
σ	Volatility coefficient of $S_t^{'}$
$\Phi_t(\cdot)$	Truncated normal distribution function of D_t

References

Araman, V., Kleinknecht, J., & Akella, R. (2003). Coordination and risk-sharing in e-business. Working paper, Stanford University, Stanford, CA.

Barnes-Schuster, D., Bassok, Y., & Anupindi, R. (2002). Coordination and flexibility in supply contracts with options. *Manufacturing and Service Operations Management*, *4*(3), 171–207.

Bonser, J., & Wu, S. (2001). Procurement planning to maintain both short-term adaptiveness and long-term perspective. *Management Science*, *47*(6), 769–786.

Bose, I., & Anand, P. (2007). On returns policies with exogenous price. *European Journal of Operational Research*, *178*(3), 782–788.

Burnetas, A., Gilbert, S., & Smith, C. (2007). Quantity discounts in single period supply contracts with asymmetric demand information. *IIE Transactions*, *39*(5), 465–480.

Cachon, G. (2003). Supply chain coordination with contracts. In S. Graves & T. de Kok (Eds.), *Handbooks in operations research and management science: Supply chain management*. Amsterdam: Elsevier, North Holland.

Cachon, G., & Lariviere, M. (2005). Supply chain coordination with revenue-sharing contracts: Strengths and limitations. *Management Science*, *51*(1), 30–44.

Cheng, F., Ettl, M., Lin, G., Tonner, M., & Yao, D. (2011). Designing flexible supply chain contracts with options. In K. Kempf, P. Keskinocak, & R. Uzsoy (Eds.), *Planning production and inventories in the extended enterprise*. New York: Springer.

Cohen, M., & Agrawal, N. (1999). An analytical comparison of long and short term contracts. *IIE Transactions*, *31*(8), 783–796.

De Giovanni, P., & Roselli, M. (2012). Overcoming the drawbacks of a revenue-sharing contract through a support program. *Annals of Operations Research*, *196*(1), 201–222.

Eppen, G., & Iyer, A. (1997). Back-up agreements in fashion buying: The value of upstream flexibility. *Management Science*, *43*(11), 1469–1484.

Fotopoulos, S., Hu, X., & Munson, C. (2008). Flexible supply contracts under price uncertainty. *European Journal of Operational Research*, *191*(1), 251–261.

Golovachkina, N., & Bradley, J. (2003). Supplier–manufacturer relationships under forced compliance contracts. *Manufacturing and Service Operations Management*, *5*(1), 67–69.

Hull, J. (2006). *Options, futures and other derivatives*. Upper Saddle River, NJ: Prentice Hall.

Kleindorfer, P., & Wu, D. (2003). Integrating long- and short-term contracting via business-to-business exchanges for capital-intensive industries. *Management Science*, *49*(11), 1579–1615.

Li, C., & Kouvelis, P. (1999). Flexible and risk-sharing supply contracts under price uncertainty. *Management Science*, *45*(10), 1378–1398.

Li, S., Murat, A., & Huang, W. (2009). Selection of contract suppliers under price and demand uncertainty in a dynamic market. *European Journal of Operational Research*, *198*(3), 830–847.

Li, X., Li, Y., & Cai, X. (2013). Double marginalization in the supply chain with uncertain supply and coordination contract design. *European Journal of Operational Research*, *226*(2), 228–236.

Macneil, I. (1980). *The new social contract: An inquiry into modern contractual relations*. New Haven, CT: Yale University Press.

Martinez-de Albeniz, V., & Simchi-Levi, D. (2005). A portfolio approach for procurement contracts. *Production and Operations Management*, *14*(1), 90–114.

Pasternack, B. (1985). Optimal pricing and return policies for perishable commodities. *Marketing Science*, *4*(2), 166–176.

Pasternack, B. (2002). Using revenue sharing to achieve channel coordination for a newsboy type inventory model. In J. Geunes, P. Pardalos, & H. Romeijn (Eds.), *Supply chain management: Models, applications and research*. Dordrecht: Kluwer Academic.

Roy, A., Sana, S., & Chaudhuri, K. (2015). Optimal pricing of competing retailers under uncertain demand—A two layer supply chain model. *Annals of Operations Research*. doi:10.1007/s10479-015-1996-0.

Sethi, S. P., Yan, H., & Zhang, H. (2004). Quantity flexibility contracts: Optimal decisions with information updates. *Decision Sciences*, *35*(4), 691–711.

Tomlin, B. (2003). Capacity investments in supply chains: Sharing the gain rather than sharing the pain. *Manufacturing and Service Operations Management*, *5*(4), 317–333.

Tsay, A., Nahmias, S., & Agrawal, N. (1999). Modeling supply chain contracts: A review. In S. Tayur, R. Ganeshan, & M. Magazine (Eds.), *Quantitative models for supply chain management*. Boston, MA: Kluwer Academic.

Wu, D., & Kleindorfer, P. (2005). Competitive options, supply contracting, and electronic markets. *Management Science*, *51*(3), 452–466.

Wu, D., Kleindorfer, P., & Zhang, J. (2002). Optimal bidding and contracting strategies for capital-intensive goods. *European Journal of Operational Research*, *137*(3), 657–676.

Zhao, Y., Choi, T., Cheng, T. C. E., & Wang, S. (2014). Mean-risk analysis of wholesale price contracts with stochastic price-dependent demand. *Annals of Operations Research*. doi:10.1007/s10479-014-1689-0.