Appendix of proofs for:

Design of the reverse channel for remanufacturing: Must profit-maximization harm the environment?

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Appendix

Section A.1: The optimal solution for In-house remanufacturing

The profit-maximization problem for the In-house strategy is:

$$
\begin{aligned}
\max_{0\le p_{r1}<\alpha p_{n1};0\le\tilde{\theta}_1\le 1} \quad \pi_1 &= (p_{n1}-w_n)D_{n1}+p_{r1}D_{r1}-[S\int_{\theta=\tilde{\theta}_1}^{1} c_1(1-\theta)f(\theta)d\theta]-\beta(1-c_1) \\
&= (p_{n1}-w_n)D_{n1}+p_{r1}D_{r1}-c_1S[1-F(\tilde{\theta}_1)-\mu+H(\tilde{\theta}_1)],\\
s.t.&\\
D_{r1} &\le S[1-F(\tilde{\theta}_1)],
\end{aligned}
$$

where $D_{n1}=1-\frac{p_{n1}-p_{r1}}{1-\alpha}$, $D_{r1}=\frac{\alpha p_{n1}-p_{r1}}{\alpha(1-\alpha)}$, and $H(\tilde{\theta}_1)=\int_0^{\theta=\tilde{\theta}_1}\theta f(\theta)d\theta$.

The constraint on D_{r1} must bind at optimality since regardless of the p_{n1} and p_{r1} the retailer loses profit if it remanufactures items that it cannot sell. So the retailer will set $\tilde{\theta}_1$ to exactly match the remanufacturing volume to the demand. Then:

$$D_{r1}=\frac{\alpha p_{n1}-p_{r1}}{\alpha(1-\alpha)} \quad = \quad S[1-F(\tilde{\theta}_1],$$

which yields $p_{r1}=\alpha p_{n1}-\alpha(1-\alpha)S[1-F(\tilde{\theta}_1)]$. Substituting this into the expression for D_{n1} provides:

$$D_{n1} \quad = \quad 1-p_{n1}-\alpha S[1-F(\tilde{\theta}_1)].$$

Then the profit-maximization problem reduces to:

$$
\begin{aligned}
\max_{0\le p_{n1}\le 1;0\le\tilde{\theta}_1\le 1} \quad \pi_1 &= (p_{n1}-w_n)[1-p_{n1}-\alpha S[1-F(\tilde{\theta}_1)]]+\\
&+[\alpha p_{n1}-\alpha(1-\alpha)S[1-F(\tilde{\theta}_1)]][S[1-F(\tilde{\theta}_1]]-\\
&-c_1S[1-F(\tilde{\theta}_1)-\mu+H(\tilde{\theta}_1)]-\beta(1-c_1).
\end{aligned}
$$

Because for any given $\tilde{\theta}_1$, $\pi_1(p_{n1}|\tilde{\theta}_1)$ is strictly concave in p_{n1}. The first-order condition of $\pi_1(p_{n1})$ with respect to p_{n1} leads to $p^*_{n1}=\frac{1+w_n}{2}$, which is independent of $\tilde{\theta}_1$. The retailer's decision problem can then be expressed as the following single-variable optimization:

$$\max_{0\le\tilde{\theta}_1\le 1} \quad \pi_1 \quad = \quad \frac{(1-w_n)^2}{4}-\frac{S\alpha(1-w_n)[1-F(\tilde{\theta}_1)]}{2}+$$

$$
\begin{aligned}
& \{\frac{\alpha(1+w_n)}{2} - S\alpha(1-\alpha)[1-F(\tilde{\theta}_1)]\}\{S[1-F(\tilde{\theta}_1)]\} - \\
& -c_1 S[1-F(\tilde{\theta}_1)-\mu+H(\tilde{\theta}_1)] - \beta(1-c_1) \\
= \ & \frac{(1-w_n)^2}{4} + S\alpha w_n - S\alpha w_n F(\tilde{\theta}_1) - \\
& -S^2\alpha(1-\alpha)[1-F(\tilde{\theta}_1)]^2 - c_1 S[1-F(\tilde{\theta}_1)-\mu+H(\tilde{\theta}_1)] - \beta(1-c_1).
\end{aligned}
$$

Using $x = \alpha w_n$ and $y = 2S\alpha(1-\alpha)$ as placeholders as noted earlier, differentiation yields:

$$
\frac{d\pi_1}{d\tilde{\theta}_1} = Sf(\tilde{\theta}_1)\{-x+y[1-F(\tilde{\theta}_1)]+c_1(1-\tilde{\theta}_1)]\}; \tag{17}
$$

$$
\frac{d^2\pi_1}{d\tilde{\theta}_1^2} = Sf(\tilde{\theta}_1)[-yf(\tilde{\theta}_1)-c_1] + Sf'[-x+y[1-F(\tilde{\theta}_1)]+c_1(1-\tilde{\theta}_1)]. \tag{18}
$$

$\pi_1(\tilde{\theta}_1)$ is concave in $\tilde{\theta}_1$ for the following reason. When $\tilde{\theta}_1 \to \tilde{\theta}_1^*$, $-x+y[1-F(\tilde{\theta}_1)]+c_1(1-\tilde{\theta}_1) = 0$. Therefore, $\frac{d^2\pi_1}{d\tilde{\theta}_1^2}|_{\tilde{\theta}_1\to\tilde{\theta}_1^*} = Sf(\tilde{\theta}_1)[-yf(\tilde{\theta}_1)-c_1]$ which is strictly negative.

From (17) , $\frac{d\pi_1}{d\tilde{\theta}_1}/(Sf(\tilde{\theta}_1)) + yF(\tilde{\theta}_1) + c_1\tilde{\theta}_1 = c_1+y-x$ and $(\tilde{\theta}_1, F(\tilde{\theta}_1))$ are both nonnegative. When $c_1+y-x<0$, then $\frac{d\pi_1}{d\tilde{\theta}_1}$ has to be negative. So the lowest possible $\tilde{\theta}_1$ will be optimal, i.e., $\tilde{\theta}_1^* = 0$. When $c_1+y-x>0$, the zero of the first-order condition will be a unique global maximum. $\tilde{\theta}_1^*$ will be the solution to the following equation:

$$
c_1\tilde{\theta}_1 + yF(\tilde{\theta}_1) = y + c_1 - x.
$$

The remainder of Table 2 follows directly.

Section A.2: Nash bargaining formulation for Outsourcing for general δ

The Nash bargaining equilibrium is the solution to a two-stage problem. The retailer's profit-maximization problem at the second stage is:

$$\max_{p_{n2}\geq w_n;p_{r2}\leq \alpha p_{n2}} \quad \pi_2 = (p_{n2}-w_n)D_{n2} + (p_{r2}-w_{r2})D_{r2}, \tag{19}$$

where $D_{n2} = 1 - \frac{p_{n2}-p_{r2}}{1-\alpha}$ and $D_{r2} = \frac{\alpha p_{n2}-p_{r2}}{\alpha(1-\alpha)}$. (19) is strictly and jointly concave in p_{n2} and p_{r2} for any w_{r2}. The retailer's best-response prices for a given w_{r2} (obtained by solving simultaneously for the p_{n2} and p_{r2} that satisfy the first-order conditions of (19)) are:

$$p_{n2}(w_{r2}) = \frac{1+w_n}{2}, \tag{20}$$

$$p_{r2}(w_{r2}) = \frac{\alpha+w_{r2}}{2}. \tag{21}$$

Substituting (20) and (21) into $D_{n2} = 1 - \frac{p_{n2}-p_{r2}}{1-\alpha}$ and $D_{r2} = \frac{\alpha p_{n2}-p_{r2}}{\alpha(1-\alpha)}$ indicates that for a given w_{r2} the demands for the new and remanufactured products are:

$$D_{n2}(w_{r2}) = \frac{1}{2}(1-\frac{w_n-w_{r2}}{1-\alpha}), \tag{22}$$

$$D_{r2}(w_{r2}) = \frac{\alpha w_n - w_{r2}}{2\alpha(1-\alpha)}. \tag{23}$$

We assume $\alpha w_n - w_{r2} \geq 0$ so that demand for the remanufactured product is non-negative. Then we step back to the first stage, in which the third-party's profit-maximization problem is:

$$\max_{w_{r2}\leq \alpha w_n;0\leq\tilde{\theta}_2\leq 1} \quad \pi_{2o} = w_{r2}D_{r2} - c_2 S[1-F(\tilde{\theta}_2) - \mu + H(\tilde{\theta}_2)], \tag{24}$$

s.t.

$$D_{r2} \leq S\int_{\theta=\tilde{\theta}_2}^{1} f(\theta)d\theta = S[1-F(\tilde{\theta}_2)]. \tag{25}$$

By the same logic that governed the profit-maximizing actions for the In-house strategy, the constraint in (25) must bind in the optimal solution, i.e., $D_{r2} = S[1-F(\tilde{\theta}_2)]$. Combining with (23) yields:

$$w_{r2} = \alpha w_n - 2S\alpha(1-\alpha)[1-F(\tilde{\theta}_2)]. \tag{26}$$

Substituting (20), (21), (22), (23), and (26) into (19) and (24) allows reformulation of the retailer and third-party profits as:

$$
\begin{aligned}
\pi_2 &= \frac{(1-w_n)^2}{4} + \frac{yS}{2}[1-F(\tilde{\theta}_2)]^2, \\
\pi_{2o} &= xS[1-F(\tilde{\theta}_2)] - yS[1-F(\tilde{\theta}_2)]^2 - c_2S[1-F(\tilde{\theta}_2) - \mu + H(\tilde{\theta}_2)],
\end{aligned}
$$

where $x = \alpha w_n$, $y = 2\alpha S(1-\alpha)$, and $H(\tilde{\theta}_2) = \int_0^{\tilde{\theta}_2} \theta f(\theta) d\theta$. These expressions are used to simplify the Nash bargaining formulation to a single-variate optimization in $\tilde{\theta}_2$.

Section A.3: The Nash bargaining equilibrium for Outsourcing when $\delta = 1$

The profit-maximization problem for the third-party is:

$$\max_{0\leq\tilde{\theta}_2\leq 1} \pi_{2o} \quad = \quad xS[1-F_2] - yS[1-F_2]^2 - c_2S[1-F_2-\mu+H_2],$$

where $x = \alpha w_n$, $y = 2\alpha(1-\alpha)S$, $F_2 = \int_0^{\tilde{\theta}_2} f(\theta)d\theta$, and $H_2 = \int_0^{\tilde{\theta}_2} \theta f(\theta)d\theta$.

The objective has these first and second derivatives:

$$\frac{d\pi_{2o}}{d\tilde{\theta}_2} \quad = \quad Sf(\tilde{\theta}_2)\{-x+2y[1-F(\tilde{\theta}_2)]+c_2(1-\tilde{\theta}_2)\};$$

$$\frac{d^2\pi_{2o}}{d\tilde{\theta}_2^2} \quad = \quad Sf(\tilde{\theta}_2)[-2yf(\tilde{\theta}_2)-c_2] + Sf'[-x+2y[1-F(\tilde{\theta}_2)]+c_2(1-\tilde{\theta}_2].$$

First we prove that $\pi_{2o}(\tilde{\theta}_2)$ is concave in $\tilde{\theta}_2$. When $\tilde{\theta}_2 \to \tilde{\theta}_2^*$, $-x+2y[1-F(\tilde{\theta}_{2o})]+c_2(1-\tilde{\theta}_2) = 0$. Therefore, $\frac{d^2\pi_{2o}}{d\tilde{\theta}_2^2}|_{\tilde{\theta}_2\to\tilde{\theta}_2^*} = Sf(\tilde{\theta}_1)[-2yf(\tilde{\theta}_1)-c_1]$ which is strictly negative.

From the first-order condition , $\frac{d\pi_{2o}}{d\tilde{\theta}_2}/(Sf(\tilde{\theta}_2))+2yF(\tilde{\theta}_2)+c_2\tilde{\theta}_2 = c_2+2y-x$ and $(\tilde{\theta}_2, F(\tilde{\theta}_2))$ are both nonnegative. When $c_2+2y-x<0$, then $\frac{d\pi_{2o}}{d\tilde{\theta}_2}$ has to be negative. So the lowest possible $\tilde{\theta}_2$ will be optimal, i.e., $\tilde{\theta}_2^* = 0$. When $c_2+2y-x>0$, the zero of the first-order condition will be a unique global maximum. $\tilde{\theta}_2^*$ will be the solution to the following equation:

$$c_2\tilde{\theta}_2 + 2yF(\tilde{\theta}_2) = 2y + c_2 - x.$$

The remainder of Table 4 follows directly.

Section B: In-house Strategy - Comparison of Uniform versus Triangular Distributions for θ

This section explores the robustness of the results in Section 4 to the distributional assumption for θ (quality of used items at time of collection). Section 4 treats θ as uniform on $[0,1]$. Here we use the triangular distribution, first with parameters $[0,1,0]$ and then with $[0,1,1]$. A θ with $T[0,1,0]$ distribution has $\mu = \frac{1}{3}$, $f(\theta) = 2(1-\theta)$, $F(\theta) = \theta(2-\theta)$, and $H(\theta) = \frac{\theta^2(3-2\theta)}{3}$. With a $T[0,1,1]$ distribution for θ, $\mu = \frac{2}{3}$, $f(\theta) = 2\theta$, $F(\theta) = \theta^2$, and $H(\theta) = \frac{2\theta^3}{3}$.

In the interest of brevity, we limit this illustration to the case in which Outsourcing involves a third-party with full leadership power ($\delta = 1$). Using the same parameter settings as in Section 4 (i.e., $\alpha = 0.8$, $w_n = 0.2$, and $S = 0.2$), we depict the congruence and conflict regions for the comparison between In-house and third-party-led Outsourcing. We use values of β that facilitate a direct comparison to the results in Section 4.2.

Table 2 and Table 4 provide the respective optimal/equilibrium outcomes for general distributions of θ. Figure 3 compares In-house and third-party-led Outsourcing with respect to retailer profitability and environmental performance with a uniformly distributed θ. Figure 10 and Figure 11 do the same for $\theta \sim Triangular[0,1,0]$ and $\theta \sim Triangular[0,1,1]$, respectively.

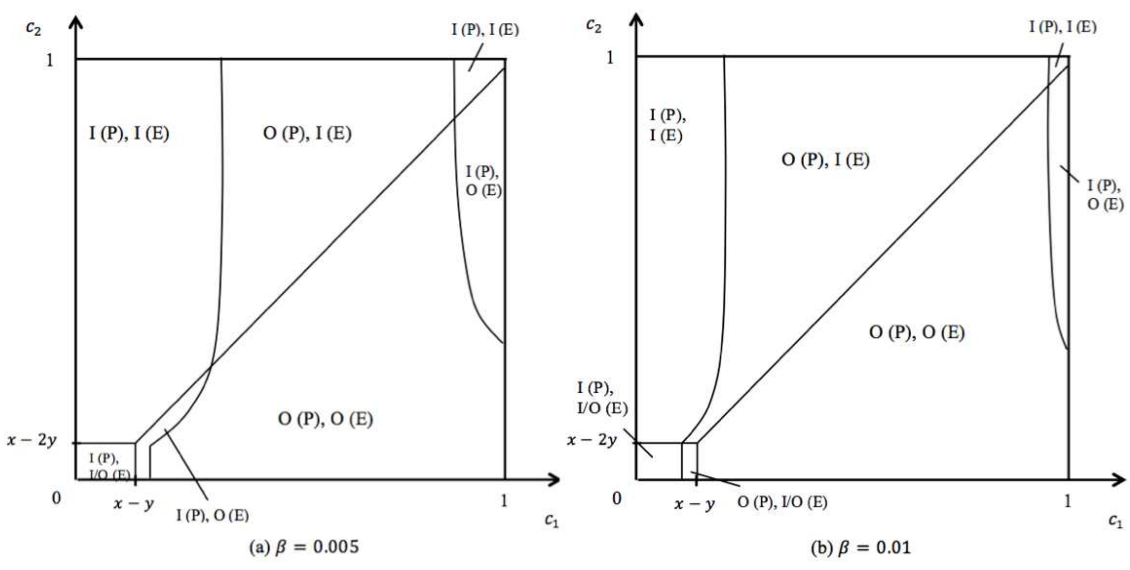

Figure 10: Retailer profit and environmental impact when $\delta = 1$ and $\theta \sim Triangular[0,1,0]$: Regions of congruence and conflict

Figure 10 and Figure 11 have the following in common with Figure 3. As the fixed-cost coefficient β increases, the regions where Outsourcing is more profitable also grow - in particular, the [*O(P), O(E)*] region (Outsourcing congruence) grows, the [*I(P),I(E)*] region (In-house congruence) shrinks, the [*I(P), O(E)*]

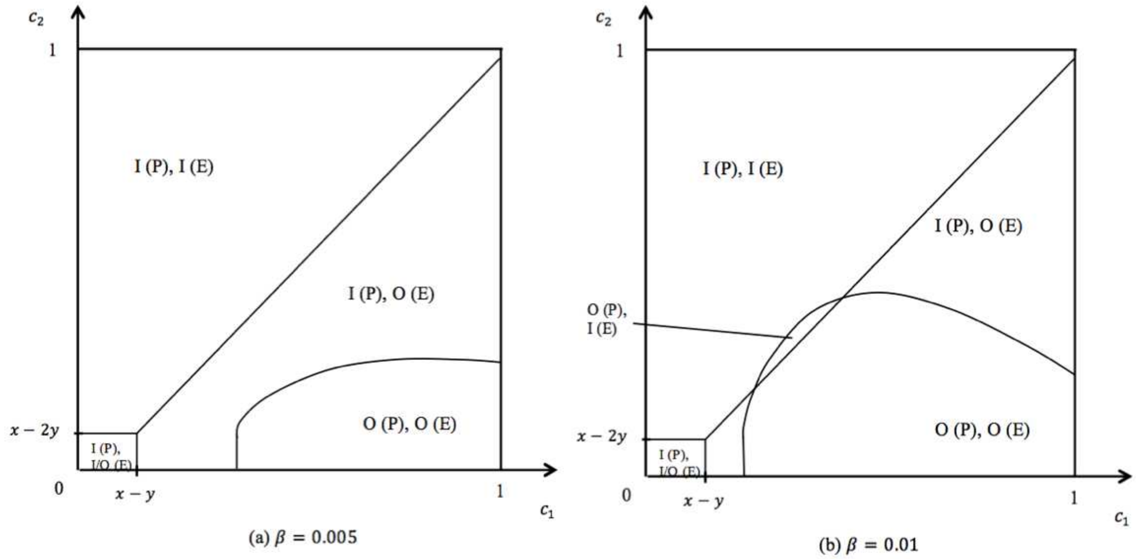

Figure 11: Retailer profit and environmental impact when $\delta = 1$ and $\theta \sim Triangular[0, 1, 1]$: Regions of congruence and conflict

conflict region shrinks, and a [*O(P), I(E)*] conflict region emerges.

Several differences are noteworthy. The *O(P)* area is larger in Figure 10 than in Figure 3. This is because, relative to $\theta \sim Uniform[0, 1]$, $\theta \sim Triangular[0, 1, 0]$ shifts the distribution of used products towards lower quality. On the other hand, the *I(P)* area is larger in Figure 11 than in Figure 3. This is because, relative to $\theta \sim Uniform[0, 1]$, $\theta \sim Triangular[0, 1, 1]$ shifts the distribution of used products towards higher quality, making In-house more profitable for the retailer.

To summarize, the Uniform distribution lies between the two extreme Triangular distributions we have considered in this section. The trends across these distributions validate our characterization of the setting in the main paper, and show some robustness to the distributional assumption for θ.

Section C.1: Proofs of Propositions 1 and 2

Table 5 is Table 2 when the quality of used products collected is $\theta \sim U[0,1]$.

	Range for c_1	
	$c_1 \in (0, x-y]$	$c_1 \in (x-y, 1)$
$\tilde{\theta}_1$	0	$1 - \frac{x}{y+c_1}$
p^*_{n1}	$\frac{1+w_n}{2}$	$\frac{1+w_n}{2}$
p^*_{r1}	$\alpha[\frac{1+w_n}{2} - S(1-\alpha)]$	$\frac{\alpha}{2} + \frac{xc_1}{2(y+c_1)}$
D^*_{n1}	$\frac{1-w_n}{2} - S\alpha$	$\frac{1}{2}[1 - \frac{(2S\alpha+c_1)w_n}{2y+c_1}]$
D^*_{r1}	S	$\frac{Sx}{y+c_1}$
π^*_1	$\frac{(1-w_n)^2}{4} + \frac{S}{2}[2x-y-c_1] - \beta(1-c_1)$	$\frac{(1-w_n)^2}{4} + \frac{Sx^2}{2(y+c_1)} - \beta(1-c_1)$

where $x = \alpha w_n$ and $y = 2\alpha(1-\alpha)S$.

Table 5: Optimal solution for In-house remanufacturing for $\theta \sim U[0,1]$

Proposition 1. In-house remanufacturing and Retailer-led Outsourcing have the following ordering with respect to retailer profit:

(a) If $c_1, c_2 \in (0, x-y]$, then if $c_2 \geq \frac{2\beta}{S} + c_1(1 - \frac{2\beta}{S})$, In-house gives the retailer greater profit; otherwise Outsourcing does;

(b) If $c_1 \in (0, x-y]$ and $c_2 \in (x-y, 1)$, then if $c_2 \geq \frac{Sx^2}{S(2x-y-c_1)-2\beta(1-c_1)} - y$, In-house gives the retailer greater profit; otherwise Outsourcing does;

(c) If $c_1 \in (x-y, 1)$ and $c_2 \in (0, x-y]$, then if $c_2 \geq (2x-y) + \frac{2\beta(1-c_1)}{S} - \frac{x^2}{y+c_1}$, In-house gives the retailer greater profit; otherwise Outsourcing does;

(d) If $c_1, c_2 \in (x-y, 1)$, then if $c_2 \geq \frac{2Sx^2c_1+4y\beta(1-c_1)(y+c_1)}{2Sx^2-4\beta(1-c_1)(y+c_1)}$, In-house gives the retailer greater profit; otherwise Outsourcing does;

where $x = \alpha w_n$ and $y = 2\alpha S(1-\alpha)$.

Proof of Proposition 1.

As noted in the main paper, the Retailer-led Outsourcing equilibrium can be obtained from Table 5 by making the following adjustments: (a) c_2 in place of c_1 since remanufacturing occurs at the third-party's variable cost; (b) $\tilde{\theta}_2$ in place of $\tilde{\theta}_1$ to match the subscript to the strategy; and (c) $\beta = 0$ because the retailer no longer maintains internal manufacturing capability.

The expressions for the optimal/equilibrium retailer profit directly yield the following:

(a) If $c_1, c_2 \in (0, x-y]$, then $\pi_1^* - \pi_2^* = \frac{S}{2}(c_2 - c_1) - \beta(1-c_1)$;

(b) If $c_1 \in (0, x-y]$ and $c_2 \in (x-y, 1)$, then $\pi_1^* - \pi_2^* = \frac{S}{2}(2x - y - c_1) - \beta(1-c_1) - \frac{Sx^2}{2(y+c_2)}$;

(c) If $c_1 \in (x-y, 1)$ and $c_2 \in (0, x-y]$, then $\pi_1^* - \pi_2^* = \frac{Sx^2}{2(y+c_1)} - \beta(1-c_1) - \frac{S}{2}(2x - y - c_2)$;

(d) If $c_1, c_2 \in (x-y, 1)$, then $\pi_1^* - \pi_2^* = \frac{Sx^2}{2(y+c_1)} - \beta(1-c_1) - \frac{Sx^2}{2(y+c_2)}$.

Straightforward algebra then produces the findings of the proposition. Q.E.D.

Proposition 2. In-house remanufacturing and Retailer-led Outsourcing have the following ordering with respect to environmental impact:

(a) If $c_1, c_2 \in (0, x-y]$, then both In-house and Outsourcing yield an equal (and "best" possible) environmental outcome;

(b) If $c_1 \in (0, x-y]$ and $c_2 \in (x-y, 1)$, then In-house is better for the environment;

(c) If $c_1 \in (x-y, 1)$ and $c_2 \in (0, x-y]$, then Outsourcing is better for the environment; and

(d) If $c_1, c_2 \in (x-y, 1)$, then if:

- $c_1 = c_2$, both In-house and Outsourcing yield an equal environmental outcome;
- $c_1 < c_2$, In-house is better for the environment; and
- $c_1 > c_2$, Outsourcing is better for the environment.

where $x = \alpha w_n$ and $y = 2\alpha S(1-\alpha)$.

Proof of Proposition 2.

$\tilde{\theta}_i$ is our metric of environmental impact, with lower values indicating a greater amount of remanufacturing. Comparing the optimal/equilibrium values of $\tilde{\theta}_1$ and $\tilde{\theta}_2$ from Table 5:

(a) If $c_1, c_2 \in (0, x-y]$, then $\tilde{\theta}_1^* = \tilde{\theta}_2^* = 0$;

(b) If $c_1 \in (0, x-y]$ and $c_2 \in (x-y, 1)$, then $\tilde{\theta}_1^* = 0$; and $\tilde{\theta}_2^* > 0$.

(c) If $c_1 \in (x-y, 1)$ and $c_2 \in (0, x-y]$, then $\tilde{\theta}_1^* > 0$ and $\tilde{\theta}_2^* = 0$;

(d) If $c_1 \in (x-y, 1)$ and $c_2 \in (x-y, 1)$, then $\tilde{\theta}_1^* - \tilde{\theta}_2^* = \frac{x}{y+c_2} - \frac{x}{y+c_1} = \frac{x(c_1-c_2)}{(y+c_1)(y+c_2)}$.

Straightforward algebra then produces the findings of the proposition. Q.E.D.

Section C.2: Proofs of Propositions 3 and 4

Table 6 is Table 4 when the quality of used products collected is $\theta \sim U[0,1]$.

	Range for c_2	
	$c_2 \in (0, x-2y]$	$c_2 \in (x-2y, 1)$
$\tilde{\theta}_2^*$	0	$1 - \frac{x}{2y+c_2}$
w_{r2}^*	$x-y$	$\frac{x}{2} + \frac{xc_2}{2[2y+c_2]}$
p_{n2}^*	$\frac{1+w_n}{2}$	$\frac{1+w_n}{2}$
p_{r2}^*	$\frac{\alpha+x-y}{2}$	$\frac{\alpha}{4}[2+w_n+\frac{w_n c_2}{2y+c_2}]$
D_{r2}^*	S	$\frac{Sx}{2y+c_2}$
D_{n2}^*	$\frac{1-w_n}{2} - S\alpha$	$\frac{1-w_n}{2} - \frac{S\alpha x}{2y+c_2}$
π_2^*	$\frac{(1-w_n)^2}{4} + \frac{Sy}{2}$	$\frac{(1-w_n)^2}{4} + \frac{x^2yS}{2[2y+c_2]^2}$
π_{2o}^*	$\frac{S}{2}[2x-2y-c_2]$	$\frac{Sx^2}{2[2y+c_2]}$

where $x = \alpha w_n$ and $y = 2\alpha(1-\alpha)S$.

Table 6: Equilibrium for Third-party-led Outsourcing of remanufacturing when $\theta \sim U[0,1]$

Proposition 3. In-house remanufacturing and Third-party-led Outsourcing have the following ordering with respect to retailer profit:

(a) When $c_1 \in (0, x-y]$ and $c_2 \in (0, x-2y]$, then if $c_1 \leq \frac{2S(x-y)-2\beta}{S-2\beta}$, In-house gives the retailer greater profit; otherwise Outsourcing does;

(b) If $c_1 \in (0, x-y]$ and $c_2 \in (x-2y, 1)$, then if $c_1 \leq \frac{S(2x-y)-2\beta}{S-2\beta} - \frac{x^2yS}{(S-2\beta)(2y+c_2)^2}$, In-house gives the retailer greater profit; otherwise Outsourcing does;

(c) If $c_1 \in (x-y, 1)$ and $c_2 \in (0, x-2y]$, then if $2\beta(1-c_1)(y+c_1) + yS(y+c_1) \leq x^2S$, In-house gives the retailer greater profit; otherwise Outsourcing does; and

(d) If $c_1 \in (x-y, 1)$ and $c_2 \in (x-2y, 1)$, then if $\frac{x^2S[(2y+c_2)^2 - y(y+c_1)]}{2(y+c_1)(2y+c_2)^2} - \beta(1-c_1) \geq 0$, In-house gives the retailer greater profit; otherwise Outsourcing does.

where $x = \alpha w_n$ and $y = 2\alpha S(1-\alpha)$.

Proof of Proposition 3.

Comparing the retailer profits from Table 5 and Table 6:

(a) When $c_1 \in (0, x-y]$ and $c_2 \in (0, x-2y]$, then $\pi_1^* - \pi_2^* = \frac{S}{2}(2x-y-c_1) - \beta(1-c_1) - \frac{Sy}{2}$;

(b) If $c_1 \in (0, x-y]$ and $c_2 \in (x-2y, 1)$, then $\pi_1^* - \pi_2^* = \frac{S}{2}(2x-y-c_1) - \beta(1-c_1) - \frac{Sx^2y}{2(2y+c_2)^2}$;

(c) If $c_1 \in (x-y, 1)$ and $c_2 \in (0, x-2y]$, then $\pi_1^* - \pi_2^* = \frac{Sx^2}{2(y+c_1)} - \beta(1-c_1) - \frac{Sy}{2}$;

(d) If $c_1 \in (x-y, 1)$ and $c_2 \in (x-2y, 1)$, then $\pi_1^* - \pi_2^* = \frac{Sx^2}{2(y+c_1)} - \beta(1-c_1) - \frac{Sx^2y}{2(2y+c_2)^2}$.

Straightforward algebra then produces the findings of the proposition. Q.E.D.

Proposition 4. In-house remanufacturing and Third-party-led Outsourcing have the following ordering with respect to environmental impact:

(a) If $c_1 \in (0, x-y]$ and $c_2 \in (0, x-2y]$, then both In-house and Outsourcing yield an equal (and "best" possible) environmental outcome;

(b) If $c_1 \in (0, x-y]$ and $c_2 \in (x-2y, 1)$, then In-house is better for the environment;

(c) If $c_1 \in (x-y, 1)$ and $c_2 \in (0, x-2y]$, then Outsourcing is better for the environment;

(d) if $c_1 \in (x-y, 1)$ and $c_2 \in (x-2y, 1)$, then if:

- $c_1 - c_1 < y$, In-house is better for the environment;
- $c_1 - c_2 = y$, both In-house and Outsourcing yield an equal environmental outcome; and
- $c_1 - c_2 > y$, Outsourcing is better for the environment.

where $x = \alpha w_n$ and $y = 2\alpha S(1-\alpha)$.

Proof of Proposition 4.

$\tilde{\theta}_i$ is our metric of environmental impact. Lower values indicate a greater amount of remanufacturing, and hence more environment-friendliness. Comparing $\tilde{\theta}_1$ and $\tilde{\theta}_2$ from Tables 5 and 6:

(a) If $c_1 \in (0, x-y]$ and $c_2 \in (0, x-2y]$, then $\tilde{\theta}_1^* = \tilde{\theta}_2^* = 0$;

(b) If $c_1 \in (0, x-y]$ and $c_2 \in (x-2y, 1)$, then $\tilde{\theta}_1^* = 0$, and $\tilde{\theta}_2^* > 0$;

(c) If $c_1 \in (x-y, 1)$ and $c_2 \in (0, x-2y]$, then $\tilde{\theta}_1^* > 0$ and $\tilde{\theta}_2^* = 0$;

(d) If $c_1 \in (x-y, 1)$ and $c_2 \in (x-2y, 1]$, then $\tilde{\theta}_1^* - \tilde{\theta}_2^* = \frac{x}{2y+c_2} - \frac{x}{y+c_1} = \frac{x}{(y+c_1)(2y+c_2)}(c_1 - c_2 - y)$.

Straightforward algebra then produces the findings of the proposition. Q.E.D.

Section D.1: The optimal solution for In-house remanufacturing when $q_r < q_n = 1$

Here we assume $q_r < q_n = 1$. The collected used products' quality is a random variable θ with finite support in the range $[0, q_r]$. The cumulative distribution function of θ is $F(\theta)$, with $F(0) = 0$ and $F(q_r) = 1$. Analysis of the consumer surplus results in the following demand functions: $D_n = 1 - \frac{p_n - p_r}{1-\alpha q_r}$, $D_r = \frac{\alpha q_r p_n - p_r}{\alpha q_r(1-\alpha q_r)}$.

The profit-maximization problem for the In-house strategy is:

$$\begin{aligned}
\max_{0 \le p_{r1} < \alpha p_{n1}; 0 \le \tilde{\theta}_1 \le 1} \quad \pi_1 &= (p_{n1} - w_n)D_{n1} + p_{r1}D_{r1} - [S \int_{\theta=\tilde{\theta}_1}^{q_r} c_1(q_r - \theta) f(\theta) d\theta] - \beta(1 - c_1) \\
&= (p_{n1} - w_n)D_{n1} + p_{r1}D_{r1} - c_1 S[q_r - q_r F(\tilde{\theta}_1) - \mu + H(\tilde{\theta}_1] - \beta(1 - c_1), \\
s.t. & \\
D_{r1} &\le S[1 - F(\tilde{\theta}_1)].
\end{aligned}$$

As with the original model, the constraint on D_{r1} must bind at optimality since regardless of the p_{n1} and p_{r1} the retailer loses profit if it remanufactures items that it cannot sell. So the retailer will set $\tilde{\theta}_1$ to exactly match the remanufacturing volume to the demand. Then:

$$D_{r1} = \frac{\alpha p_{n1} - p_{r1}}{\alpha q_r(1 - \alpha q_r)} \quad = \quad S[1 - F(\tilde{\theta}_1],$$

which yields $p_{r1} = \alpha p_{n1} - \alpha q_r(1 - \alpha q_r)S[1 - F(\tilde{\theta}_1)]$. Substituting this into the expression for D_{n1} provides:

$$D_{n1} \quad = \quad 1 - p_{n1} - \alpha q_r S[1 - F(\tilde{\theta}_1)].$$

For any fixed $\tilde{\theta}_1$, π_1 is strictly concave in p_{n1}. The first-order condition leads to $p^*_{n1} = \frac{1+w_n}{2}$, which is independent of $\tilde{\theta}_1$. The retailer's decision problem can then be expressed as the following single-variable optimization:

$$\begin{aligned}
\max_{0 \le \tilde{\theta}_1 \le 1} \quad \pi_1 \quad = \quad & \frac{(1 - w_n)^2}{4} + \frac{1}{2} S(1 - F(\tilde{\theta}_1))(2\alpha q_r w_n - 2S\alpha q_r(1 - \alpha q_r)(1 - F(\tilde{\theta}_1))) \\
& - c_1 S[q_r - q_r F(\tilde{\theta}_1) - \mu + H(\tilde{\theta}_1)] - \beta(1 - c_1).
\end{aligned}$$

Using $x = \alpha q_r w_n$ and $y = 2S\alpha q_r(1-\alpha q_r)$ as placeholders, differentiation yields:

$$\frac{d\pi_1}{d\tilde{\theta}_1} = Sf(\tilde{\theta}_1)\{-x + y[1-F(\tilde{\theta}_1)] + c_1(q_r - \tilde{\theta}_1)\};$$

$$\frac{d^2\pi_1}{d\tilde{\theta}_1^2} = Sf(\tilde{\theta}_1)[-yf(\tilde{\theta}_1) - c_1] + Sf'[-x + y[1-F(\tilde{\theta}_1)] + c_1(1-\tilde{\theta}_1)].$$

The reason $\pi_1(\tilde{\theta}_1)$ is concave in $\tilde{\theta}_1$ is as follows. When $\tilde{\theta}_1 \to \tilde{\theta}_1^*$, $-x + y[1-F(\tilde{\theta}_1)] + c_1(q_r - \tilde{\theta}_1) = 0$. Therefore, $\frac{d^2\pi_1}{d\tilde{\theta}_1^2}|_{\tilde{\theta}_1 \to \tilde{\theta}_1^*} = Sf(\tilde{\theta}_1)[-yf(\tilde{\theta}_1) - c_1 q_r]$ which is strictly negative.

When $c_1 q_r + y - x < 0$, $\frac{d\pi_1}{d\tilde{\theta}_1}$ has to be negative. This is because from the first-order condition, $\frac{\frac{d\pi_1}{d\tilde{\theta}_1}}{Sf(\tilde{\theta}_1)} + yF(\tilde{\theta}_1) + c_1\tilde{\theta}_1 = c_1 q_r + y - x < 0$; and $\tilde{\theta}_1$ and $F(\tilde{\theta}_1))$ are both non-negative. So $\frac{d\pi_1}{d\tilde{\theta}_1} < 0$. Then the lowest possible $\tilde{\theta}_1$ will be optimal, i.e., $\tilde{\theta}_1^* = 0$. When $c_1 q_r + y - x \geq 0$, the zero of the first-order condition will be a unique global maximum. $\tilde{\theta}_1^*$ will be the solution to the following equation:

$$c_1\tilde{\theta}_1 + yF(\tilde{\theta}_1) = c_1 q_r + y - x.$$

The optimal solutions when $q_r < 1$ and θ follows general distribution are shown in Table 7.

	Range for c_1	
	$c_1 \in (0, \frac{x-y}{q_r}]$	$c_1 \in (\frac{x-y}{q_r}, 1)$
$\tilde{\theta}_1^*$	0	Solution of $c_1\tilde{\theta}_1 + yF(\tilde{\theta}_1) = c_1 q_r - x + y$
p_{n1}^*	$\frac{1+w_n}{2}$	$\frac{1+w_n}{2}$
p_{r1}^*	$\frac{\alpha q_r + x - y}{2}$	$\frac{\alpha q_r + x - y[1-F(\tilde{\theta}_1^*)]}{2}$
D_{n1}^*	$\frac{1-w_n}{2} - \alpha q_r S$	$\frac{1-w_n}{2} - \alpha q_r S[1-F(\tilde{\theta}_1^*)]$
D_{r1}^*	S	$S[1-F(\tilde{\theta}_1^*)]$
π_1^*	$\frac{(1-w_n)^2}{4} + \frac{S}{2}(2x-y) - c_1 S(q_r - \mu) - \beta(1-c_1)$	$\frac{(1-w_n)^2}{4} + \frac{S[1-F(\tilde{\theta}_1^*)]}{2}[2x - y[1-F(\tilde{\theta}_1^*)]]$ $-c_1 S[q_r - q_r F(\tilde{\theta}_1^*) - \mu + H(\tilde{\theta}_1^*)] - \beta(1-c_1)$

where $x = \alpha q_r w_n$, $y = 2\alpha q_r(1-\alpha q_r)S$, $H(\tilde{\theta}_1^*) = \int_0^{\tilde{\theta}_1^*} \theta f(\theta) d\theta$, and μ is the mean of θ.

Table 7: Optimal solution for the In-house remanufacturing strategy when $q_r < 1$

Table 8 is Table 7 when the quality of used products collected is $\theta \sim U[0, q_r]$.

	Range for c_1	
	$c_1 \in (0, \frac{x-y}{q_r}]$	$c_1 \in (\frac{x-y}{q_r}, 1)$
$\tilde{\theta}_1$	0	$q_r - \frac{\alpha q_r w_n}{c_1+2S\alpha(1-\alpha q_r)}$
p_{n1}^*	$\frac{1+w_n}{2}$	$\frac{1+w_n}{2}$
p_{r1}^*	$\alpha q_r[\frac{1+w_n}{2} - S(1-\alpha q_r)]$	$\frac{\alpha q_r}{2}(1 + \frac{c_1 \alpha q_r w_n}{c_1+2S\alpha(1-\alpha q_r)}$
D_{n1}^*	$\frac{1-w_n}{2} - S\alpha q_r$	$\frac{1-w_n}{2} - \frac{S\alpha^2 q_r w_n}{c_1+2S\alpha(1-\alpha q_r)}$
D_{r1}^*	S	$\frac{S\alpha w_n}{c_1+2S\alpha(1-\alpha q_r)}$
π_1^*	$\frac{(1-w_n)^2}{4} + \frac{S}{2}[2\alpha q_r w_n - 2S\alpha q_r(1-\alpha q_r)]$ $-c_1 S\frac{q_r}{2} - \beta(1-c_1)$	$\frac{(1-w_n)^2}{4} + \frac{S\alpha^2 q_r w_n^2}{2(c_1+2S\alpha(1-\alpha))}$ $-\beta(1-c_1)$

Table 8: Optimal solution for In-house remanufacturing when $q_r < 1$ and $\theta \sim U[0, q_r]$

Section D.2: The equilibrium for Outsourcing of remanufacturing when $q_r < q_n = 1$

The equilibrium for Retailer-Led Outsourcing

The equilibrium is the same as the optimal solution for the In-house strategy stated in Appendix D.1, except that (a) c_2 replaces c_1; (b) $\tilde{\theta}_2$ replaces $\tilde{\theta}_1$; and (c) $\beta = 0$.

The equilibrium for Third-party-Led Outsourcing

The equilibrium when $q_r < 1$ and θ has a general distribution is shown in Table 9.

	Range for c_2	
	$c_2 \in (0, \frac{x-2y}{q_r}]$	$c_2 \in (\frac{x-2y}{q_r}, 1)$
$\tilde{\theta}_2^*$	0	Solution of $c_2\tilde{\theta}_2 + 2yF(\tilde{\theta}_2) = c_2 q_r - x + 2y$
w_r^*	$x - y$	$x - y[1 - F(\tilde{\theta}_2^*)]$
p_{n2}^*	$\frac{1+w_n}{2}$	$\frac{1+w_n}{2}$
p_{r2}^*	$\frac{\alpha q_r + x - y}{2}$	$\frac{\alpha q_r + x - y[1-F(\tilde{\theta}_2^*)]}{2}$
D_{n2}^*	$\frac{1-w_n}{2} - \alpha q_r S$	$\frac{1-w_n}{2} - \alpha q_r S[1 - F(\tilde{\theta}_2^*)]$
D_{r2}^*	S	$S[1 - F(\tilde{\theta}_2^*)]$
π_2^*	$\frac{(1-w_n)^2}{4} + \frac{yS}{2}$	$\frac{(1-w_n)^2}{4} + \frac{yS}{2}[1 - F(\tilde{\theta}_2^*)]^2$
π_{2o}^*	$S(x-y) + c_2 S(q_r - \mu)$	$S[1 - F(\tilde{\theta}_2^*)]\{x - y[1 - F(\tilde{\theta}_2^*)]\} - c_2 S[q_r - F(\tilde{\theta}_2^*)q_r - \mu + H(\tilde{\theta}_2^*)]$

where $x = \alpha q_r w_n$, $y = 2\alpha q_r(1-\alpha q_r)S$, $H(\tilde{\theta}_2^*) = \int_0^{\tilde{\theta}_2^*} \theta f(\theta) d\theta$, and μ is the mean of θ.

Table 9: Nash bargaining equilibrium for the Outsourcing remanufacturing strategy when $\delta = 1$ and $q_r < 1$

Table 10 is Table 9 when the quality of used products collected is $\theta \sim U[0, q_r]$.

	Range for c_2	
	$c_2 \in (0, \frac{x-2y}{q_r}]$	$c_2 \in (\frac{x-2y}{q_r}, 1)$
$\tilde{\theta}_2^*$	0	$q_r - \frac{\alpha q_r w_n}{c_2+4S\alpha(1-\alpha q_r)}$
w_{r2}^*	$\alpha q_r w_n - 2S\alpha q_r(1-\alpha q_r)$	$\alpha q_r w_n - \frac{2S\alpha^2 q_r(1-\alpha q_r)w_n}{c_2+4S\alpha(1-\alpha q_r)}$
p_{n2}^*	$\frac{1+w_n}{2}$	$\frac{1+w_n}{2}$
p_{r2}^*	$\frac{\alpha q_r+\alpha q_r w_n-2S\alpha q_r(1-\alpha q_r)}{2}$	$\frac{1}{2}(\alpha q_r + \alpha q_r w_n - \frac{S\alpha^2 q_r(1-\alpha q_r)w_n}{c_2+4S\alpha(1-\alpha q_r)})$
D_{r2}^*	S	$\frac{S\alpha w_n}{c_2+4S\alpha(1-\alpha q_r)}$
D_{n2}^*	$\frac{1-w_n}{2} - S\alpha q_r$	$\frac{1-w_n}{2} - \frac{S\alpha^2 q_r w_n}{c_2+4S\alpha(1-\alpha q_r)}$
π_2^*	$\frac{(1-w_n)^2}{4} + S^2\alpha q_r(1-\alpha q_r)$	$\frac{(1-w_n)^2}{4} + \frac{S^2\alpha^3 q_r(1-\alpha q_r)w_n^2}{(c_2+4S\alpha(1-\alpha q_r))^2}$
π_{2o}^*	$S\alpha q_r(w_n - 2S(1-\alpha q_r)) - \frac{Sc_2 q_r}{2}$	$\frac{Sc_2(1-q_r^2)}{2q_r} + \frac{S\alpha^2 q_r w_n^2}{2(c_2+4S\alpha(1-\alpha q_r))}$

Table 10: Equilibrium for Outsourcing of remanufacturing when $\delta = 1$, $q_r < 1$, and $\theta \sim U[0, q_r]$

Proposition 5. The impact of q_r is as follows:

(a) Under In-house and Outsourcing strategies ($i = 1$ and 2, respectively), increases in relative quality for remanufactured products (q_r) leads to increases in: the retail price (p_{ri}) and demand (D_{ri}) for the remanufactured product, and retailer profit (π_i).

(b) Under Third-party-led Outsourcing, if the third-party's remanufacturing cost is sufficiently low (i.e., $c_2 \in (0, (x-2y)/q_r])$, the third-party's profit increases with q_r; otherwise (i.e., $c_2 \in ((x-2y)/q_r, 1))$, the third-party's profit will first increase and then decrease with q_r.

Proof of Proposition 5.

We are unable to derive closed-form solutions when θ has a general distribution. So for the subsequent analysis we consider a uniformly distributed θ. Table 11 shows how q_r impacts the optimum of the In-house strategy (based on the solutions in Table 8). Table 11 shows how q_r impacts the equilibrium for Retailer-led Outsourcing. Table 12 shows how q_r impacts the equilibrium of Third-party-led Outsourcing (based on the equilibrium shown in Table 10). Straightforward differentiation produces these comparative statics. Q.E.D.

Parameter	Key Decisions					
	θ_1^*	p_{n1}^*	p_{r1}^*	D_{n1}^*	D_{r1}^*	π_1^*
$q_r \uparrow$	NC/$\uparrow$	NC	$\uparrow$	$\downarrow$	NC/$\uparrow$	$\uparrow$

The directional relationships are abbreviated as follows, and apply over all $c_1 \in (0, 1)$ unless otherwise noted: (a) NC indicates no change; (b) NC/$\uparrow$ indicates NC when $c_1 \in (0, (x - y)/q_r]$ and $\uparrow$ when $c_1 \in ((x - y)/q_r, 1)$.

Table 11: Comparative statics for optimal solution for In-house remanufacturing

Parameter	Key Decisions							
	θ_2^*	w_{r2}^*	p_{n2}^*	p_{r2}^*	D_{n2}^*	D_{r2}^*	π_2^*	π_{2o}^*
$q_r \uparrow$	NC/$\uparrow$	$\uparrow$	NC	$\uparrow$	$\downarrow$	NC/$\uparrow$	$\uparrow$	$\uparrow$/ ($\uparrow$ then $\downarrow$)

The directional relationships are abbreviated as follows, and apply over all $c_2 \in (0, 1)$ unless otherwise noted: (a) NC indicates no change; (b) NC/$\uparrow$ indicates NC when $c_2 \in (0, (x - 2y)/q_r]$ and $\uparrow$ when $c_2 \in ((x - 2y)/q_r, 1)$; (c) $\uparrow$/ ($\uparrow$or $\downarrow$) indicates $\uparrow$ when $c_2 \in (0, (x - 2y)/q_r]$ and ($\uparrow$ then $\downarrow$) when c_2 varies within the rage of $((x - 2y)/q_r, 1)$.

Table 12: Comparative statics for equilibrium for Outsourcing remanufacturing when $\delta = 1$

Section D.3: The optimal solution for In-house remanufacturing, adding salvage value g

The profit-maximization problem for the In-house strategy is:

$$\begin{aligned}
\max_{0\le p_{r1}<\alpha p_{n1};0\le\tilde{\theta}_1\le 1} \quad \pi_1 &= (p_{n1}-w_n)D_{n1}+p_{r1}D_{r1}-[S\int_{\theta=\tilde{\theta}_1}^{1} c_1(1-\theta)f(\theta)d\theta]+S\int_{0}^{\theta=\tilde{\theta}_1} gf(\theta)d\theta-\beta(1-c_1)\\
&= (p_{n1}-w_n)D_{n1}+p_{r1}D_{r1}-c_1S[1-F(\tilde{\theta}_1)-\mu+H(\tilde{\theta}_1)]+gSF(\tilde{\theta}_1)-\beta(1-c_1),\\
s.t. &\\
D_{r1} &\le S[1-F(\tilde{\theta}_1)],
\end{aligned}$$

where $D_{n1}=1-\frac{p_{n1}-p_{r1}}{1-\alpha}$, $D_{r1}=\frac{\alpha p_{n1}-p_{r1}}{\alpha(1-\alpha)}$, and $H(\tilde{\theta}_1)=\int_0^{\theta=\tilde{\theta}_1}\theta f(\theta)d\theta$.

As in previous sections, the constraint on D_{r1} will bind at optimality, i.e., the retailer will set $\tilde{\theta}_1$ to exactly match the remanufacturing volume to the demand. Then:

$$D_{r1}=\frac{\alpha p_{n1}-p_{r1}}{\alpha(1-\alpha)} \quad = \quad S[1-F(\tilde{\theta}_1],$$

which yields $p_{r1}=\alpha p_{n1}-\alpha(1-\alpha)S[1-F(\tilde{\theta}_1)]$. Substituting this into the expression for D_{n1} provides:

$$D_{n1} \quad = \quad 1-p_{n1}-\alpha S[1-F(\tilde{\theta}_1)].$$

Then the profit-maximization problem reduces to:

$$\begin{aligned}
\max_{0\le p_{n1}\le 1;0\le\tilde{\theta}_1\le 1} \quad \pi_1 &= (p_{n1}-w_n)[1-p_{n1}-\alpha S[1-F(\tilde{\theta}_1)]]+\\
&\quad +[\alpha p_{n1}-\alpha(1-\alpha)S[1-F(\tilde{\theta}_1)]][S[1-F(\tilde{\theta}_1]]\\
&\quad -c_1S[1-F(\tilde{\theta}_1)-\mu+H(\tilde{\theta}_1)]+gSF(\tilde{\theta}_1)-\beta(1-c_1).
\end{aligned}$$

For any $\tilde{\theta}_1$, $\pi_1(p_{n1}|\tilde{\theta}_1)$ is strictly concave in p_{n1}. The first-order condition leads to $p^*_{n1}=\frac{1+w_n}{2}$, which is independent of $\tilde{\theta}_1$. The retailer's decision problem can then be expressed as the following single-variable optimization:

$$\begin{aligned}
\max_{0\le\tilde{\theta}_1\le 1} \quad \pi_1 &= \frac{(1-w_n)^2}{4}+S\alpha w_n-S\alpha w_nF(\tilde{\theta}_1)\\
&\quad -S^2\alpha(1-\alpha)[1-F(\tilde{\theta}_1)]^2-c_1S[1-F(\tilde{\theta}_1)-\mu+H(\tilde{\theta}_1)]+gSF(\tilde{\theta}_1)-\beta(1-c_1).
\end{aligned}$$

Using $x = \alpha w_n$ and $y = 2S\alpha(1-\alpha)$ as placeholders, the profit becomes:

$$\begin{aligned}\max_{0\le\tilde{\theta}_1\le 1} \quad \pi_1 &= \frac{(1-w_n)^2}{4} + Sx(1-F(\tilde{\theta}_1) - \frac{Sy}{2}[1-F(\tilde{\theta}_1)]^2 - c_1S(1-\mu) \\ &\quad +c_1SF(\tilde{\theta}_1) - c_1SH(\tilde{\theta}_1) - gSF(\tilde{\theta}_1) - \beta(1-c_1).\end{aligned}$$

The differentiation yields:

$$\frac{d\pi_1}{d\tilde{\theta}_1} = Sf(\tilde{\theta}_1)\{-x + y[1-F(\tilde{\theta}_1)] + c_1(1-\tilde{\theta}_1)] + g\};$$

$$\frac{d^2\pi_1}{d\tilde{\theta}_1^2} = Sf(\tilde{\theta}_1)[-yf(\tilde{\theta}_1) - c_1] + Sf'[-x + y[1-F(\tilde{\theta}_1)] + c_1(1-\tilde{\theta}_1) + g].$$

$\pi_1(\tilde{\theta}_1)$ is concave in $\tilde{\theta}_1$ for the following reason. When $\tilde{\theta}_1 \to \tilde{\theta}_1^*$, $-x+y[1-F(\tilde{\theta}_1)]+c_1(1-\tilde{\theta}_1)+g = 0$. Therefore, $\frac{d^2\pi_1}{d\tilde{\theta}_1^2}|_{\tilde{\theta}_1\to\tilde{\theta}_1^*} = Sf(\tilde{\theta}_1)[-yf(\tilde{\theta}_1) - c_1]$ which is strictly negative.

When $c_1 + y + g - x < 0$, $\frac{d\pi_1}{d\tilde{\theta}_1}$ has to be negative. This is because $\frac{d\pi_1}{d\tilde{\theta}_1}/(Sf(\tilde{\theta}_1)) + yF(\tilde{\theta}_1) + c_1\tilde{\theta}_1 = c_1+y+g-x$ and $\tilde{\theta}_1$ and $F(\tilde{\theta}_1))$ are both nonnegative. So the lowest possible $\tilde{\theta}_1$ will be optimal, i.e., $\tilde{\theta}_1^* = 0$. When $c_1 + y + g - x > 0$, the zero of the first-order condition will be a unique global maximum. $\tilde{\theta}_1^*$ will be the solution to the following equation:

$$c_1\tilde{\theta}_1 + yF(\tilde{\theta}_1) = y + c_1 + g - x.$$

We examine this condition for different ranges of g and c_1.

- if $g > 0$,

  if $g < x - y$ and $c_1 < x - y - g$, then $\tilde{\theta}_1^* = 0$;

  if $g < x - y$ and $c_1 > x - y - g$, then $\tilde{\theta}_1^* \Leftarrow c_1\tilde{\theta}_1 + yF(\tilde{\theta}_1) = y + c_1 + g - x$;

  if $g > x - y$ then $c_1 + g + y - x > 0$, and then $\tilde{\theta}_1^* \Leftarrow c_1\tilde{\theta}_1 + yF(\tilde{\theta}_1) = y + c_1 + g - x$;

- if $g < 0$ ($x - y - g$ must be positive),

  if $c_1 < x - y - g$, then $\tilde{\theta}_1^* = 0$;

  if $c_1 > x - y - g$, then $\tilde{\theta}_1^* \Leftarrow c_1\tilde{\theta}_1 + yF(\tilde{\theta}_1) = y + c_1 + g - x$.

To summarize the above cases, regardless of whether $g \ge 0$ or $g < 0$, the optimal solution is such that:

(a) as long as $c_1 > \max\{x-y-g, 0\}$, then $\tilde{\theta}_1^* \Leftarrow c_1\tilde{\theta}_1 + yF(\tilde{\theta}_1) = y + c_1 + g - x$; (b) when $g < x - y$ and $c_1 < x - y - g$, then $\tilde{\theta}_1^* = 0$. Table 13 presents these findings.

	Range for c_1	
	$c_1 \in (0, x-y-g]$	$c_1 \in (x-y-g, 1)$
$\tilde{\theta}_1^*$	0	Solution of $c_1\tilde{\theta}_1 + yF(\tilde{\theta}_1) = c_1 + g - x + y$
p_{n1}^*	$\frac{1+w_n}{2}$	$\frac{1+w_n}{2}$
p_{r1}^*	$\frac{\alpha+x-y}{2}$	$\frac{\alpha+x-y[1-F(\tilde{\theta}_1^*)]}{2}$
D_{n1}^*	$\frac{1-w_n}{2} - \alpha S$	$\frac{1-w_n}{2} - \alpha S[1-F(\tilde{\theta}_1^*)]$
D_{r1}^*	S	$S[1-F(\tilde{\theta}_1^*)]$
π_1^*	$\frac{(1-w_n)^2}{4} + \frac{S}{2}(2x-y) - c_1S(1-\mu) - \beta(1-c_1)$	$\frac{(1-w_n)^2}{4} + \frac{S[1-F(\tilde{\theta}_1^*)]}{2}[2x - y[1-F(\tilde{\theta}_1^*)]]-$ $-c_1S[1-F(\tilde{\theta}_1^*) - \mu + H(\tilde{\theta}_1^*)] + gSF(\tilde{\theta}_1^*) - \beta(1-c_1)$

where $x = \alpha w_n$, $y = 2\alpha(1-\alpha)S$, $H(\tilde{\theta}_1^*) = \int_0^{\tilde{\theta}_1^*} \theta f(\theta) d\theta$, and μ is the mean of θ. We assume $g < x - y$.

Table 13: Optimal solution for In-house remanufacturing with non-zero salvage value

Table 14 is Table 13 when the quality of used products collected is $\theta \sim U[0,1]$.

	Range for c_1	
	$c_1 \in (0, x-y-g]$	$c_1 \in (x-y-g, 1)$
$\tilde{\theta}_1$	0	$1 - \frac{x-g}{y+c_1}$
p_{n1}^*	$\frac{1+w_n}{2}$	$\frac{1+w_n}{2}$
p_{r1}^*	$\alpha[\frac{1+w_n}{2} - S(1-\alpha)]$	$\frac{\alpha}{2} + \frac{xc_1+yg}{2(y+c_1)}$
D_{n1}^*	$\frac{1-w_n}{2} - S\alpha$	$\frac{1-w_n}{2} - \frac{S\alpha(x-g)}{y+c_1}$
D_{r1}^*	S	$\frac{S(x-g)}{y+c_1}$
π_1^*	$\frac{(1-w_n)^2}{4} + \frac{S}{2}[2x-y-c_1] - \beta(1-c_1)$	$\frac{(1-w_n)^2}{4} + \frac{S}{2(y+c_1)}[(x-g)^2 + 2g(y+c_1)] - \beta(1-c_1)$

where $x = \alpha w_n$ and $y = 2\alpha(1-\alpha)S$. We assume $g < x - y$.

Table 14: Optimal solution for In-house remanufacturing with non-zero salvage value when $\theta \sim U[0,1]$

Section D.4: The equilibrium for Outsourcing of remanufacturing, with non-zero salvage value g

The equilibrium for Retailer-led Outsourcing ($\delta = 0$)

The equilibrium is the same as in In-house optimum stated in Appendix D.3, except that (a) c_2 replaces c_1; (b) $\tilde{\theta}_2$ replaces $\tilde{\theta}_1$; and (c) $\beta = 0$.

The equilibrium for Third-party-led Outsourcing ($\delta = 1$)

	Range for c_2	
	$c_2 \in (0, x - 2y - g]$	$c_2 \in (x - 2y - g, 1)$
$\tilde{\theta}_2^*$	0	Solution of $c_2\tilde{\theta}_2 + 2yF(\tilde{\theta}_2) = c_2 + g - x + 2y$
w_r^*	$x - y$	$x - y[1 - F(\tilde{\theta}_2^*)]$
p_{n2}^*	$\frac{1+w_n}{2}$	$\frac{1+w_n}{2}$
p_{r2}^*	$\frac{\alpha+x-y}{2}$	$\frac{\alpha+x-y[1-F(\tilde{\theta}_2^*)]}{2}$
D_{n2}^*	$\frac{1-w_n}{2} - \alpha S$	$\frac{1-w_n}{2} - \alpha S[1 - F(\tilde{\theta}_2^*)]$
D_{r2}^*	S	$S[1 - F(\tilde{\theta}_2^*)]$
π_2^*	$\frac{(1-w_n)^2}{4} + \frac{yS}{2}$	$\frac{(1-w_n)^2}{4} + \frac{yS}{2}[1 - F(\tilde{\theta}_2^*)]^2$
π_{2o}^*	$S(x - y) + c_2S(1 - \mu)$	$S[1 - F(\tilde{\theta}_2^*)]\{x - y[1 - F(\tilde{\theta}_2^*)]\} - c_2S[1 - F(\tilde{\theta}_2^*) - \mu + H(\tilde{\theta}_2^*)] + SfF(\tilde{\theta}_2^*)$

where $x = \alpha w_n$, $y = 2\alpha(1 - \alpha)S$, $H(\tilde{\theta}_2^*) = \int_0^{\tilde{\theta}_2^*} \theta f(\theta) d\theta$, and μ is the mean of θ. We assume $g < x - y$.

Table 15: Nash bargaining equilibrium for the Outsourcing remanufacturing strategy when $\delta = 1$, with non-zero salvage value

Table 16 is Table 15 when the quality of used products collected is $\theta \sim U[0, 1]$.

	Range for c_2	
	$c_2 \in (0, x - 2y - g]$	$c_2 \in (x - 2y - g, 1)$
$\tilde{\theta}_2^*$	0	$1 - \frac{x-g}{2y+c_2}$
w_{r2}^*	$x - y$	$x - \frac{y(x-g)}{2y+c_2}$
p_{n2}^*	$\frac{1+w_n}{2}$	$\frac{1+w_n}{2}$
p_{r2}^*	$\frac{\alpha+x-y}{2}$	$\frac{1}{2}(\alpha + x - \frac{y(x-g)}{2y+c_2})$
D_{r2}^*	S	$\frac{S(x-g)}{2y+c_2}$
D_{n2}^*	$\frac{1-w_n}{2} - S\alpha$	$\frac{1-w_n}{2} - \frac{S\alpha(x-g)}{2y+c_2}$
π_2^*	$\frac{(1-w_n)^2}{4} + \frac{Sy}{2}$	$\frac{(1-w_n)^2}{4} + \frac{(x-g)^2yS}{2(2y+c_2)^2}$
π_{2o}^*	$\frac{S}{2}[2x - 2y - c_2]$	$\frac{S}{2(2y+c_2)}[(x - g)^2 + 2g(2y + c_2)]$

where $x = \alpha w_n$ and $y = 2\alpha(1 - \alpha)S$. We assume $g < x - y$.

Table 16: Equilibrium for Outsourcing of remanufacturing when $\theta \sim U[0, 1]$, with non-zero salvage value

Proposition 6. The impact of per-unit salvage value g is as follows:

(a) When $c_1 \in (0, x - y - g]$ for In-house; $c_2 \in (0, x - y - g]$ for Retailer-led Outsourcing; or $c_2 \in (0, x - 2y - g]$ for Third-party-led Outsourcing, there is no impact of g on the optimal/equilibrium solutions.

(b) When $c_1 \in (x - y - g, 1)$ for In-house; $c_2 \in (x - y - g, 1)$ for Retailer-led Outsourcing; or $c_2 \in (x - 2y - g, 1)$ for Third-party-led Outsourcing, a higher g leads to higher retail price (p_{ri}), higher demand for new product (D_{ni}), and lower demand for remanufactured product (D_{ri}). Under In-house and Retailer-led Outsourcing, the retailer's profit (π_i) increases with g. Under Third-party-led Outsourcing, the retailer's profit decreases with g while the third-party's profit (π_{2o}) increases with g.

Proof of Proposition 6.

These comparative statics follow from straightforward differentiation. Table 17 shows the properties of In-house and Retailer-led Outsourcing, while Table 18 addresses Third-party-led Outsourcing. Q.E.D.

Parameter	Key Decisions					
	θ_1^*	p_{n1}^*	p_{r1}^*	D_{n1}^*	D_{r1}^*	π_1^*
$g \uparrow$	NC/$\uparrow$	NC	NC / $\uparrow$	NC/$\uparrow$	NC/$\downarrow$	$\uparrow$

The directional relationships are abbreviated as follows, and apply over all $c_1 \in (0, 1)$ unless otherwise noted: (a) NC indicates no change; (b) NC/$\uparrow$ (or $\downarrow$) indicates NC when $c_1 \in (0, x - y - g]$ and $\uparrow$ (or $\downarrow$) when $c_1 \in (x - y - g, 1)$.

Table 17: Comparative statics for optimal solution for In-house remanufacturing

Parameter	Key Decisions							
	θ_2^*	w_{r2}^*	p_{n2}^*	p_{r2}^*	D_{n2}^*	D_{r2}^*	π_2^*	π_{2o}^*
$g \uparrow$	NC/$\uparrow$	NC/$\uparrow$	NC	NC/$\uparrow$	NC/$\uparrow$	NC/$\downarrow$	NC/$\downarrow$	NC/$\uparrow$

The directional relationships are abbreviated as follows, and apply over all $c_2 \in (0, 1)$ unless otherwise noted: (a) NC indicates no change; (b) NC/$\uparrow$ (or $\downarrow$) indicates NC when $c_2 \in (0, x - 2y - g]$ and $\uparrow$ (or $\downarrow$) when $c_2 \in (x - 2y - g, 1)$.

Table 18: Comparative statics for equilibrium for Outsourcing of remanufacturing when $\delta = 1$