Price Discrimination via Information Provision*

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Abstract

We study price discrimination where different prices are offered as a bundle with different levels of information about a product. The seller’s price discrimination induces high valuation buyers to purchase a good without information and low valuation buyers to purchase with information. Our analysis highlights several interesting results about price discrimination: (i) the seller’s choice of information provision is the combination of full information and no information. (ii) Products can be cheaper without information provision than with information provision (iii) as a result of price discrimination, prices can be more dispersed as buyers’ valuations become largely similar, and (iv) the high valuation buyers purchase a damaged good and may earn negative surplus. Furthermore, we investigate under which circumstances price discrimination is more profitable than uniform pricing. We show that a decline in transportation costs which facilitate price discrimination can be welfare reducing.

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1 Introduction

Price discrimination is often associated with information differentiation. One example is that many traditional merchants operate brick-and-mortar (offline) stores as well as online sites. A crucial difference between online shopping and offline shopping is the accessibility of information about products. In the offline store, people can learn better about the product, for example, by reading parts of novels, by trying on clothes, and so forth. On the other hand, when they shop online, it may be more difficult for consumers to decide whether the products really match their preferences. Another example is travel agency sites such as priceline.com or hotwire.com. These websites offer two options to buyers: buyers can choose hotels and flights with detailed information (transparent goods), or they can do so without knowing the brand and location of hotels or the brand and schedule of flights (opaque goods). Namely, they are offered both bundles which include different levels of information about the services at different prices. In addition, advance-purchase discounts or buy-now discounts can also be thought of as the type of price discrimination based on information provision because consumers are offered the discounts at the expense of a chance of evaluating the product fully or updating a previous valuation.

In these examples, buyers have two kinds of private information. One is their prior valuation for a good or service. The other is a signal that they receive about how well the product fits their taste. To receive a signal or process information, the buyers have to pay transportation costs, or alternatively, have to exert costly effort. Once the buyers receive a signal, they update their valuations. In addition, the seller can control the level of information by choosing a marketplace or a product design.

We provide a simple model of price discrimination through information differentiation which incorporates all these economic forces. We show that the self-selection is incentive compatible only when high valuation buyers purchase the product with less information and low valuation buyers purchase the one with more information. The intuition is as follows. When a buyer purchases a good or a service without information, he has to take some risks of ending up being mismatched with it. The buyer who has a high expected valuation, thereby
becoming sufficiently optimistic, will face relatively less risks, and so he may decide to buy the product even without further information. Knowing this, a seller is able to separate high valuation buyers from low valuation buyers by offering a cheaper price together with less information.

Information-driven price discrimination generates several interesting results. The seller’s optimal choice of information provision is the combination of full information and no information. Our model thus explains why products are often cheaper without information provision than with information provision, and similarly, why online prices are lower than offline prices.\(^1\) In addition, the optimal choice of information provision results in that a lower price may be offered to the \textit{ex ante} higher valuation buyer. Price discrimination can be strengthened as consumers are more homogeneous in their preferences. That is, the prices widens as the buyers’ valuations are closer to each other. Also, the result can be interpreted as that the high valuation buyers purchase a damaged good and may earn negative surplus.

We also study the conditions under which price discrimination is more profitable than selling only online or offline. Price discrimination is found to be profitable when the consumer heterogeneity is neither too large nor too small. This is intuitive because either the compensation to the low type or information rent to the high type is too costly, otherwise. Alternatively, price discrimination is profitable when transportation (effort) cost is small enough. This result implies that price discrimination is introduced when it is relatively easy for consumers to visit offline stores. On the other hand, our analysis shows that price discrimination may lead to lower consumer surplus and social welfare. Taken together, a decline in transportation cost can be welfare reducing.

Our paper presents a model of second-degree price discrimination where a seller offers a menu of options and lets buyers select what they want (Mussa and Rosen (1978); Maskin and Riley (1984)). There are several papers which study price discrimination in an environment where buyers are initially uncertain of their valuations and learn their preferences by addi-

\(^1\)Brynjolfsson and Smith (2000) find that prices on the Internet are 9-16% lower than prices in conventional outlets. See also Carlton and Chevalier (2001), Goolsbee (2001), Brown and Goolsbee (2002), and Chevalier and Goolsbee (2003) for empirical studies on online prices.
tional information. The seminal paper, Lewis and Sappington (1994) studies how information provision affects second-degree price discrimination of offering menus of different prices and quantities. Miravete (1996) compares \textit{ex ante} two-part tariffs and \textit{ex post} two-part tariffs in telecommunication industry. Courty and Li (2000) study sequential screening through refund policy. Grubb (2009) also studies a similar issue but focuses on the case that consumers are overconfident in that they overestimate the precision of their demand forecasts. In these papers, contracts are signed when consumers have partial private information, that is, before they learn their valuations. Compared to these papers, our paper differs in the sense that the provision of different levels of information is the screening mechanism itself. In other words, buyers' self selection arises by their purchase of different goods with different levels of information.\footnote{Bar-Isaac et al. (2010) explore information provision and gathering issues in a similar environment, but do not allow prices to differ.}

In this sense, the closest paper to ours is Nocke, Peitz and Rosar (2011) which studies how advance-purchase discounts can serve to price discriminate. In an intertemporal setting where consumers' uncertainty is resolved over time, advance-purchasing can be thought of as purchasing with less information. Our paper complements this literature by incorporating endogenous provision of information and studies the seller's optimal choice of prices and information provisions. Our simple model also allows us to analyze the effect of price discrimination on welfare.

In what follows, for ease of exposition, our description of the model will follow mostly the offline and online scenario. Section 2 introduces the basic model. Section 3 analyzes the cases where either only online or only offline stores are available. In Section 4, the seller is allowed to price discriminate by having both types of stores, and we derive the optimal price discrimination. In Section 5, we then compare the profits of three cases: selling only online, only offline, and at both stores. We will show which is most profitable to the seller. Section 6 extends the model by relaxing several assumptions. Section 7 concludes.
2 Basic Model

Seller. There is a monopoly seller with a single product, which can be sold at offline and/or online stores. A significant difference between shopping at online and at offline stores is the informativeness of the signal that a buyer may receive. In other words, the monopolist offers a product with different levels of information: one with more information and another with less information. We assume that production cost is zero.\(^3\)

Buyers. There is a continuum of buyers with a unit demand. Each buyer’s match value, \(v\), for the product is either \(v_H\) with probability \(\theta\) or \(v_L\) with probability \((1 - \theta)\), where \(\theta \in [0, 1]\). We normalize buyers’ valuation as \(v_H = 1\) and \(v_L = 0\), then the buyers’ expected valuation for the product is simply \(\theta\). We consider two types of buyers (each with mass equal to one): type-\(i\) buyers have \(\theta = \theta_i\), where \(i \in \{L, H\}\) and \(\theta_L < 1/2 < \theta_H\).\(^4\) In words, type-H buyers are more optimistic about their match values, \(v\), than type-L buyers. The buyers’ type is private information, and thus it is not observable to the seller. This simple setup with only two types enables us to conduct comparative statics analysis on the optimal prices and draw welfare implications.

Information. Once a buyer arrives at a store, he may observe a binary private signal on his match value: a good signal \(s_H\) or a bad signal \(s_L\). The signal provides information about the good’s match value in the sense of Blackwell. Information in our model is real information, as in Johnson and Myatt (2006), which allows consumers to learn of their true match with the product.\(^5\)

The probability of the signal being \(s_k\) follows the conditional probability distributions

\[
Pr(s_k|v_l) = \alpha \text{ for } l = k \quad \text{and} \quad Pr(s_k|v_l) = 1 - \alpha \text{ for } l \neq k,
\]

where \(l, k \in \{L, H\}\). Note that

\(^3\)The production cost can be added to the model, but our main results remain the same. To focus on the effect of information provision on prices, we prefer not to include it.

\(^4\)This assumption greatly simplifies the analysis of the seller’s choice of information provision. This assumption is imposed only for technical simplicity. Our main results are robust to the relaxation of this assumption.

\(^5\)In contrast, hype information is basic publicity for the product; a consumer might learn of the product’s existence, price, availability, and any objective quality. Alternatively, information in our model can be thought of as the direct (first-hand) information which potential buyers can obtain when inspecting products by themselves at a store. Among others, see Ghose (2009) for empirical studies on product-level uncertainty in online markets.

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\( \alpha \) represents the precision or informativeness of the signal. Without loss of generality, we consider \( \alpha \in \left[ \frac{1}{2}, 1 \right] \). Given the prior belief \( \theta_i \), the probability of receiving signal \( s_k \) is

\[
\Pr(s_k; \theta_i, \alpha) = \theta_i \Pr(s_k|v_H) + (1 - \theta_i) \Pr(s_k|v_L).
\]

Let \( \phi_k(\theta_i, \alpha) \equiv \Pr(v_H|s_k) \) be the buyer’s posterior belief about \( v = v_H \), after receiving the signal \( s_k \). Then, Bayes’ rule leads to

\[
\phi_H(\theta_i, \alpha) \equiv \Pr(v_H|s_H) = \frac{\alpha \theta_i}{\alpha \theta_i + (1 - \alpha)(1 - \theta_i)}, \quad \text{and}
\]

\[
\phi_L(\theta_i, \alpha) \equiv \Pr(v_H|s_L) = \frac{(1 - \alpha) \theta_i}{\alpha(1 - \theta_i) + (1 - \alpha) \theta_i}.
\]

Type-\( i \) buyer’s posterior valuation of the good is a mean-preserving spread from a degenerate distribution to a two-point distribution: it will be \( \phi_H(\theta_i, \alpha) \) with the probability \( \Pr(s_H) \) and \( \phi_L(\theta_i, \alpha) \) with the probability \( \Pr(s_L) \).

**Online vs. Offline.** The buyers are able to personally examine a product in offline stores. It is thus easier for the seller to provide information so that the buyers can find out how well the product fits his tastes. In online stores, however, the buyers only can inspect the product by browsing the pictures on the website, which might prevent them from figuring out the product’s exact characteristics.\(^7\) At the same time, understanding or processing this information is also costly to the buyers. The buyers have to incur effort costs of learning the realization of their valuations. In our scenario, this can be understood as transportation costs to get to the offline store.

To capture these features in a simple way, we assume that the signal that the buyer receives from the offline stores is more informative than from the online stores, i.e., \( \alpha_N < \alpha_F \), where \( \alpha_N \) and \( \alpha_F \) are the informativeness of the signals from the online and the offline stores, respectively.

\(^6\)If \( \alpha \) is less than \( \frac{1}{2} \), then the buyers would regard \((1 - \alpha)\) as an effective signal, which provides the same level of information.

\(^7\)We assume that it is impossible for the buyers to return the product once they purchase it. Of course, the product return can be an issue in this online and offline scenario. But, in other examples such as advance discount purchase or travel agency websites, the product return is not an issue.
respectively. Also, the buyers pay \( t_F \) for visiting offline stores.\(^8\) To rule out a trivial case, let us assume that \( \alpha_N < \bar{\alpha} \), where \( \bar{\alpha} \) is a possible maximum level of information at the online store, where \( \bar{\alpha} \) is defined as \( (1/2 + \bar{\alpha})\bar{\alpha} = \text{Max}\{\theta_H, 2\theta_L\} \).\(^9\) Otherwise, the first-best outcome can be achieved by providing full information with setting the highest price, i.e., \( \alpha_N = 1 \) and \( p_N = 1 \). The assumption implies that information provision beyond a certain level is prohibitively costly for the seller at the online store.

If the buyer shops at the online store, his posterior valuation is either \( \phi_H(\theta_i, \alpha_N) \) when receiving a good signal or \( \phi_L(\theta_i, \alpha_N) \) when a bad signal. Then, type-\( i \) buyer’s net utility is

\[
U_i = \begin{cases} 
\phi_H(\theta_i, \alpha_N) - p_N & \text{with a good signal} \\
\phi_L(\theta_i, \alpha_N) - p_N & \text{with a bad signal}
\end{cases}
\]

if buys at the online store,

where the online price is denoted by \( p_N \). Similarly, if he shops at the offline store, his posterior valuation is \( \phi_H(\theta_i, \alpha_F) \) when receiving a good signal or \( \phi_L(\theta_i, \alpha_F) \) when a bad signal. However, the important difference is that his purchase decision is conditional on receiving a good signal offline. Otherwise, consumers do not have incentives to pay transportation costs. Let us denote the offline price by \( p_F \). Now, type-\( i \) buyer’s net utility from the purchase offline is written as

\[
U_i = \Pr(s_H; \theta_i, \alpha_F)(\phi_H(\theta_i, \alpha_F) - p_F) - t_F, \quad \text{if buys at the offline store.}
\]

We further assume \( t_F \) is small enough to ensure that all the possible cases are considered in the analysis;

\[
\theta_L \geq 2t_F.
\]

This assumption guarantees that the seller has an incentive to sell to type-L buyers, i.e.,

\[
\min_{\alpha_F, p_F} \Pr(s_H; \theta_L, \alpha_F)(\phi_H(\theta_L, \alpha_F) - p_F) \geq t_F.
\]

Otherwise, type-L buyers may not be able to purchase even at \( p_F = 0 \).

\(^8\)Forman et al. (2009) empirically examine the trade-off between the benefits of online and offline shopping and show that offline transportation costs matter.

\(^9\)The LHS of the inequality is derived from \( [\Pr(s_H; \theta_L, \tilde{\alpha}) + \Pr(s_H; \theta_H, \tilde{\alpha})] \phi_H(\theta_L, \tilde{\alpha}) \) by setting \( \theta_H = 1 \) and
Timeline. The timing of the game is as follows. First, the monopolist determines marketplace(s) to sell her product: selling only online, only offline, or both online and offline. Second, the seller designs a mechanism. That is, the seller quotes price(s) which immediately become observable to all potential buyers. At the same time, the seller generates an informative signal at each marketplace. Lastly, the buyers make a purchase decision: where to go shopping and whether to buy or not.

First Best. Let us consider the first-best outcome as a benchmark. When the type of buyers is known to the seller, the optimal level of information provision and corresponding prices are $\alpha_N = 1/2$ and $p_N = \theta_i$. Then, the seller’s profit is

$$\pi^{FB} = \theta_H + \theta_L.$$  

This is essentially perfect (first degree) price discrimination. The seller does not provide any information and extracts each type of buyers’ expected surplus.

$\theta_L = 1/2$. This is the profit when the seller chooses $p_N = \phi_H(\theta_L, \tilde{\alpha})$. See the proof of Lemma 1.
3 Online or Offline

To begin with, we study the case where the product is available only at either online or offline stores. The seller is thus not able to price discriminate. Information provision leads to an increase in the dispersion of demand. As a result, in either regime, the seller’s price choice is to decide whether to serve only type-H buyers or both types of buyers.

**Only online store.** If the seller charges $\phi_H(\theta_i, \alpha_N) \geq p_N > \phi_L(\theta_i, \alpha_N)$, then type-$i$ buyers will purchase only when receiving a good signal. If he charges $\phi_L(\theta_i, \alpha_N) \geq p_N$, then all buyers will purchase the good. Since there are two types of buyers in our model, it turns out that there are four types of posterior valuations after receiving a signal: $\phi_H(\theta_H, \alpha_N)$, $\phi_L(\theta_H, \alpha_N)$, $\phi_H(\theta_L, \alpha_N)$, and $\phi_L(\theta_L, \alpha_N)$. We can certainly rank $\phi_H(\theta_H, \alpha_N)$ as the highest and $\phi_L(\theta_L, \alpha_N)$ as the lowest, but the rank between $\phi_L(\theta_H, \alpha_N)$ and $\phi_H(\theta_L, \alpha_N)$ is determined by $\alpha_N$. In this environment, the following lemma shows the seller’s optimal choice of information provision and price.

**Lemma 1** The seller’s optimal choice of information level is $\alpha_N = 1/2$. The optimal price and corresponding profits are

\[ p^*_N = \theta_H \text{ and } \pi^*_N = \theta_H, \quad \text{if } \theta_H > 2\theta_L, \]  
\[ p^*_N = \theta_L \text{ and } \pi^*_N = 2\theta_L, \quad \text{if } \theta_H \leq 2\theta_L. \]

**Proof.** In the Appendix. ■

Since the seller’s optimal choice of information provision is no information, the buyer’s valuation does not change and he or she always ends up buying the product. Thus, the seller can simply charge the expected gross utility of the targeted type of buyers, $\theta_i$.\(^{10}\)

\(^{10}\)One may think that no provision of information at online stores does not seem to be supported empirically, because many online stores provide images and photos of products with detailed product description. However, recall that information in our model is real information. See footnote ??.
**Only offline store.** Now, the type-$i$ buyer goes to the offline for shopping if
\[ \Pr(s_H; \theta, \alpha_F)(\phi_H(\theta, \alpha_F) - p_F) - t_F \geq 0. \]
Only type-H buyers will go shopping at the offline store if the price is
\[ \phi_H(\theta, \alpha_F) - \frac{t_F}{\Pr(s_H; \theta, \alpha_F)} \geq p_F > \phi_H(\theta_L, \alpha_F) - \frac{t_F}{\Pr(s_H; \theta_L, \alpha_F)}, \]
but both types of buyers will do so if
\[ \phi_H(\theta_L, \alpha_F) - \frac{t_F}{\Pr(s_H; \theta_L, \alpha_F)} \geq p_F. \]

**Lemma 2** The seller’s optimal choice of information level is $\alpha_F = 1$. Thus, the optimal price and the corresponding profits are now
\[
\begin{align*}
\bar{p}_F^* &= 1 - t_F/\theta_H \quad \text{and} \quad \bar{\pi}_F^* = \theta_H - t_F, \quad \text{if } \theta_H > \theta_L^2/t_F, \quad \text{(Case 3)} \\
p_F^* &= 1 - t_F/\theta_L \quad \text{and} \quad \pi_F^* = (\theta_H + \theta_L)(1 - t_F/\theta_L), \quad \text{if } \theta_H \leq \theta_L^2/t_F. \quad \text{(Case 4)}
\end{align*}
\]

**Proof.** In the Appendix. ■

In the offline store case, the seller targets only the buyers who receive a good signal. As a result, the seller’s choice of information provision is now full information, $\alpha_F = 1$. The optimal price is essentially comprised of the buyer’s gross utility net of transportation cost. Also, only buyers receiving a good signal will decide to buy the product and all others will walk away empty-handed. It turns out that the seller provides full information at the online store and no information at the offline store. It is well-known in the literature that the seller’s profits are U-shaped in the dispersion of demand.

**Proposition 1** If $\theta_L \geq 3t_F$ and $(\theta_L + t_F)/(1 - t_F/\theta_L) < \theta_H < (\theta_L/t_F)(\theta_L - t_F)$, the seller’s profit is greater at the offline store than at the online. Otherwise, the seller’s profit is greater at the online than at the offline store.

**Proof.** In the Appendix. ■
This proposition summarizes how the seller chooses the price and the provision of information together when price discrimination is not feasible. In particular, the seller’s decision of information provision is not monotonic in the degree of consumer heterogeneity. When buyers’ information acquisition costs are rather small ($\theta_L \geq 3t_F$), the seller has three possible alternatives. When the heterogeneity of consumers is large enough, the seller chooses to target only type-H buyers at the online store without information provision. When the heterogeneity is small enough, the seller chooses to serve all consumers at online stores without information provision. However, when the heterogeneity is intermediate, the seller prefers now to provides full information at offline stores.

The intuition for this result is as follows. The online store does not achieve the first best outcome because it cannot price discriminate. On the other hand, the offline store has to adjust the price downward to compensate for buyers’ transportation costs. Note that the offline store could be able to achieve the first best if $t_F = 0$, without price discrimination. This trade-off determines at which marketplace the seller’s profit is greater. Thus, if transportation costs are relatively large, i.e., $\theta_L < 3t_F$, the online store strictly dominates the offline store. However, when the transportation costs are small, the offline store can make a greater profit when $\theta_H$ is intermediate. When $\theta_H$ is either large enough or small enough, the deviation of the online prices from the first best becomes marginal and the online store makes a greater profit.

4 Second-Degree Price Discrimination: Online and Offline

Let us consider the case that the seller runs both online and offline stores and quotes prices at each store. There are possibly two ways to induce self-selection in terms of buyers’ choices of stores. First, the self-selection may occur in the way that type-L buyers shop at the online store and type-H buyers at the offline store.

**Proposition 2** There exists no pair of prices, $(p_N, p_F)$, which induce type-L buyers to shop at online stores and type-H buyers at offline stores.
**Proof.** In the Appendix.

The intuition of the proof is as follows. Without information, the purchase of a good or service may end up being a mismatch with the buyer. Type-H buyers are more optimistic about the match value of the product, and thus they are more likely to prefer shopping at the online store with no further information to shopping at the offline store than type-L buyers are. Hence, whenever type-L buyers find the online price a better deal than the offline price, so do type-H buyers. There does not exist an incentive compatible set of prices such that type-H buyers shop offline and type-L buyers shop online.

The other possible scenario is that type-L buyers shop at the offline store but type-H buyers at the online store. In this case, for the seller’s profit maximization problem, we analyze the following truthful direct revelation mechanism.

\[
\max_{\alpha_F, \alpha_N, \phi_F, \phi_N} \pi_{NF} = \Pr(s_H; \theta_L, \alpha_F) \cdot p_F + p_N
\]  

subject to

\[
[IR_L] \quad \Pr(s_H; \theta_L, \alpha_F) (\phi_H(\theta_L, \alpha_F) - p_F) - t_F \geq 0
\]

\[
[IR_H] \quad \phi_L(\theta_H, \alpha_N) - p_N \geq 0
\]

\[
[IC_L] \quad \Pr(s_H; \theta_L, \alpha_F) (\phi_H(\theta_L, \alpha_F) - p_F) - t_F \geq \phi_H(\theta_L, \alpha_N) - p_N
\]

\[
[IC_H] \quad \phi_L(\theta_H, \alpha_N) - p_N \geq \Pr(s_H; \theta_H, \alpha_F) (\phi_H(\theta_H, \alpha_F) - p_F) - t_F
\]

The first two individual-rationality \([IR]\) constraints ensure that the buyers’ utility should be non-negative. When type-H buyers purchase online, they may receive a good signal or a bad signal. The seller wants to serve all type-H buyers. Thus, \([IR_H]\) says that type-H buyers purchase even when they draw a bad signal. The other two constraints are the incentive-compatible \([IC]\) constraints. \([IC_L]\) requires type-L buyers to choose the offline over the online store. The RHS of this constraint is the type-L buyer’s utility if he draws a good signal and purchases online. That is, the seller does not want to sell even to those who receive a good signal for the self-selection. Likewise, \([IC_H]\) ensures type-H buyers choose the online over the
off-line store.

As usual, \([IR_H]\) and \([IC_L]\) are always satisfied and the maximization problem reduces to maximizing (1) subject to \([IR_L]\) and \([IC_H]\). This is illustrated in Figure 2. The shaded area is the feasible set of prices for price discrimination, which is determined by the three constraints, \([IC_H]\), \([IC_L]\), and, \([IR_L]\). We represent the downward sloping iso-profit curve as the locus of price combinations along which expected profit is constant. Thus, the seller’s profit is maximized at the prices where \([IR_L]\) and \([IC_H]\) are binding.

**Proposition 3** Suppose the seller induces self-selection in the way that type-H buyers purchase online and type-L buyers purchase off-line. The seller chooses \(\alpha_F = 1\) and \(\alpha_N = 1/2\). There exists an optimal pair of prices,

\[
\hat{p}_F = 1 - t_F/\theta_L \quad \text{and} \quad \hat{p}_N = \theta_H(1 - t_F/\theta_L) + t_F,
\]

for which type-H buyers shop online and type-L buyers at the off-line stores. The corresponding
profits are

\[ \hat{\pi}_{NF} = \theta_H + \theta_L - t_F \frac{\theta_H}{\theta_L}. \]  

(3)

**Proof.** In the Appendix. ■

The seller’s optimal choice of information provision is the combination of full information and no information. As shown by Lewis and Sappington (1994) and Johnson and Myatt (2006), it is well-known in the literature that the monopolist prefers either full information or no information. This all-or-nothing result in information provision continues to hold also in the case where the seller price discriminates through different levels of information. An interesting point is that full information provided to type-L buyers is the distortion from the first-best outcome. In the standard model of adverse selection, the contract for a less efficient agent is downwardly distorted (underprovision of quality or quantity) relative to the first best outcome.

As a result of the seller’s choice of \( \alpha_F = 1 \) and \( \alpha_N = 1/2 \), type-L buyers, who shop offline, purchase the good with probability \( \theta_L \), i.e., only when they receive a good signal after examining the product at the store. On the other hand, type-H buyers make the purchase with probability 1 with receiving information rent,

\[ U_H - U_L = \frac{\theta_H - \theta_L}{\theta_L} t_F \]  

(4)

Thus, the problem is greatly reduced to

\[ \max_{p_F, p_N} \pi_{NF} = \theta_L \cdot p_F + p_N \quad \text{s.t.} \quad \begin{cases} [IR'_L] & p_F \leq 1 - t_F/\theta_L \\ [IC'_H] & t_F \geq p_N - \theta_H p_F \end{cases} \]

Solving \([IR'_L]\) and \([IC'_H]\) together, we immediately obtain the prices (2) and the seller’s profit (3). From Proposition 3, we find some interesting observations on the optimal prices and welfare implications, which are summarized in the following three corollaries.

**Corollary 1** There is a cutoff value \( \bar{t}_F \) such that the optimal price offered at the online store
is lower than that at the offline store if $t_F < \bar{t}_F$, and vice versa.

\[
\hat{p}_N \leq \hat{p}_F, \text{ as } t_F \leq \bar{t}_F = \frac{(1 - \theta_H)\theta_L}{1 - \theta_H + \theta_L}
\]

**Corollary 2** When $t_F < \bar{t}_F$, $(\hat{p}_F - \hat{p}_N)$ is decreasing in $(\theta_H - \theta_L)$.

**Corollary 3** The type-H buyers with low match value will earn a negative surplus.

Price discrimination through information provision shows several interesting and novel results. Note first that, without informational discrepancies between online and offline stores, the price at the offline store is supposed to be lower than at the online store due to a transportation cost. That is, the price at the offline store should be adjusted to compensate the transportation cost. However, when shopping at the online store, buyers have to take risks of buying an undesirable product with zero match value, and the risks are larger to type-L buyers than type-H buyers. Thus, the seller can separate the two types by lowering the price at the online store where no further information on products is available. Such incentives to separate out the two types of buyers push down the price at the online store as information rent for type-H buyers. This is the reason why type-H buyers may pay a lower price than type-L buyers when $t_F$ is relatively small.\footnote{A similar observation was made by our companion paper, Bang, Kim, and Yoon (2011). The phenomenon is named as "reverse price discrimination". The paper studies the third-degree price discrimination in the environment where consumers are uncertain of the quality of goods and services.} But, when $t_F$ is large enough, the offline price for L-type buyers becomes lower to compensate their transportation costs.

Second, the price difference between offline and online, $\hat{p}_F - \hat{p}_N$, is getting larger as the degree of consumer heterogeneity decreases. Interestingly, price discrimination is waning as buyers have more divergent beliefs (expected valuations). The explanation of this result is rather subtle. Recall that type-H buyers are offered a lower price than type-L buyers under $t_F < \bar{t}_F$. Thus, as $\theta_H$ rises, only $\hat{p}_N$ offered to type-H buyers increases. Next, as $\theta_L$ rises, $\hat{p}_N$ increases slower than $\hat{p}_F$ because informational rent should be given to type-H buyers.

Lastly, type-H buyer may end up having a negative surplus. Since type-H buyers purchase the product without knowing how well it matches their preferences, they will suffer from a
loss with probability \(1 - \theta_H\), although they are given \textit{ex ante} informational rent. Their purchase may not be socially desirable either with probability \(1 - \theta_H\) if a seller has to incur positive production costs. But, type-L buyer’s purchase is socially optimal since type-L buyers are perfectly informed of the characteristics of the product at the offline and only people who find a good match will buy the product.

In addition, interestingly, the incentive compatible screening requires the high valuation buyer to choose the damaged product. When we think of the level of information as a part of the product, concealing important characteristics of the product can be regarded as providing a damaged good, in particular, when hiding information can be costly.

5 Optimal Choice of Marketplace

In this section, we compare the optimal profits in three cases: selling at the online, at the offline, and at both stores. In particular, we are interested in knowing when price discrimination is profitable.

\textbf{Proposition 4} (a) If \( \frac{\theta_L}{1 - \theta_L/\theta_F} \leq \theta_H \leq 2\theta_L \) or \( 2\theta_L < \theta_H \leq \frac{\theta_L^2}{\theta_F} \), then it is optimal for the seller to operate both online and offline stores and to employ price discrimination.

(b) If \( \theta_H > \max\{2\theta_L, \frac{\theta_L^2}{\theta_F}\} \), then it is optimal to operate only an online store and to sell only to type-H buyers.

(c) If \( \theta_H < \min\{2\theta_L, \frac{\theta_L}{1 - \theta_L/\theta_F}\} \), then it is optimal to operate only an online store and to sell to both types of buyers.

\textbf{Proof.} In the Appendix. ■

Given \( \theta_L \), the seller wants to use price discrimination when \( \theta_H \) is neither too large nor too small relative to \( \theta_L \) (case a). That is, price discrimination is profitable when the heterogeneity of consumers is intermediate. This can be understood by considering two kinds of costs in implementing price discrimination. One is related to the compensation of transportation costs for type-L buyers’ purchases at the offline store. The other one is involved with information
rent that should be given to type-H buyers. When the heterogeneity of consumers is small (case c), the compensation of transportation costs is relatively large. On the other hand, when the heterogeneity of consumers is large (case b), price discrimination is again too costly because information rent, \((\theta_H - \theta_L) t_F / \theta_L\), are increasing in \(\theta_H\) and decreasing \(\theta_L\).

Proposition 4 can be rewritten in terms of transportation costs. This is illustrated in Figure 3. What is the impact of a change in transportation costs on the optimal prices? It is generally ambiguous because the cutoff value of \(t_F\) under which price discrimination is introduced is different depending on the consumer heterogeneity.

**Corollary 4**

(a) If \(\theta_H > 2\theta_L\), as \(t_F\) increases beyond \(\theta_L^2 / \theta_H\), only type H-buyers purchases, and they pay a higher price. Consumer surplus falls.

(b) If \(\theta_H \leq 2\theta_L\), as \(t_F\) increases beyond \(\theta_L(\theta_H - \theta_L) / \theta_H\), the buyers pay a lower price and consumer surplus rises.

Price discrimination is introduced when \(t_F\) is relatively small. When \(\theta_H > 2\theta_L\), as \(t_F\) increases, the seller chooses to target only type-H buyers at \(\bar{p}_N = \theta_H\). It can be shown that \(\bar{p}_N > \hat{p}_N\). Type-L buyers are not able to purchase without price discrimination, and type-H buyers have to pay a higher price. Thus, consumer surplus decreases. On the other hand, when \(\theta_H \leq 2\theta_L\), the seller chooses to serve both types of buyers at the price \(\bar{p}_N = \theta_L < \min\{\hat{p}_N, \hat{p}_F\}\). In this case, it is readily clear that price discrimination leads to lower social
welfare because transportation costs must be paid by type-L buyers. As a result, a fall in transportation costs can reduce social welfare because it facilitates price discrimination.

6 Extensions

In this section, we relax several assumptions underlying our model: (i) no cost of information provision and (ii) no information elicitation. Since our main interest is to see the effect of each assumption on price discrimination, we confine our attention to the case of price discrimination. In each case, we retain the other assumption to isolate the effect of each assumption.

Bounds on information level. Until now, the provision of information has been flexible. But, a more realistic situation might be that the seller has to incur some costs to provide information. Also, hiding all the information may be costly because consumers can ask other users to tell their level of satisfaction. Then, a natural question is how the cost of information provision or hiding changes our results. The simplest way to study these issues is that we can restrict the parameter range of information level into $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, where $1/2 < \underline{\alpha} < \overline{\alpha} < 1$. A higher cost of information provision or hiding can be thought of as shrinking the parameter range.

Since $\pi_{NF}$ in (6) is increasing in $\alpha_F$ and decreasing in $\alpha_N$, it is immediate that the seller’s choice of information level is $\alpha_N = \underline{\alpha}$ and $\alpha_F = \overline{\alpha}$, thus the profit definitely falls. The interesting part is the impact on the prices. If the seller is not able to provide full information, $[IR_L]$ is tightened and $[IC_H]$ is relaxed. This implies that as information provision is more costly, the offline price $p_F$ falls, whereas the online price $p_N$ rises. On the other hand, due to the cost of information hiding, $[IC_H]$ is tightened. In this case, only the online price $p_N$ falls. Somewhat interestingly, the addition of the cost of information provision at the offline store reduces the offline price. Likewise, the cost of information hiding at the online store reduces the online price.

Buyers’ milking strategy. Until now, buyers can only purchase at the store where they
receive the signal. If buyers are sophisticated enough, however, the buyers might consider visiting one store to get information about the product and then purchasing from the other store (*milking strategy*).\footnote{This issue is only relevant in the scenario of online and offline stores. In other examples such as advance purchase discounts and travel agency websites, it is impossible to examine a transparent good for the purchase of an opaque good at a discounted price.} The buyers will be tempted to do so especially when they know that the other store is offering a lower price than the store from which they gather the product information. In order to prevent buyers pursuing milking strategies, the seller’s choice of prices must satisfy the following "arbitrage (milking)-proof" conditions.

\[
[AP_L] \quad \Pr(s_H; \theta_L, \alpha_F)(\phi_H(\theta_L, \alpha_F) - p_F) \geq \Pr(s_H; \theta_L, \alpha_F)(\phi_H(\theta_L, \alpha_F) - p_N - s) \quad (5)
\]

\([AP_L]\) implies that, for type-L buyers, buying at the offline store is better than switching to the online store after examining the product at the offline store. The RHS in (5) says that when type-L buyer draws a good signal at the offline store, he goes to shop online by paying a small switching cost \(s\). This constraint boils down to \(p_F - p_N \leq s\), which is intuitively appealing because type-L buyers would marginally compare (i) the benefits from paying a lower price at the online store with (ii) the extra transportation costs incurred from an additional visit to the online store. \([AP_L]\) tells us that even if the buyer arrives at
the offline store and finds the product perfectly matching his tastes, the price advantage of
the online over the offline store is not prominent enough to provoke him to make another
trip to the online store.\footnote{13} Then, as illustrated in Figure 4 (b), $[IC_H]$ and $[AP_L]$ are now
binding for the profit maximization problem. It is clear that the seller has incentives to
reduce information provision offline because a smaller $\alpha_F$ relaxes both $[IC_H]$ and $[AP_L]$. As
a result, the "milking-proof" condition reduces both prices, $p_F$ and $p_N$.\footnote{14}

7 Concluding Remarks

In this paper, we propose a model of second-degree price discrimination where different prices
are offered as a bundle with different levels of information about a product. Price discrimi-
nation through information discrimination is not uncommon. There are many occasions in
which buyers are uncertain about their match value of the product, and they may update
their expected valuations in a Bayesian way after observing a signal coming from the product.
At the same time, a seller is able to control the precision of the signal and offer differentiated
products in terms of information.

Our model suggests that high valuation buyers are induced to purchase the good or service
without information because they face less risks of ending up being mismatched. Who is more
willing to book a hotel without knowing its exact location and facilities? Who is more willing
to purchase a newly introduced product in advance or clothes at online stores without trying
on? They are those who are more optimistic and thus have higher expected valuations.

In this setting, we find several interesting results compared to the canonical model of
price discrimination. We show that online prices are often cheaper than offline prices. Also,

\footnote{13}Similarly, we can think of $[AP_H]$ which assures that type-H buyers do not visit the offline store for the
purpose of collecting information before making a purchase at the online store. But it is trivial that $[AP_H]$
implied by $[IC_H]$ and $[AP_L]$ together.

\footnote{14}There can be some ways for the seller to prevent buyers from using a milking strategy. One immediate way
is to manipulate offline transportation costs either by relocating offline stores to remote sites or by reducing
the number of offline stores. An alternative way is that she can differentiate products sold in online and
offline stores. If the seller provides a slightly different version in online from offline, the product information
buyers gather from offline stores will not perfectly show the match value of the product being sold in online
stores. This would give an interesting explanation for marketplace-specific product versioning by which some
products are only made available for either online or offline purchase.
the high valuation buyers may be offered a lower price than the low valuation buyers because information rent is given to the high valuation buyers in the form of a lower price without information. As a result of price discrimination, prices are more dispersed as buyers’ valuations become largely similar. Also, since the low valuation buyers purchase the good with full information while the high valuation buyers purchase the good without information, the result can be interpreted in a way that the high valuation buyers purchase a damaged good and may earn negative surplus.

Lastly, we also find that price discrimination is profitable when the heterogeneity of consumers is intermediate. This is because price discrimination can be implemented at the cost of paying information rent and compensating buyers’ effort (transportation) cost. In addition, we show how price discrimination affects consumer surplus. When the heterogeneity of consumers is small, consumers are worse-off with price discrimination, and vice versa. One implication of this result is that consumer surplus and social welfare is non-monotonic in transportation costs.

8 Appendix

The proof of Lemma 1.

Proof. The seller’s choice of the price is one of \( \phi_H(\theta_H, \alpha_N) \), \( \phi_L(\theta_H, \alpha_N) \), \( \phi_H(\theta_L, \alpha_N) \), and \( \phi_L(\theta_L, \alpha_N) \). (i) First, if \( p_N = \phi_H(\theta_H, \alpha_N) \), only type-H buyers who receive a good signal buy and \( \pi_N = \Pr(s_H; \theta_H, \alpha_N)\phi_H(\theta_H, \alpha_N) = \alpha_N \theta_H \). But since \( \alpha_N < 1 \), this is dominated by \( \alpha_N = 1/2 \) and \( p_N = \theta_H \).

(ii) Second, if \( p_N = \phi_L(\theta_H, \alpha_N) > \phi_H(\theta_L, \alpha_N) \), only type-H buyers purchase and so \( \pi_N = \phi_L(\theta_H, \alpha_N) \) which is decreasing in \( \alpha_N \). In the case of \( p_N = \phi_L(\theta_H, \alpha_N) \leq \phi_H(\theta_L, \alpha_N) \), \( \pi_N = \phi_L(\theta_H, \alpha_N)(1 + \Pr(s_H; \theta_L, \alpha_N)) \), because all type-H buyers and type-L buyers who receive a good signal purchase the good. The profit can be shown to be decreasing in \( \alpha_N \). Thus, the seller clearly chooses \( \alpha_N = 1/2 \) in either case.

(iii) Third, if \( p_N = \phi_H(\theta_L, \alpha_N) \), then \( \pi_N = [\Pr(s_H; \theta_L, \alpha_N) + \Pr(s_H; \theta_H, \alpha_N)] \phi_H(\theta_L, \alpha_N) \), which is increasing in \( \alpha_N \). But, according to our assumption that \( \alpha_N < \bar{\alpha} \), this is dominated
by $\alpha_N = 1/2$.

(iv) Lastly, if $p_N = \phi_L(\theta_L, \alpha_N)$, both H- and type-L buyers purchase. In this case, $\pi_N = 2\phi_L(\theta_L, \alpha_N)$ is also decreasing in $\alpha_N$.

By inserting $\alpha_N = 1/2$, when the seller charges $\theta_L < p_N \leq \theta_H$, only type-H buyers will purchase. But, when she charges $p_N \leq \theta_L$, both types of buyers will buy. ■

The proof of Lemma 2.

**Proof.** The seller’s choice of price is one of $\phi_H(\theta_H, \alpha_F) - t_F/\Pr(s_H; \theta_H)$, and $\phi_H(\theta_L, \alpha_F) - t_F/\Pr(s_H; \theta_L)$. (i) First, if $p_F = \phi_H(\theta_H, \alpha_F) - t_F/\Pr(s_H; \theta_H)$, only type-H buyers who receive a good signal will buy and $\pi_F = \Pr(s_H; \theta_H)(\phi_H(\theta_H, \alpha_F) - t_F/\Pr(s_H; \theta_H)) = \alpha_F \theta_H - t_F$.

(ii) Next, if $p_F = \phi_H(\theta_L, \alpha_F) - t_F/\Pr(s_H; \theta_L)$, both type-H and type-L buyers who receive a good signal choose to buy. That is, $\Pr(s_H; \theta_H) + \Pr(s_H; \theta_L)$ proportion of buyers make the purchase at that price. Then, $\pi_F = (\Pr(s_H; \theta_H) + \Pr(s_H; \theta_L))(\phi_H(\theta_L, \alpha_F) - t_F/\Pr(s_H; \theta_L))$. This can be rewritten as $\pi_F = \alpha_F \theta_L - t_F + \Pr(s_H; \theta_H)(\phi_H(\theta_L, \alpha_F) - t_F/\Pr(s_H; \theta_L))$. It can be easily shown that both $\Pr(s_H; \theta_H)$ and $\frac{\alpha_F \theta_L - t_F}{\alpha_F \theta_L + (1 - \alpha_F)(1 - \theta_L)}$ are increasing in $\alpha_F$. Thus, the seller chooses $\alpha_F = 1$ in both cases.

By inserting $\alpha_F = 1$, only type-H buyers will go shopping at the offline store if the price is $1 - t_F/\theta_L < p_F \leq 1 - t_F/\theta_H$, but both types of buyers will do so if $p_F \leq 1 - t_F/\theta_L$. ■

The proof of Proposition 1.

**Proof.** We consider the following four cases. (i) When $\theta_H > 2\theta_L$ and $\theta_H > \theta_L^2/t_F$, we immediately get $\theta_H = \pi^*_N > \pi^*_F = \theta_H - t_F$.

(ii) When $\theta_H > 2\theta_L$ and $\theta_H \leq \theta_L^2/t_F$, $\pi^*_N - \pi^*_F = (\theta_H/\theta_L + 1)t_F - \theta_L$. $\pi^*_N \geq \pi^*_F$ corresponds to $\theta_H \geq (\theta_L/t_F)(\theta_L - t_F)$. Given the range of parameter values, we must have $\theta_L < 3t_F$ for $\pi^*_N > \pi^*_F$. This can be shown by evaluating when $2\theta_L > (\theta_L/t_F)(\theta_L - t_F)$ holds.

(iii) When $\theta_L \leq 2\theta_L$ and $\theta_H \leq \theta_L^2/t_F$, $\pi^*_N \leq \pi^*_F$ corresponds to $\theta_L(\theta_L + t_F)/(\theta_L - t_F) \geq \theta_H$. Given the range of parameter values, we have to look at two different regions to determine which inequality holds; $\theta_H < \theta_L^2/t_F$ is relevant for $\theta_L < 2t_F$, whereas $\theta_H < 2\theta_L$ is relevant for $\theta_L \geq 2t_F$. First, when $\theta_L < 2t_F$, it can be shown that $\theta_L(\theta_L + t_F)/(\theta_L - t_F) > \theta_L^2/t_F$. This
can be rewritten as $\theta_L^2 - 2t_F\theta_L - t_F^2 < 0$. This is true for $\theta_L < 2t_F$. Next, when $\theta_L \geq 2t_F$, $\theta_L(\theta_L + t_F)/(\theta_L - t_F) > 2\theta_L$ holds only when $\theta_L < 3t_F$.

(iv) When $\theta_H \leq 2\theta_L$ and $\theta_H > \theta_L^2/t_F$, $\bar{\pi}_N - \bar{\pi}_F = 2\theta_L + t_F - \theta_H > 0$ because $2\theta_L > \theta_H > \theta_H - t_F$. ■

The proof of Proposition 2.

Proof. Let us assume that there exists a pair of prices, $(\hat{p}_N, \hat{p}_F)$, where type-L buyers shop at online stores and type-H buyers at offline stores. Then, it must be true for $\hat{p}_N$ and $\hat{p}_F$ that

$$\phi_L(\theta_L, \alpha_N) - \hat{p}_N \geq \Pr(s_H; \theta_L, \alpha_F)(\phi_H(\theta_L, \alpha_F) - \hat{p}_F) - t_F$$

and

$$\Pr(s_H; \theta_H, \alpha_F)(\phi_H(\theta_H, \alpha_F) - \hat{p}_F) - t_F \geq \phi_H(\theta_H, \alpha_N) - \hat{p}_N.$$

Note these conditions ensure that type-L prefers online to offline stores but type-H prefers offline to online stores. That is, these are incentive compatibility constraints for type-L and type-H buyers, respectively. When type-H buyers purchase online, information is readily available and they may receive a good signal or a bad signal. The second constraint implies that they prefer to shop offline despite drawing a good signal at the online store. Combining two conditions, we get

$$\phi_H(\theta_H, \alpha_N) - \phi_L(\theta_L, \alpha_N) + [\Pr(s_H; \theta_H, \alpha_F) - \Pr(s_H; \theta_L, \alpha_F)]\hat{p}_F \leq \alpha_F(\theta_H - \theta_L)$$

This is not feasible for $\theta_L < \theta_H$ because $Min_{\alpha_N} [\phi_H(\theta_H, \alpha_N) - \phi_L(\theta_L, \alpha_N)] \geq Max_{\alpha_F} \alpha_F(\theta_H - \theta_L)$. ■

The proof of Proposition 3.

Proof. First, $[IR_H]$ is always satisfied. $[IC_H]$ and $[IR_L]$ together imply

$$\phi_L(\theta_H, \alpha_N) - p_N \geq \Pr(s_H; \theta_H, \alpha_F)(\phi_H(\theta_H, \alpha_F) - p_F) - t_F$$

$$\geq \Pr(s_H; \theta_L, \alpha_F)(\phi_H(\theta_L, \alpha_F) - p_F) - t_F \geq 0.$$
Next, we solve the problem and later show that the solutions satisfy \([IC_L]\) constraint. The binding \([IR_L]\) provides \(p_F = \phi_H(\theta_L, \alpha_F) - \frac{t_F}{\phi_L(\theta_L, \alpha_F)}\). The binding \([IC_H]\) yields \(p_N = \phi_L(\theta_H, \alpha_N) - \alpha_F \theta_H + \Pr(s_H; \theta_H, \alpha_F)p_F + t_F\). Substituting these two into the seller’s profit function, we obtain the following relaxed problem,

\[
\max_{\alpha_F, \alpha_N} \pi_{NF} = \phi_L(\theta_H, \alpha_N) - \alpha_F(\theta_H - \theta_L) + \frac{(\alpha_F \theta_L - t_F)(\alpha_F \theta_H + (1 - \alpha_F)(1 - \theta_H))}{\alpha_F \theta_L + (1 - \alpha_F)(1 - \theta_L)} - (\theta_H - \theta_L). \tag{6}
\]

Only the first term depends on \(\alpha_N\). Since \(\phi_L(\theta_H, \alpha_N)\) is decreasing in \(\alpha_N\), the seller chooses \(\alpha_N = 1/2\). Next, by differentiating with respect to \(\alpha_F\), we obtain the first-order condition,

\[
\frac{(\alpha_F \theta_L - t_F)(\theta_H - \theta_L)}{(\alpha_F \theta_L + (1 - \alpha_F)(1 - \theta_L))^2} + \frac{(\alpha_F \theta_H + (1 - \alpha_F)(1 - \theta_H))}{\alpha_F \theta_L + (1 - \alpha_F)(1 - \theta_L)} \theta_L - (\theta_H - \theta_L).
\]

The second-order condition becomes

\[
2\frac{-\theta_L^2 + (1 + 2t_F)\theta_L - t_F}{(\alpha_F \theta_L + (1 - \alpha_F)(1 - \theta_L))^3} (\theta_H - \theta_L) \tag{7}
\]

The sign of this is solely determined by the term in the numerator. For \(2t_F \leq \theta_L < 1/2\), the numerator is always positive, i.e., \(-\theta_L^2 + (1 + 2t_F)\theta_L - t_F > 0\). Thus, the profit function is convex in \(\alpha_F\). But, note that its slope is positive when \(\alpha_F = 1\). To show that \(\alpha_F = 1\) is optimal, we now have to check if \(\pi_{NF}(\alpha_F = 1) > \pi_{NF}(\alpha_F = 1/2)\). We obtain \(\pi_{NF}(\alpha_F = 1) = \theta_H + \theta_L - t_F \frac{\theta_H}{\theta_L}\) and \(\pi_{NF}(\alpha_F = 1/2) = \theta_H/2 + \theta_L - t_F\), respectively. Thus, \(\theta_L \geq 2t_F, \pi_{NF}(\alpha_F = 1) - \pi_{NF}(\alpha_F = 1/2) = \frac{\theta_L - 2t_F}{2\theta_L} \theta_H + t_F > 0\), and the seller’s optimal choice is indeed \(\alpha_F = 1\).

Finally, after inserting \(\alpha_F = 1\) and \(\alpha_N = 1/2\) into \([IC_H]\), we get \(p_N = \theta_H(1 - t_F/\theta_L) + t_F\). It is immediate that these solutions satisfy \([IC_L]\). □

The proof of Proposition 4.

**Proof.** Below the comparison between \(\pi_{NF}^*, \pi_{N}^*, \pi_{F}^*,\) and \(\hat{\pi}_{F}^*\) follows from Proposition 1.

(i) If \(\theta_H \leq 2\theta_L - t_N\), then \(\max\{\pi_{F}^*, \pi_{N}^*\} = \pi_{N}^*\). Since \(\pi_{F}^* < \pi_{N}^*\) and \(\pi_{F}^* < \pi_{NF}\), now we should compare \(\pi_{N}^*\) and \(\pi_{NF}\). It can be easily shown that \(\pi_{N}^* \leq \pi_{NF}\) when \(\frac{\theta_L - t_N}{t_F/\theta_L} \leq \theta_H\).
If \( 2\theta_L < \theta_H \), then \( \max \{ \pi_N^*, \pi_N^* \} = \pi_N^* \). Similarly, if we compare \( \pi_N^* \) and \( \widehat{\pi}_{NF} \), then we get \( \pi_N^* \leq \widehat{\pi}_{NF} \) when \( \theta_H \leq \theta_L^2/t_F \).

(ii) If \( \theta_H > \max \{ 2\theta_L, \theta_L^2/t_F \} \), then \( \max \{ \pi_N^*, \pi_N^* \} = \pi_N^* \) and \( \max \{ \pi_F^*, \pi_F^* \} = \pi_F^* \). Since \( \pi_F^* < \pi_N^* \), we compare \( \pi_N^* \) and \( \widehat{\pi}_{NF} \). It is straightforward to see \( \widehat{\pi}_{NF} < \pi_N^* \) for \( \theta_H > \theta_L^2/t_F \).

(iii) If \( \theta_H \leq 2\theta_L \), then \( \max \{ \pi_N^*, \pi_N^* \} = \pi_N^* \). Since \( \pi_F^* < \pi_N^* \) and \( \pi_F^* < \widehat{\pi}_{NF} \), we compare \( \pi_N^* \) and \( \widehat{\pi}_{NF} \), then we get \( \widehat{\pi}_{NF} < \pi_N^* \) for \( \theta_H < \frac{\theta_L - 1}{1 - 1/F/\theta_L} \).

References


