Commodity Currencies and Monetary Policy

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August 2018

Abstract

Countries that specialize in commodity exports often exhibit a correlation between the relevant commodity price and the value of their currency. We explore a natural but little-studied explanation for this correlation. An increase in the commodity price leads to increases in the future values of the international differential in policy interest rates. The tightening of expected future monetary policy relative to the US then leads to an immediate appreciation. We show theoretically that this correlation depends on the stance of monetary policy. We then derive a statistical model that embodies this mechanism and test the over-identifying restrictions for Australia, Canada, and New Zealand. For all three countries, controlling for the effect of commodity prices in predicting current and future monetary policy leaves them no significant, remaining role in statistically explaining exchange rates.

JEL classification: F31, F41, E52.

Keywords: commodity currency, exchange rate, monetary policy

1. Introduction

Countries that specialize in commodity exports often exhibit a correlation between the relevant commodity price and the value of their currency. We explore a natural but little-studied explanation for this correlation. The mechanism is this: An increase in the export commodity price leads participants in the foreign exchange market to expect a tightening of domestic monetary policy relative to policy in the US. The exchange rate reacts immediately to the change in expected future policy, inducing a correlation with the commodity price.

Australia, Canada, and New Zealand often are cited as developed economies with commodity currencies. For these countries there is an obvious correlation between an export commodity price (or an index of them) and the value of the currency. Commentators typically remark on this correlation at high frequency but it also is evident in the monthly or quarterly data that macroeconomists usually study.

However, most models of the exchange rate treat it as determined in financial markets given the enormous daily volume of transactions. The BIS triennial survey (2016) reports daily average turnover for the currencies of these three countries (against the USD) for April 2016 in billions of US dollars (followed by the rank by these volumes among countries): AUD: 262 (4); CAD: 218 (5); NZD: 78 (11). These daily volumes of foreign exchange transactions dwarf daily trade flows, daily GDP, or daily transactions volumes in the corresponding stock exchanges (the ASX, TSX, and NSX respectively). For example, the average daily value of transactions on the TSX in 2016 was 4.7 billion USD. And quoting Canada's 2016 annual exports by dividing by the number of trading days on the TSX (258) gives 1.84 billion USD per day. These differences in scale suggest that exchange-rate movements cannot easily be explained by transactions in goods or equity markets.

We study commodity prices as predictors of future monetary policy. We measure relative monetary policy using the difference between the central bank's policy interest rate and the US federal funds rate. Forecasts of future monetary policy of course cannot be observed directly, so the paper derives ways to measure their projections on commodity prices. We then present a test of the hypothesis that those prices contribute to exchange-rate move-

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ments only for this reason. We measure commodity prices first by the export commodity price index and then separately by its largest component, which comprises prices of energy for Canada, base metals for Australia, and dairy products for New Zealand. For all three countries, controlling for the effect of commodity prices in predicting current and future monetary policy leaves them no significant, remaining role in explaining exchange rates.

Section 2 next outlines a model of a small, open economy, illustrating how the response of the nominal exchange rate to a commodity-price shock depends on the monetary-policy rule. Section 3 describes the countries and series we study. Section 4 then highlights some related empirical research. We explain the econometric restrictions and construct tests in sections 5 and 6.

2. A Model of Exchange Rates and Commodity Prices

The empirical hypothesis is that the exchange-rate response to commodity price shocks is linked primarily to the stance of monetary policy. In this section, we construct a model consistent with this hypothesis. Our objective is not to establish a definitive explanation of the characteristics of commodity currencies, but rather just to demonstrate that a currency appreciation following spikes in commodity prices may be a natural implication of inflation-targeting monetary policy, and would not occur under alternative policy rules.

The model is quite standard. Consider a small, open economy, with two goods, traded and non-traded. Uribe and Schmitt-Grohé (2017, chapter 8) introduce this TNT model. Traded goods come in two types, manufactures and commodities. In both cases, prices are determined exogenously on world markets. Households consume both traded and non-traded goods. For simplicity, we assume that the two traded goods are perfect substitutes in consumption. Household preferences are given by

$$\sum_{t=0}^{\infty} \beta^t [U(C_{T,t}) + U(C_{N,t})], \tag{1}$$

where $C_{T,t}$ ($C_{N,t}$) represents consumption of the traded (non-traded) good. The household's budget constraint is

$$P_{T,t}C_{T,t} + P_{N,t}C_{N,t} + \frac{S_t B_{t+1}^*}{(1+i_{t+1}^*)} + \frac{B_{t+1}}{1+i_{t+1}} = P_{N,t}Y_{N,t} + P_{T,t}Y_{T,t} + P_{R,t}Y_R + B_t^* S_t + B_t, \tag{2}$$

where $P_{T,t}$ is the price of the traded good. We assume that $P_{T,t} = S_t$ (where S_t is the nominal exchange rate, here defined as the price of foreign currency in term of domestic currency), so that the law of one price holds for traded goods, and we assume the foreign price of traded goods is normalized to unity. $P_{N,t}$ is the price of the non-traded good. B_t^* represents the foreign-currency-denominated, one-period, risk-free bond holdings while B_t is the holding of domestic currency bonds. i_t^* is the nominal interest rate on foreign currency bonds, set exogenously in the foreign financial market, while i_t is the home currency nominal interest rate on domestic currency bonds and is set by the domestic central bank.

$Y_{N,t}$ is the output of the non-traded good. $Y_{T,t}$ is the exogenous output of traded goods, while $P_{R,t} = X_{R,t}^* S_t$, where $X_{R,t}^*$ is the world-determined price of the resource good, and Y_R is the exogenous output of the (tradable) resource good.

The domestic household's optimal division of consumption between traded and non-traded goods satisfies the condition:

$$P_{T,t}U'(C_{N,t}) = P_{N,t}U'(C_{T,t}). \tag{3}$$

The optimal holding of foreign-currency bonds (using condition $P_{T,t} = S_t$) is represented by the Euler equation:

$$U'(C_{T,t}) = \beta E_t U'(C_{T,t+1})(1 + i_{t+1}^*), \tag{4}$$

while the optimal holding of home-currency bonds satisfies:

$$\frac{U'(C_{T,t})}{S_t} = \beta E_t U'(C_{T,t+1})\frac{(1 + i_{t+1})}{S_{t+1}}. \tag{5}$$

We make two assumptions about monetary policy. In each case, monetary policy follows an interest-rate rule. But in the first case, (Case A) the interest rate rule is geared towards exchange-rate stability. This is represented by the following exchange-rate targeting rule:

$$1 + i_{t+1} = \exp(\nu_t)\frac{1}{\beta}\left(\frac{S_{t-1}}{S_t}\right)^{-\sigma_s}, \tag{6}$$

where σ_s determines the size of the interest-rate response to exchange-rate changes, and ν_t is a mean-zero, white noise shock to the monetary rule. Note that this is not a fixed exchange rate rule. Shocks to the foreign interest rate i_t^*, or shocks ν_t will effect the exchange rate. But as we see below, this rule does have the implication that the exchange rate will not respond to news about future commodity prices.

The second monetary rule (Case B) is geared towards targeting inflation in domestic goods prices:

$$1 + i_{t+1} = \exp(\nu_t) \frac{1}{\beta} \left(\frac{P_{N,t}}{P_{N,t-1}} \right)^{\sigma_\pi}. \tag{7}$$

This is a natural monetary rule to follow in an environment where all price stickiness is in the non-traded goods sector.

We assume that the non-traded goods price is sticky. It is set one period in advance, and adjusts after a shock. Within a period, output of non-traded goods is determined by demand (*i.e.* perfectly elastic supply), given the fixed price. After adjustment, non-traded goods output is in perfectly inelastic supply, fixed at an exogenous natural rate Y_N. This assumption is not necessary. We could easily endogenize labour supply and have the flexible-price level of non-traded output determined by equilibrium in the labour market, without changing any of our results. As noted, the output of of both types of traded goods is exogenous.

We now illustrate our main result: The exchange-rate response to a commodity price shock is determined by the stance of monetary policy. We will show this in a particularly transparent way. Take the following event. Start in equilibrium where Y_N is at its natural rate. Then, assume that at time 0, there is an announced permanent increase in X_R^* for time 1. This will increase the present value of traded goods income. There is a fixed output of traded goods and a constant foreign-currency interest rate. In addition, let $\beta(1+i^*) = 1$, so that households wish to smooth their consumption of traded goods over time. We can work out the impact on current traded goods consumption from time 0 onwards using the Euler equation in traded-goods consumption:

$$U'(C_{T,t}) = \beta U'(C_{T,t+1})(1 + i^*), \tag{8}$$

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combined with the intertemporal budget constraint for traded goods:

$$\sum_{t=0}^{\infty}\Big(\frac{1}{1+i^*}\Big)^t C_{T,t} = \sum_{t=0}^{\infty}\Big(\frac{1}{1+i^*}\Big)^t (Y_{T,t} + X_{R,t}^* Y_R). \tag{9}$$

This implies that there is a permanent rise in C_T, starting at $t = 0$. Denote the new value of traded goods consumption as C_T'. Since the rise in $X_{R,1}^*$ increases the demand for traded goods from time 0 onward, this will raise the demand for non-traded goods, using (3). Given the unexpected rise in $X_{R,1}^*$, the price of the non-traded good in period 0 is fixed, and cannot respond until period 1. The response of output in the non-traded goods sector will depend on the monetary rule.

Note that, from period 1 onwards, non-traded output is fixed at Y_N due to the condition:

$$U'(Y_N) = \frac{P_{N,t+1}}{S_{t+1}} U'(C_T'). \tag{10}$$

The relative price of non-traded goods is uniquely determined by this condition, from time 1 onwards.

What happens to the output of non-traded goods in period 0, and to the exchange rate? First consider monetary policy A. Combining (4), (5), and (6), assuming $\nu_t = 0$ we have, from time $t = 0$ onwards:

$$\frac{1}{\beta}\Big(\frac{S_{t-1}}{S_t}\Big)^{-\sigma_s} = (1+i^*)\frac{S_{t+1}}{S_t}. \tag{11}$$

Given that $\beta(1 + i^*) = 1$, this gives a unique solution for the exchange rate $S_t = S_{t-1}$. Critically, the exchange rate does not respond to the announced commodity price increase, either in time 0 or in future periods. In time 1, the real appreciation is achieved by a rise in the non-traded goods price.

Figure 1 illustrates this result, for period 0. It shows that in period 0, the rise in C_T will shift out the demand for non-traded goods. Given the fixed price of non-traded goods, $Y_{N,0}$ rises. From period 1 onwards, output of non-traded goods goes back to its natural rate, and the price of the non-traded good rises to satisfy (10)).

Now focus on monetary policy B. Combining (4), (5), and (7), again assuming $\nu_t = 0$, we have, from time $t = 0$ onwards:

$$\frac{1}{\beta}\left(\frac{P_{N,t}}{P_{N,t-1}}\right)^{\sigma_s} = (1+i^*)\frac{S_{t+1}}{S_t}. \tag{12}$$

At time 0, the left-hand side of (12) is predetermined (and $P_{N1} = P_{N,-1}$ by our assumption of an initial steady state in the non-traded goods sector), so therefore it must be that $S_1/S_0 = 1$. From time 1 onwards, we have:

$$\frac{1}{\beta}\left(\frac{P_{N,1}}{P_{N,0}}\right)^{\sigma_s} = (1+i^*). \tag{13}$$

Equations (12) and (13) give us a unique value of $P_{N,t} = P_{N,-1}$ for all $t \geq 0$. The non-traded goods price does not move at all in response to the announced commodity price increase, either at $t = 0$ nor in any future period. This means that (10) must be satisfied by an exchange-rate appreciation. So S_t falls permanently from time $t = 1$ onwards. But we have established already that $S_1/S_0 = 1$. Thus, the domestic currency must immediately appreciate at time 0 after the announced commodity price increase. This in fact implies that the demand for non-traded goods will shift back to its original level, and output of non-traded goods in period 0 will not increase at all, unlike the response under monetary rule A. Figure 2 illustrates this response.

Hence, under monetary policy A, there is no immediate response of the exchange rate (real or nominal) to the commodity price shock. But under monetary policy B, which we can think of as a variant of an inflation targeting rule, there is an immediate (and permanent) nominal and real exchange-rate appreciation.

Policy B might be replaced with a more familiar CPI targeting regime. In this case, we would still see a time 0 appreciation, but it would be smaller than under rule B. An appreciation would reduce the time zero CPI, leading to a fall in the time 0 domestic interest rate. As a result, the UIRP condition would require an anticipated appreciation, so the time zero appreciation would be less than that described above. But qualitatively, we would still have the contrast between a CPI rule and rule A, where the exchange rate would not respond at all to commodity price shocks.

3. Statistical Background for Canada, Australia, and New Zealand

Our study focuses on Canada, Australia, and New Zealand. These are the same countries studied by Chen and Rogoff (2003). They have floating exchange rates, have targetted inflation for most of the period we study, and export commodities. An underlying idea is that the world commodity prices will be exogenous to monetary policy in these countries, a feature that should help identification. Monetary policy in these countries was not constrained by the zero lower bound, with the exception of Canada during 2009–2010, so their standard indicators of monetary policy are relevant.

To streamline notation we label the commodity price $X_{R,t}^*$ simply as X_t and label its logarithm x_t. To measure this we use the national, commodity price indexes of central banks, where possible. Central banks say that they track these series so it seems natural to assume that participants in foreign exchange markets do so too. We thus use the commodity price indexes of the Bank of Canada and the Reserve Bank of Australia. The Reserve Bank of New Zealand does not publish such an index, but a private bank (ANZ) has done so since 1986 and its index is widely tracked. Each series is at monthly frequency. We use the versions in USD then deflate by the US CPI.

The Bank of Canada's commodity price index (BCPI) is a chain Fisher index. As of 2018, the largest components were the prices of West Texas intermediate oil (20.7%) and metals and minerals (19.5%). The Reserve Bank of Australia's Index of Commodity Prices (ICP) is a Laspeyres index, with weights periodically updated. The largest components are the prices of iron ore (with a weight of 32.7%) and metallurgical coal (with a weight of 16.4%). Weights in the ANZ index also are based on shares of commodity exports. For 2018 the largest components are the prices of dairy products (38%), beef (10.9%), and lamb (10.3%).

Chen and Rogoff (2003, section 4.2) explain why the export commodity price index may be a better way to measure shocks to a country's terms of trade than the terms of trade itself. The latter is affected by price stickiness (and the nature of pass-through) which affects the correlation with the exchange rate and also limits the degree to which the terms of trade can respond to shocks within a month.

The exchange rate, S_t (with log s_t), is the value of the USD in local currency, also monthly. Thus a decrease is a domestic appreciation. We measure the stance of monetary policy relative to that in the US by the difference between the policy interest rate in the home country and the US interest rate: $d_t \equiv i_t - i_t^*$. During the zero lower bound period, we use shadow rates designed to reflect unconventional monetary policy. Wu and Xia (2016) provide US shadow rates while MacDonald and Popiel (2016) provide Canadian ones, in each case constructed from a term structure model. The estimation below instruments d_t and so defends against some forms of measurement error. The appendix collects the definitions of each series.

In figures 3–5 the upper panels show the monthly values of the commodity price indexes and values of the local currencies in USD ($1/S_t$) since 1986 for each country. The lower panels show the interest differentials. The commodity price indexes and currency values appear positively correlated. That correlation certainly features in commentary on the value of each currency.

Table 1 presents statistics on the properties of the three series. First, it reports the correlation coefficient between the growth rates in s_t and x_t (the change in the log). These are negative: -0.46 for Canada, -0.34 for Australia, and -0.18 for New Zealand. A commodity-price increase thus is associated with a nominal appreciation. We report the correlation for growth rates to allow for the possibility that the commodity prices are nonstationary. The next column presents augmented Dickey-Fuller tests that suggest they are. The null hypothesis of a unit root cannot be rejected for each series $\{x_t\}$. Floating exchange rates often are viewed as non-stationary (in fact as random walks, in numerous studies) and the next column confirms that for the log exchange rates $\{s_t\}$. Comparing the test statistics for the two series, and looking at figures 1–3, suggests they are similarly persistent. The fifth column of table 1 then presents ADF tests for unit roots in the interest differential. These tests too do not reject the null hypothesis of a unit root at the 5% level for each country.

The final columns of table 1 presents residual-based tests for cointegration between s_t and x_t, s_t and d_t, and all three series. Here the evidence is more mixed. The evidence of

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cointegration is strongest for Australia and weakest for Canada.

Table 2 presents trace tests for the cointegration rank in the vector $\{x_t, d_t, s_t\}$. The tests statistics are based on VARs with either 3 or 6 lags. We compare the test statistics to the 95% asymptotic quantile following the top-down recommendation of Juselius (2006, chapter 8). For Canada and Australia there is evidence for either two unit roots or one unit root depending on the lag length. For New Zealand the test suggests a single unit root when the lag length is 3 months.

Conflicts among tests for cointegration are not news. But overall the evidence suggests modelling the variables as non-stationary and cointegrated, with a single unit root. Section 5 outlines an econometric test consistent with this description. But first section 4 sets this research in context.

4. Related Empirical Research

4.1 Commodity prices and exchange rates

A range of studies have examined these series, with commodity prices measured with an index or else using the price of an individual export commodity such as oil. At high frequency Ferraro, Rogoff, and Rossi (2015) document a correlation between daily changes in the CAD-USD nominal exchange rate and daily oil price changes. Berg, Guérin, and Imura (2016) do the same, and find separate roles for an energy commodity price index and a non-energy commodity price index, especially at higher frequencies. Not all studies find a significant correlation though. For example, Akram (2004) shows that the Norwegian krone value against a European basket is not closely correlated with the oil price.

Other studies of this bivariate statistical relationship look at other energy exporters and also at emerging market economies. For example, Lizardo and Mollick (2010) find oil prices statistically explain movements in the value of the USD against major currencies from 1970 to 2008. In particular, increases in the real price of oil lead to a significant depreciation of the USD against net oil exporter currencies, while currencies of oil importers depreciate relative to the USD. Bodart, Candelon, and Carpantier (2012) find an overall

effect for a large group of emerging market economies using panel restrictions to enhance precision.

A number of studies also examine the statistical relationship between the real exchange rate and commodity prices. For example, the influential study by Chen and Rogoff (2003) studies Australia, New Zealand, and Canada. They find a strong correlation and also cointegration for Australia and New Zealand. Cashin, Céspedes, and Sahay (2004) examine the correlation and cointegration between the real exchange rate and x_t (measured with export prices) for 58 countries. They find evidence of a correlation for about a third of them.

4.2 Commodity price predictability

A wide range of models of the nominal exchange rate characterize it as the present discounted value of a stream of future, expected fundamentals. An implication of present-value models is that the exchange rate should Granger-cause those fundamentals. Engel and West (2005) and Engel, Mark and West (2007) find some evidence for this, for fundamentals from the monetary model of the exchange rate for example. But they note that the present-value relationship is not the only explanation for such Granger-causality. For example, the exchange rate may help predict the interest rate if monetary policymakers react to the exchange rate with a lag. Thus, it may make sense to focus on predicting the commodity price in a small, open economy, because that is more likely to be exogenous.

Chen, Rogoff, and Rossi (2010) report a striking finding: nominal exchange rates (for commodity currencies) help forecast commodity prices. They study the nominal exchange rates of Canada, Australia, New Zealand, South Africa, and Chile (each relative to the US dollar) along with export-earnings-weighted commodity prices for each country, at quarterly frequency. They find this effect using (a) in-sample Granger-causality tests that allow for time-varying parameters and (b) out-of-sample forecasting with rolling windows. The results hold one quarter ahead and at longer horizons up to two years. The exchange rate predicts commodity prices better than futures prices do. And the reverse effect is not present: the exchange-rate changes cannot be predicted.

They interpret this pattern using a present-value model. To see how this works,

suppose that the commodity price x evolves jointly with some other exogenous variable. Also suppose the exchange-rate is forward-looking and depends on forecasts of future values of x. Then s_t will Granger-cause x_t (because it contains information on the additional exogenous variable) as long as there is some persistent, unobserved component. They argue that this pattern supports the PV model because x is exogenous to these small, open economies. In contrast, s Granger-causing interest rates or inflation could just be measuring a slow reaction of policy, as Engel and West (2005) also noted.

4.3 Exchange-rate predictability

Rossi (2013) surveys the large research literature on nominal exchange-rate forecasting. She reports that linear models are the most successful and that results vary based on the set of predictors, the sample period, the forecast evaluation method, and the forecast horizon. The random walk model remains a resilient benchmark. Models based on Taylor rules are the only ones with consistently significant out-of-sample forecasting ability at short horizons. But there is little evidence that monetary fundamentals help forecast exchange rates. Commodity prices are included in the fundamentals that Rossi considers. She concludes there is little evidence that they help forecast exchange rates, a result documented at different frequencies by Chen, Rogoff, and Rossi (2010) and Ferraro, Rogoff, and Rossi (2015).

Cheung *et al* (2017) provided updated evidence on prediction with a range of exchange-rate models. These include models with central-bank reaction functions that make monetary policy endogenous, a key development in the 2000s. Their study includes one commodity currency: the Canadian dollar. Overall they find that it is difficult to improve on the random walk model (based on mean-squared error of forecasts) especially at short horizons, though the findings vary by time period and currency.

Several studies focus on daily data on exchange rates and commodity prices. Kohlscheen, Avalos, and Schrimpf (2017) find a strong correlation between changes in the nominal exchange rate and a daily index of export commodity prices for 11 countries. But they also confirm that there is little evidence of out-of-sample prediction using lagged commodity prices. However, Zhang, Galbraith, and Dufour (2015) find evidence of

Granger-causality running from commodity prices to exchange rates (and more so than in the opposite direction) over daily horizons in high-frequency data.

Engel and West (2004) show that the log exchange rate can follow a random walk in a present-value model with (a) observed fundamentals that follow a random walk, (b) unobserved fundamentals that follow a random walk, or (c) fundamentals that are $I(1)$ but not a pure random walk combined with a discount factor near 1. And a discount factor near 1 produces volatility in s greater than in fundamentals. Engel, Mark, and West (2007) provide an overview of models of the nominal exchange rate. They stress the importance of expected future monetary policy, which reacts endogenously to macroeconomic indicators. They reiterate the finding that a random walk in s does not refute present-value models.

5. Present-Value Restrictions

The goal of this section is to derive a test of the hypothesis that commodity prices may affect the nominal exchange rate through their effect on monetary policy. To do this, we adopt a present-value model in which the nominal exchange rate depends on current and expected future monetary policy, relative to policy in the US. This framework allows for a multi-period version of the mechanism we studied in section 2, essentially operating through the UIP condition. But it also allows consistency with the statistical properties of in section 3 and with the evidence surveyed in section 4. For example, it can be consistent with the exchange rate helping forecast commodity prices and with the exchange rate following a random walk.

Our hypothesis has two components. First, commodity price movements sometimes lead to a reaction from monetary policy. This is because of their effect on domestic inflation, as under monetary policy B in the theoretical example. Although there may be some persistence in the policy rate due to interest-rate smoothing, future monetary policy can be partly forecasted with commodity prices. Section 3 showed that those prices are highly persistent, which may enhance their role in forecasts. Second, the nominal exchange rate responds to both current and expected future policy interest rates, so it reacts immediately to the commodity price index.

Recall that s_t is the log exchange rate, x_t the log, real commodity price, and $d_t \equiv i_t - i_t^*$ the differential in policy interest rates relative to the US. The exchange rate is measured in local currency (so that a decrease is an appreciation). It is described by the traditional, present-value, monetary model of the exchange rate, with relative monetary policy measured by d_t:

$$s_t = \alpha(1-\beta)d_t + \beta \mathrm{E}_t s_{t+1} = \alpha(1-\beta)\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j d_{t+j}, \tag{14}$$

(with the transversality condition implicit). Engel and West (2005) and Engel, Mark, and West (2007) outline a variety of models that yield this equation. They also provide a range of empirical evidence on this approach. They stress the need to allow for the endogeneity of monetary policy. In our example, monetary policy is expected to respond to the commodity price and so the current exchange rate does so too. We expect α to be negative and below find $\hat{\alpha}$ to be negative: An increase in expected future i, relative to US i^*, leads to a fall in s, an appreciation of the domestic currency.

Next, suppose that the international interest differential reacts to the commodity price and to the current value of the exchange rate:

$$d_t = \gamma_s s_t + \gamma_x x_t + \epsilon_{dt}, \tag{15}$$

where ϵ_{dt} is a martingale difference series. This is not intended as a complete description of the relative policy rule, for it excludes other variables such as inflation and also excludes dynamics from interest-rate smoothing. Rather, we use it to show that one can estimate and test the present-value model even though d_t reacts to s_t and x_t. At monthly frequency it makes sense to allow for monetary policy to react to both commodity prices and the exchange rate (among other variables). Thus s_t and d_t are determined simultaneously.

Our focus will be on estimating the parameters of the present-value model (14) (α and β) and on testing for the role of x_t in the forecasts in it. We do not try to identify the parameters of the interest-differential reaction function (15). But we use a fully solved statistical example to illustrate why our methods make sense. The example begins with the commodity price index following an autonomous random walk:

$$x_t = x_{t-1} + \epsilon_{xt}. \tag{16}$$

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Using this law of motion and equations (14) and (15), the guess-and-verify method shows that the projections of the two endogenous variables on x_t are:

$$P(s_t|x_t) = \frac{\alpha\gamma_x}{1 - \alpha\gamma_s}x_t$$
$$P(d_t|x_t) = \frac{\gamma_x}{1 - \alpha\gamma_s}x_t. \tag{17}$$

This simplest example illustrates two points. First, d_t and s_t are each cointegrated with x_t. The cointegrating vectors also are given by equations (17) regardless of higher-order dynamics. For example, interest-rate smoothing might add a lagged, value d_{t-1} to equation (15) but that would not affect the long-run relationships (17).

Second, this long-run information can be used to identify α. The coefficients (17) show that α is given by the ratio of the two cointegrating vectors. This is a classic source of identification via an exclusion restriction: d_t reacts to x_t directly but s_t does not and that distinction identifies α, the effect of d_t on s_t. That exclusion restriction is simply a restatement of equation (14).

We next extend this example by allowing for higher-order dynamics in $\{x_t\}$. Section 3 reported that each commodity price series appears to contain a unit root. Unlike nominal exchange rates, though, these series are *not* well described as random walks. We find that the univariate dynamics of each series are well described by a first-order autoregression in growth rates:

$$\Delta x_t = \omega_0 + \omega_x \Delta x_{t-1} + \epsilon_{xt}. \tag{18}$$

The estimates $\hat{\omega}_x$ (and HAC standard errors) are for Canada 0.26 (0.09), for Australia 0.49 (0.05), and for New Zealand 0.32 (0.07). Thus each country's coefficient is positive and statistically greater than zero. Suppose then that the commodity price index evolves autonomously following the AR(2) model (AR(1) in growth rates), omitting the constant term:

$$x_t = (1 + \omega_x)x_{t-1} - \omega_x x_{t-2} + \epsilon_{xt}, \tag{19}$$

with the parameter constraint so that x_t contains a unit root.

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Write the projections of the endogenous variables on $\{x_t, x_{t-1}\}$ with to-be-determined coefficients as follows:

$$P(s_t|x_t, x_{t-1}) = a_0 x_t + a_1 x_{t-1}$$
$$P(d_t|x_t, x_{t-1}) = b_0 x_t + b_1 x_{t-1}. \tag{20}$$

The coefficients are given recursively by:

$$a_0 = \alpha(1-\beta)\gamma_x \left[1 - \alpha(1-\beta)\gamma_s - \beta(1+\omega_x) + \frac{\beta^2 \omega_x}{1 - \alpha(1-\beta)\gamma_s}\right]^{-1}$$

$$a_1 = \frac{-\beta\omega_x}{1 - \alpha(1-\beta)\gamma_s} a_0 \tag{21}$$

$$b_0 = \gamma_s a_0 + \gamma_x$$

$$b_1 = \gamma_s a_1.$$

We use this second solved example to make three further observations. First, even in this relatively simple example, where x_t evolves autonomously, the cross-equation restrictions are quite complicated. They will be more complicated if one adds other sources of dynamics, such as interest-rate smoothing and forecasting information from other variables including the exchange rate itself. For this reason we do not estimate the solved model but instead use a limited-information method to test the hypothesis that x_t is correlated with s_t because it helps forecast the present-value of d_t.

Second, the Engel-West (2005) theorem applies. As $\beta \to 1$ the exchange rate follows a random walk. In the system of projection coefficients (21), as $\beta \to 1$ the coefficient $a_1 \to -\omega_x a_0$ and the application of l'Hôpital's rule shows that $a_0 \not\to 0$, so that

$$s_t = a_0 x_t - \omega_x a_0 x_{t-1} + \epsilon_{st}, \tag{22}$$

where the error term arises because of additional information used by forecasters. Thus s_t depends on the quasi-difference $(x_t - \omega_x x_{t-1})$ which is a random walk given the law of motion for x_t (19). Thus the solution is consistent with the evidence that $\{s_t\}$ follows a random walk while $\{x_t\}$ does not.

Third, the discount factor β now appears in the system of coefficients (21), which shows that higher-order dynamics in x_t can be used to identify β. Equivalently, there

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are two intruments and so one can identify the two parameters, α and β. Higher-order dynamics in x_t, while sufficient, are not necessary for identification though. If the interest-rate reaction function (15) is extended to include lagged commodity prices (with lag length greater than 1) then those will be valid instruments even if the commodity price follows a random walk. Whether the source of dynamics is autonomous (in x_t) or in policy (in the reaction of d_t to x_t), both current and lagged commodity prices become valid instruments and so allow tests based on over-identifying restrictions. The next section applies this standard idea from linear, rational-expectations models in a stationary transformation of the present-value model (14). We are not seeking a complete test of the PV model and to find fitted values implied by it but rather to test whether the correlation between the exchange rate and the commodity price is given by the mechanism here. For that reason we focus on instrumental variables methods that directly test that prediction.

The theoretical example in section 2 suggested that the response of the exchange rate to the commodity price shock depends on features of the economy including the monetary policy rule and the nature of price-stickiness. Identifying the components of a fully-solved, New Keynesian model may be difficult, as shown by Cochrane (2011) for the Taylor rule, Nason and Smith (2008) for the New Keynesian Phillips curve, and Uribe and Schmitt-Grohé (2017, chapter 8) for the impact of a terms-of-trade shock on domestic inflation. Fortunately, understanding these components is not necessary for our test. The test applies whatever the mechanism by which x_t affects d_t.

6. Estimates and Tests

A survey of sources on present-value methods with nonstationary variables suggests that there is no consensus on what method to use for estimation. For example, Kilian and Lütkepohl (2016) outline a range of methods. We adopt a two-step procedure that is consistent and that uses tools that have been extensively studied by Monte Carlo methods and yet are very simple to apply. This choice should aid replication and extension by other researchers.

The first step follows Campbell and Shiller's (1987) application of the Granger-Engle two-step method. Define a new variable $y_t \equiv s_t - \alpha d_t$ and then rewrite the present value

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(14) as:

$$y_t = \alpha \beta E_t \Delta d_{t+1} + \beta E_t y_{t+1} = \alpha E_t \sum_{j=1}^{\infty} \beta^j \Delta d_{t+j}. \tag{23}$$

It is easy to see that if d_t is $I(1)$ then d_t and s_t are cointegrated and $s_t - \alpha d_t$ is $I(0)$ so the variables in this second present value are stationary, which facilitates inference.

We estimate the cointegrating relationship between s_t and d_t (with coefficient $\tilde{\alpha}$) and generate $\tilde{y}_t = s_t - \tilde{\alpha} d_t$. (A constant term is included but not reported.) In this environment it is well known that coefficients in the levels regressions will be estimated super-consistently. It also is well known that there may be bias in such estimates if dynamics are omitted. We estimate $\tilde{\alpha}$ first by fully modified ordinary least squares as developed by Phillips and Hansen (1990). This involves corrections for endogeneity and serial correlation that reduce the bias in OLS estimation of the static regression. Estimation uses the two cointegrating relationships (17) by FM-OLS with 3 lags. The estimate $\tilde{\alpha}$ is the ratio of the two cointegrating vectors.

In table 3, the first column lists the countries and the second column gives the estimates $\tilde{\alpha}$ with their standard errors. The estimates are negative and statistically significant for each country, with t-statistics of 2.3 for Canada, 4.7 for Australia, and 4.4 for New Zealand. Finding a significant value in this first step is a *sine qua non* for proceeding to the next step.

The second step then involves estimation with stationary variables, using standard IV tools. The present-value model implies that:

$$\tilde{y}_t = \tilde{\alpha} \beta E_t \Delta d_{t+1} + \beta E_t \tilde{y}_{t+1} + \epsilon_{yt}. \tag{24}$$

Here the error term ϵ_{yt} allows for the possibility of unobserved fundamentals or measurement error. We estimate the discount factor β by continuously updated GMM using (stationary) instruments z_t. Estimation uses the constructed measure $\tilde{y}_t$ and is consistent given the super-consistency of the first step. With one parameter to estimate, adopting more than one instrument provides a test based on over-identifying restrictions (and adds precision to the estimate $\hat{\beta}$).

This method differs from those of Campbell and Shiller (1987) and Campbell (1987) in two respects. First, we estimate the difference equation because we wish to estimate and test the value of β, while given their applications they did not and instead focused on fully-solved examples while forecasting with a VAR. Second, we focus on the question of whether commodity prices are correlated with the exchange rate because those prices forecast differential monetary policy, and so assess that directly, rather than using a fully solved forecasting model and restricted VAR. Thus we include variables such as Δx_t in the instrument set, report if they are relevant, and then test the overidentifying restrictions.

The third column of table 3 gives the instruments, z_t, which consist of current and lagged values of Δx_t. The fourth column then reports the p-value from an F-test of the relevance of the instruments in forecasting the endogenous regressors in equation (24). Those statistics show that the instruments are highly relevant for Canada and New Zealand (where p-values are below 0.05) but *not* for Australia, where we therefore have weak instruments (discussed further below).

The fifth column gives the estimated discount factor $\hat{\beta}$ along with its standard error. The sixth column then reports the p-value from a one-tailed t-test of $H_0 : \beta = 1$ against $H_A : \beta < 1$. There is stronger evidence against this null hypothesis (in favour of a discount factor below one) for Canada than for Australia or New Zealand. The final column in table 3 reports the p-value from the J-test of the over-identifying restrictions given each instrument set. None of the sets of restrictions is rejected at the 5% level of significance.

To summarize, we find a significant, long-run relationship between s_t and d_t for each country. We then control for the current policy differential and its discounted, expected future values as forecasted with commodity price changes. We find that the commodity price terms do forecast these present-value terms for Canada and New Zealand, but only weakly do so for Australia. And there is some evidence that β is less than one especially for Canada. Most importantly, once we control for these present value terms in the exchange-rate equation there is no evidence of a significant correlation between the residuals and the commodity price terms.

For Australia, where the instruments are weak, we conduct an Anderson-Rubin (1949)

test that is valid in that case. Rewrite equation (24) by taking the future values Δd_{t+1} and $\tilde{y}_{t+1}$ (or equivalently s_{t+1}) to the left-hand side (without forecasting them) and by adding some list of other variables u_t on the right-hand side:

$$\tilde{y}_t - \tilde{\alpha}\beta_0\Delta d_{t+1} - \beta_0\tilde{y}_{t+1} \equiv s_t - \tilde{\alpha}(1-\beta_0)d_t - \beta_0 s_{t+1} = \delta u_t + \epsilon_{yt}. \qquad (25)$$

To create this composite variable on the left-hand side of the equation, we need to choose a value for β, labelled β_0. We cannot use this regression to estimate that value. But it can be used to test any value for this discount factor. To test the hypothesis that $\beta = \beta_0$ we simply perform a traditional F-test of the hypothesis that $\delta = 0$, so that the auxiliary variables u_t are insignificant. The logic is that if we happen to select the correct value for β, then the two explanatory variables in the present-value model will reproduce the time-series pattern in the exchange rate s_t, and there will be no systematic pattern in the residuals that will be detected by including other macroeconomic variables, u_t.

We use $u_t = \{\Delta x_t, \Delta x_{t-1}\}$. We also use $u_t = \{x_t, ..., x_{t-4}\}$ to allow the regression to select a stationary combination of those levels, with inference still valid as shown by Sims, Stock, and Watson (1990). We then run the regression (25) on a grid of values of β_0 between 0 and 1. For each such value we record the F-statistic associated with the restriction that none of the variables in u_t enters the equation and calculate the corresponding p-value.

The results (not shown) are very simple to report: The Anderson-Rubin (AR) test does not reject the restrictions for any value of β. The range of values for which the F-statistic falls below the 5% critical value of the F distribution (equivalently the p-values lie above 0.05) constitutes the 95% percent confidence interval for β. In this case, all the values we considered qualify so the confidence interval is very wide, in contrast with those implied by the rows for Australia in table 3 which are not robust to weak identification. This means we cannot identify the discount factor even with this robust method. Equivalently, though, we cannot reject the null hypothesis that the x-variables are irrelevant. In fact, the AR test does not reject $H_0 : \beta = 0$: Once we control for current, relative monetary policy, there is no correlation between the exchange rate and commodity prices.

It is possible that the central bank in each country reacts not to the overall index of export commodity prices but instead to its largest component *and* that the foreign

exchange market expects it to do so. In that case, our test power in table 3 may be low because the overall index is a noisy measure of this component. And this feature might explain why the overall commodity price index x_t is a weak instrument in the Australian data. To examine that possibility, we repeat the calculations in table 3 but now with x_t measured by the energy component for Canada, the base metals component for Australia, and the dairy products component for New Zealand.

The results are in table 4. Again $\tilde{\alpha}$ is negative for each country, though it is estimated less precisely than in table 3. The t-statistics are 1.2 for Canada, 2.3 for Australia, and 2.8 for New Zealand. All point estimates $\hat{\beta}$ are less than 1, and overall there is more evidence against $H_0 : \beta = 1$. The test for instrument relevance shows that the instruments now are stronger for Australia than they were in table 3. The base metals component is a better predictor of Australian relative monetary policy than the overall commodity price index is. Finally, the J-test p-values are above 0.05 (though for Australia they are below 0.10). At the 5% significance level one cannot reject the null hypothesis that the commodity price index components are correlated with the nominal exchange rate only because they forecast indicators of relative monetary policy.

The results in table 4 are of course not independent of those in table 3, for the components in table 4 are central to the indexes in table 3. But even performing our test across separate commodity price indicators may make it difficult to control test size (equivalently raise the risk of false rejections), given the relatively small number (389–401) of monthly observations possible in this study. We hope that the simplicity of the method will allow researchers to apply it for other countries or time periods.

7. Conclusion

We examined a natural but under-studied explanation for a correlation between a country's commodity price index x_t and its nominal exchange rate s_t: s_t is determined by current and expected future values of an indicator of relative monetary policy and that indicator reacts to the commodity price index. A theoretical model of a small, open economy showed that this correlation depends on the stance of monetary policy and is induced, for example, by inflation targetting.

We studied three countries selected according to the criteria that (a) they have long histories of floating exchange rates, mainly under inflation targetting and (b) they have widely-followed export commodity price indexes. Our hypothesis is that the central bank follows this index in setting monetary policy and so practitioners in the foreign exchange market react to it too.

For Canada, Australia, and New Zealand we can identify α, the long-run effect of the policy interest-rate differential on the exchange rate, and estimate it with precision even though policy also reacts to the exchange rate. For Canada and New Zealand we then can also identify the discount factor β using commodity prices as instruments while for Australia these instruments are weak. The J-test for Canada and New Zealand and the Anderson-Rubin test for Australia show that there is no significant correlation between the exchange rate and the commodity price once we control for measures of current and expected future monetary policy, as predicted from commodity prices. When we instead measure x_t with the largest component of each country's index, we find p-values above 0.05 for the J-test. Thus, these narrower commodity prices again have no significant correlation with residuals from the present-value model.

Data Appendix

1. Canada

x: The commodity price x is the log of the Bank of Canada's monthly commodity price index, BCPI (label M.BCPI) expressed in real terms by division by the US CPI, series CPIAUCSL from FRED. Table 4 then measures x using the energy component of this index (label M.ENER).

d: The main monetary policy indicator is the difference between the overnight interest rate in Canada and the effective federal funds rate in the US. At the US ZLB from December 2008 to November 2015 we instead use the shadow rate constructed by Wu and Xia (2016). At the Canadian ZLB from April 2009 to June 2010 we use the shadow rate constructed by MacDonald and Popiel (2016).

s: The exchange rate is the log of the monthly average price of the USD in CAD, series EXCAUS from FRED.

2. Australia

x: The commodity price x is the log of the Reserve Bank of Australia's monthly commodity price index, in USD, (series GRCPAIUSD), expressed in real terms by division by the US CPI. Table 4 then measures x using the base metals component of this index (series GRCPBMUSD).

d: The policy indicator for Australia is the monthly average of the cash rate target after August 1990 when it was introduced. Before that we use the interbank overnight cash rate. The source is `f01hist.xls` from the RBA. Then d subtracts the effective federal funds rate described above.

s: The exchange rate is the log of the monthly average price of the USD in AUD, series EXUSAL from FRED

3. New Zealand

x: The commodity price x is the log of the ANZ commodity price index, in USD, expressed in real terms by division by the US CPI. Table 4 then measures x using the dairy products component of this index. The source is `www.anz.co.nz/about-us/economic-markets-research/commodity-price-index/`

d: The policy rate is the official cash rate (OCR) from March 1999 when it was introduced. Before that we use the overnight interbank cash rate. The source is `hb2-monthly.xls` from the RBNZ. Then d subtracts the effective federal funds rate described above.

s: The exchange rate is the log of the monthly average price of the USD in NZD, series EXUSNZ from FRED.

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Figure 1. Period 0, Before IT *

Demand for non-traded goods

$$U'(C_T) \times P_N = U'(C_N)$$

P_N

Y_N

* Demand for non-traded goods shifts out, P_N is predetermined

Figure 2. Period 0, With IT *

Demand for non-traded goods

$U'(C_T)xP_N=U'(C_N)$

P_N

Y_N

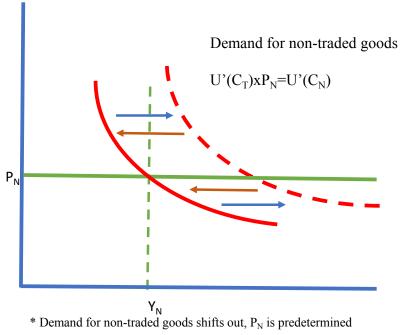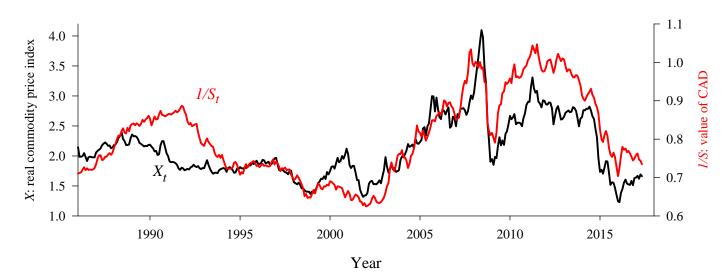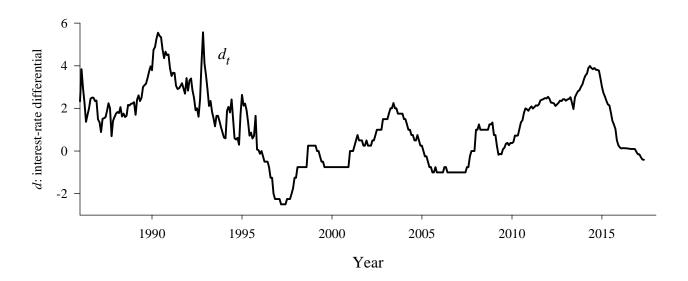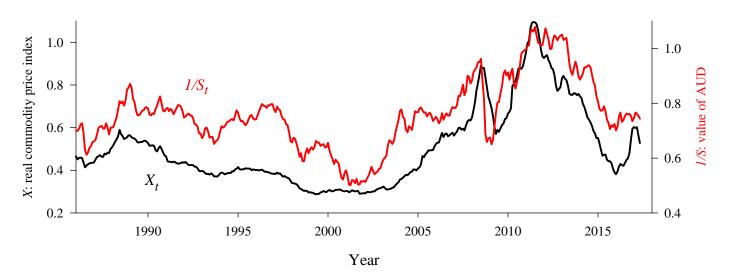* Demand for non-traded goods shifts out, P_N is predetermined

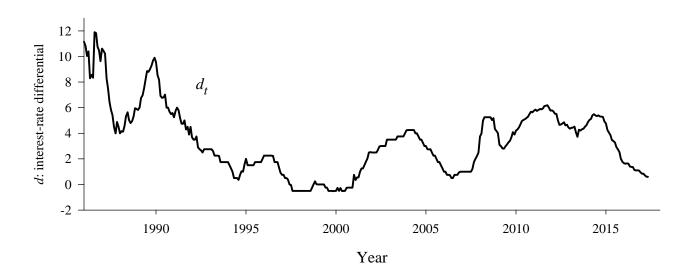# Figure 3: Commodity Prices, Exchange Rates, and Interest Rates
Canada

Notes: In the upper panel the black line (left axis) shows the Bank of Canada's commodity price index (in USD), divided by the US CPI, monthly. The red line (right axis) shows the monthly average value of the currency measured in US dollars. The lower panel shows the policy interest-rate differential $d = i-i*$.

Figure 4: Commodity Prices, Exchange Rates, and Interest Rates
Australia

Notes: In the upper panel the black line (left axis) shows the Reserve Bank of Australia's commodity price index (in USD), divided by the US CPI, monthly. The red line (right axis) shows the monthly average value of the currency measured in US dollars. The lower panel shows the policy interest-rate differential $d = i$-$i*$.

Figure 5: Commodity Prices, Exchange Rates, and Interest Rates
New Zealand

Notes: In the upper panel the black line (left axis) shows the ANZ commodity price index (in USD), divided by the US CPI, monthly. The red line (right axis) shows the monthly average value of the currency measured in US dollars. The lower panel shows the policy interest-rate differential $d = i$-i^*.
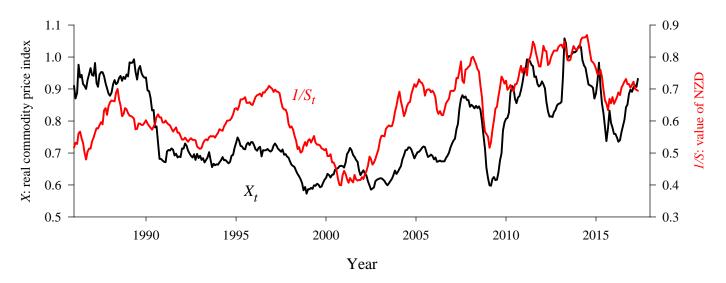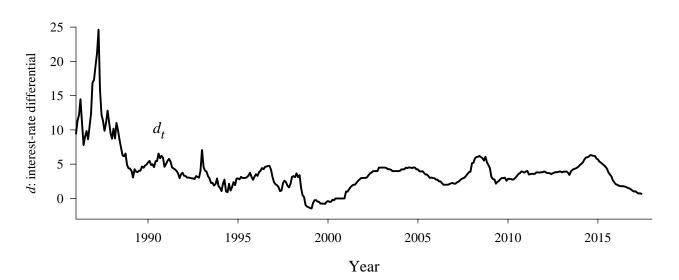
Table 1: Statistics for Commodity Prices, Exchange Rates, and Interest Differentials 1986–2018

Country	$r_{\Delta x_t, \Delta s_t}$	ADF_x	ADF_s	ADF_d	$\text{CADF}_{s,x}$	$\text{CADF}_{s,d}$	$\text{CADF}_{s,d,x}$
Canada	-0.46	-2.37	-1.71	-1.88	-2.77	-1.86	-3.14
Australia	-0.34	-1.62	-2.36	-2.61*	-4.60***	-2.83	-4.57***
New Zealand	-0.18	-2.33	-2.32	-2.70*	-2.94*	-2.35	-2.98

Notes: x is the log, real commodity price index, s the log nominal exchange rate in USD, and d the interest-rate differential. There are 389 observations from 1986:1 to 2018:5. r is the correlation coefficient. ADF is the augmented Dickey-Fuller t-statistic with an an intercept and 6 lags. CADF is the residual-based test statistic for cointegration, also with 6 lags. The symbols *, **, and *** denote significance at the 10%, 5% and 1% levels. Overall, a unit root cannot be rejected at the 5% level for any series. But there is clear evidence of cointegration only for Australia. Statistics are similar if the sample instead begins in 2000:1.

Table 2: Trace Tests of Cointegration Rank

$$\{x_t, d_t, s_t\}$$

Country	r	$3-r$	$\tau(3-r)$ 3 lags	$\tau(3-r)$ 6 lags	$C_{0.95}$
Canada	1	2	32.2	27.2†	29.8
	2	1	12.7†	8.9	15.4
	3	0	2.9	3.4	3.8
Australia	1	2	25.2†	33.0	29.8
	2	1	6.9	11.8†	15.4
	3	0	1.8	2.9	3.8
New Zealand	1	2	36.1	32.0	29.8
	2	1	14.2†	16.4	15.4
	3	0	4.0	4.1	3.8

Notes: r is the cointegrating rank and $3-r$ is the number of unit roots. τ is the trace test statistic and $C_{0.95}$ is the 95% asymptotic quantile. Following the top-down rule, a † shows the row at which $\tau < C_{0.95}$ for the first time.

Table 3: Estimates and Tests
(Export Commodity Price Indexes)

$$\tilde{y}_t \equiv s_t - \tilde{\alpha}d_t$$

$$\tilde{y}_t = \tilde{\alpha}\beta\mathrm{E}_t\Delta d_{t+1} + \beta\mathrm{E}_t\tilde{y}_{t+1} + \epsilon_{yt}$$

Country	$\tilde{\alpha}$ (se)	z_t	Relevance p	$\hat{\beta}$ (se)	$\beta = 1$ p	J-test p
Canada	-0.302 (0.131)	$\Delta x_t, \Delta x_{t-1}$	0.02	0.949 (0.027)	0.03	0.20
		$\Delta x_t, ..., \Delta x_{t-3}$	0.02	0.962 (0.017)	0.01	0.45
Australia	-0.121 (0.026)	$\Delta x_t, \Delta x_{t-1}$	0.25	1.017 (0.065)	0.39	0.30
		$\Delta x_t, ..., \Delta x_{t-3}$	0.51	1.022 (0.064)	0.36	0.67
New Zealand	-0.071 (0.016)	$\Delta x_t, \Delta x_{t-1}$	0.02	0.927 (0.056)	0.10	0.41
		$\Delta x_t, ..., \Delta x_{t-3}$	0.01	0.973 (0.031)	0.20	0.28

Notes: x is the log, real commodity price index, s the log nominal exchange rate in local currency, and d the policy interest-rate differential. $T = 389$ from 1986:1 to 2018:5 for New Zealand and $T = 401$ from 1985:1 to 2018:5 for Canada and Australia. The cointegrating coefficient $\tilde{\alpha}$ is estimated from the system (17) by FM-OLS with 3 lags. The discount factor $\hat{\beta}$ is estimated by continuously updated GMM with instruments z_t. The p-values apply to the first-stage F-test of instrument relevance, the t-test of $H_0 : \beta = 1$ vs $H_A : \beta < 1$, and the J-test of the over-identifying restrictions. Constants are included in each equation but not shown.

Table 4: Estimates and Tests
(Price Index Components)

$$\tilde{y}_t \equiv s_t - \tilde{\alpha}d_t$$

$$\tilde{y}_t = \tilde{\alpha}\beta\mathrm{E}_t\Delta d_{t+1} + \beta\mathrm{E}_t\tilde{y}_{t+1} + \epsilon_{yt}$$

Country	$\tilde{\alpha}$ (se)	z_t	Relevance p	$\hat{\beta}$ (se)	$\beta = 1$ p	J-test p
Canada (energy)	-0.533 (0.457)	$\Delta x_t, \Delta x_{t-1}$	0.02	0.979 (0.015)	0.07	0.31
		$\Delta x_t, ..., \Delta x_{t-3}$	0.03	0.984 (0.009)	0.05	0.66
Australia (base metals)	-0.197 (0.087)	$\Delta x_t, \Delta x_{t-1}$	0.11	0.922 (0.053)	0.07	0.09
		$\Delta x_t, ..., \Delta x_{t-3}$	0.08	0.961 (0.026)	0.07	0.07
New Zealand (dairy)	-0.144 (0.051)	$\Delta x_t, \Delta x_{t-1}$	0.11	0.917 (0.057)	0.07	0.11
		$\Delta x_t, ..., \Delta x_{t-3}$	0.09	0.961 (0.031)	0.11	0.10

Notes: x is a log, real commodity price index component: energy for Canada, base metals for Australia, and dairy products for New Zealand. s is the log nominal exchange rate in local currency, and d the policy interest-rate differential. $T = 389$ from 1986:1 to 2018:5 for New Zealand and $T = 401$ from 1985:1 to 2018:5 for Canada and Australia. The cointegrating coefficient $\tilde{\alpha}$ is estimated from the system (17) by FM-OLS with 3 lags. The discount factor $\hat{\beta}$ is estimated by continuously updated GMM with instruments z_t. The p-values apply to the first-stage F-test of instrument relevance, the t-test of $H_0 : \beta = 1$ vs $H_A : \beta < 1$, and the J-test of the over-identifying restrictions. Constants are included in each equation but not shown.