Managing Capital Flows in the Presence of External Risks*

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Abstract

We introduce external risks, in the form of shocks to the level and volatility of world interest rates, into a small open economy model subject to the risk of sudden stops—large recessions together with abrupt reversals in capital inflows—and characterize optimal macroprudential policy in response to these shocks. In the model, collateral constraints create a pecuniary externality that leads to "overborrowing" and sudden stops that arise when the constraints bind. The typical sudden stop generated by the model replicates existing empirical evidence for emerging market economies: Low and stable external interest rates reinforce "overborrowing" and lead to greater exposure to crises typically accompanied by abrupt increases in interest rates and a persistent rise in their volatility. We solve for the optimal policy and argue that the size of a tax on international borrowing that implements the policy depends on two factors, the incidence and the severity of potential future crises. We show quantitatively that these taxes respond to both the level and volatility of interest rates even though optimal decisions in the competitive equilibrium do not respond substantially to changes in volatility, and that the size of the optimal tax is non-monotonic with respect to external shocks.

JEL classification: E3, E6, F3, F4, F6, G1, G2

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1 Introduction

Large and volatile capital flows across countries carry risks. The 2008 Global Financial Crisis provided clear evidence of how global factors can shape these flows and their inherent short-run risks and highlighted the possibility of using macroprudential policies to reduce the size and frequency of crises associated with these flows. The potential need for this type of policies has motivated the recent development of frameworks to analyze their benefits and has established grounds for the optimal use of these instruments.[1] However, despite the evident risks associated with more volatile capital flows—in large part due to increased uncertainty in the global economy—the implications of uncertainty in external shocks for optimal macroprudential policy and its implementation have not been studied.[2]

In this paper, we introduce external risks in the form of shocks to the level and volatility of world interest rates into a model of financial crises suitable for the analysis of optimal macroprudential policy, and characterize how optimal policy should respond to external shocks.[3] The quantitative framework consists of a small open economy facing an external borrowing limit that depends on the value of a domestic non-tradable asset. External risks arise from two sources: shocks to the level of interest rates and the existence of multiple stochastic regimes in the variance of interest rates at which the economy borrows. We show that, in the model, low and stable world interest rates reinforce "overborrowing" and lead to greater exposure to sudden stops typically accompanied by abrupt increases in interest rates and a persistent rise in external interest rate volatility. These predictions of the model are in line with existing empirical evidence for emerging market economies (EMEs). We solve for the optimal policy and show that the size of a tax on international borrowing that implements the policy depends on two factors, which we define as the *incidence*—reflecting the likelihood and harm—and the *severity*—reflecting the magnitude of the pecuniary externality caused by the collateral constraint—of potential future sudden stops.[4] Quantitatively, we show

[1] See, for example Bianchi (2011) and Bianchi and Mendoza (2013) or Korinek and Mendoza (2013) for a survey of recent contributions.

[2] For example, Mackowiak (2007), Chang and Fernández (2013), and Ahmed and Zlate (2013) assess and highlight the relevance of global risks, external to countries' fundamentals. Johri et al. (2015) highlight the role of global uncertainty in shaping default decisions. Bianchi et al. (2016) do consider one particular type of external shock, to liquidity, and study macroprudential policy when there are multiple stochastic regimes in the level of world interest rates.

[3] We follow previous literature by Uribe and Yue (2006), Neumeyer and Perri (2005), and Fernández-Villaverde et al. (2011) and refer to shocks to world interest rates as external because we consider them to be driven by factors other than countries' fundamentals.

[4] We will refer to these terms throughout the paper and will define each precisely after we describe

that capital controls are contingent on both the level and volatility of interest rates even though optimal decisions in the competitive equilibrium do not respond significantly to changes in volatility regimes. More strikingly, we also show that the size of the optimal capital control is non-monotonic with respect to the volatility of external interest rates. For instance, contrary to conventional wisdom, for certain borrowing levels it is optimal to reduce taxes on international borrowing when interest rate volatility rises.[5] Our results shed light on the optimal use and, most importantly, on the implementation of macroprudential policy in a particularly relevant moment when many EMEs have shown concerns about volatility in global markets, partly due to the uncertainty in the decisions of advanced economies regarding their countercyclical macroeconomic policy.

The 2008 Global Financial Crisis and the transfers of capital across countries that it generated led to policy and academic circles to focus on the use of macroprudential policy tools to prevent and minimize the costs associated with capital flows. A number of policy studies have called for a more active management of capital flows because of the risks associated with these flows' size and volatility.[6,7] An emerging policy paradigm includes policies such as capital controls, but more generally macroprudential tools, to prevent and minimize the ex post costs associated with the risks carried by capital flows and concurrent financial crises. In parallel with the changes in the policy agenda, significant advances have been made in establishing a theoretical framework suitable for quantitative analyses of the underlying mechanisms and implementation of the aforementioned policy recommendations.[8] The rationale for policy intervention in this framework arises because of a pecuniary externality induced by a collateral constraint on international borrowing in which the collateral is valued at market prices—either the price of an asset or the real exchange rate—that themselves depends upon aggregate external indebtedness.

the model.

[5]This conventional wisdom comes from the idea that higher volatility in world interest rates directly translates into more volatile capital flows thay carry intrinsic risks. The following statement provides an example: "There is now a growing recognition that the short-term nature and inherent volatility of global capital flows are problematic." (Christine Lagarde, https://www.imf.org/external/np/speeches/2016/020416.htm)

[6]See Ostry et al. (2011) and Dell'Ariccia et al. (2012) as analytical background for IMF (2012), the organization's institutional view, and the policy proposals in IMF (2013), Chapter 4.

[7]The policy agenda has identified both sources of risk as posing significant policy challenges for all countries, but especially for EMEs.

[8]Korinek (2011) and Korinek and Mendoza (2013) provide surveys of the literature. This framework has relied on models traditionally exploited to study the positive side of large and abrupt capital outflows in EMEs—also known as sudden stops—to analyze normative aspects of policy. Mendoza and Smith (2002) and Mendoza (2010) explore the positive aspects of this framework.

The framework previously mentioned and the mechanism that it entails, which leads to "overborrowing" due to a pecuniary externality, has become a benchmark providing the rationale for the use of macroprudential policy. Relying on this framework, recent work has focused on studying the design and implementation of macroprudential policies when "overborrowing"—that is, when the size of capital flows becomes too large—arises because of country-specific factors such as negative income shocks.[9] However, the design and implementation of these policies are much less clear for countries facing substantial risks due to external factors, and even more unclear once we account for the fact that business cycles in EMEs, including sudden stops nested within these cycles, are significantly shaped by global forces that are independent of a country's fundamentals.[10] More specifically, there is a vast literature that has focused on the effects of shocks to external interest rates at which EMEs borrow and that has clearly documented significant effects of shocks to this variable on real economic activity and capital flows for EMEs. Furthermore, these studies have identified that not only the first, but also the second moment of these shocks matter for EMEs business cycles.[11] Given the association of changes in output, interest rates, and volatility with capital flow reversals in the data, it becomes relevant to consider the role of these external factors in the design and implementation of optimal policy.[12,13]

Against this backdrop, we introduce shocks to the level and volatility of external interest rates in a suitable framework to study the qualitative and quantitative features of optimal macroprudential policy to manage capital flows, and we use the proposed model to analyze the implications of these shocks for the competitive equilibrium and the design of optimal macroprudential policy.[14] The small open economy model that we

[9]Thus, the normative implications of these studies are directly related to policy intervention due to the size of capital flows rather than their volatility. See Bianchi (2011) for a detailed account of how "overborrowing" arises.

[10]See Mackowiak (2007) and Chang and Fernández (2013).

[11]For business cycles in general, Neumeyer and Perri (2005) and Uribe and Yue (2006) consider the effects of shocks to the level of interest rates, while Fernández-Villaverde et al. (2011) focus on the effects of shocks to their volatility. Ricardo Reyes-Heroles and Tenorio (2017) focus on the effects of both shocks for sudden stops in particular. See Ahmed and Zlate (2013) for the effects on capital flows.

[12]In recent work, Bianchi et al. (2016) consider the design of optimal policy when the mean of world interest rates follows a regime-switching process, but they do not consider stochastic volatility. Johri et al. (2015) study the implications of stochastic volatility in a model of sovereign default.

[13]Furthermore, recent studies have shown that an environment of high volatility is most likely to continue for EMEs throughout the unwinding of the countercyclical policies implemented in advanced economies, which stresses the importance of considering volatility shocks in policy (for example, Aizenman et al., 2014; Eichengreen and Gupta, 2014).

[14]The framework we consider is within a type of model that is now considered a benchmark to analyze macroprudential policy. For a survey of these models, see Korinek and Mendoza (2013).

propose is akin to the models by Jeanne and Korinek (2010) and Bianchi and Mendoza (2013). The small open economy is populated by a continuum of households whose only source of income is the payoff of a risky asset. The asset's shares cannot be traded across borders, but the households can lend or borrow from abroad in the form of non-contingent riskless bonds. However, the key friction in the model arises from the fact that borrowing is constrained by household's holdings of the risky asset valued at market prices.

Our model differs from previous work in two main respects. First, we provide a microfoundation of the collateral constraint based on contractual imperfections that take into account the asymmetry that arises in this type of models because the risky asset cannot be traded across borders. Even though the qualitative implications of the constraint for the competitive equilibrium are the same as those in existing models, we show how the conditions that characterize the equilibrium are different when the collateral constraint binds. Second, we extend models of macroprudential policy by allowing the interest rate at which the economy borrows, the external interest rate, to follow a stochastic process with time-varying volatility. This extension implies that households also face refinancing risks because of shocks to the level and volatility of the interest rate, which they take into account when making optimal consumption and saving decisions. Hence, these shocks have implications for the incidence and severity of sudden stops. In these models, a sudden stop in capital inflows occurs when households borrow up to the point where the borrowing constraint binds. The binding collateral constraint leads to an abrupt deleveraging of households, which reduces current consumption and causes a drop in asset prices, further reducing the value of collateral and tightening the borrowing constraint.

We characterize the model's competitive equilibrium and discuss how the pecuniary externality leads to "overborrowing" and an amplification mechanism, leading to large financial crises. Shocks to the level and volatility of external interest rates have implications for the incidence and severity of crises in equilibrium. Low and stable interest rates incentivize households to borrow more, thus increasing the incidence of a binding collateral constraint, even though such rates also decrease the risks of refinancing debt to hedge against other types of shocks. Regarding the severity of crises, this feature is affected by how shocks to interest rates affect asset prices in equilibrium. After characterizing the competitive equilibrium, we consider the problem of a social planner that internalizes the effects of borrowing on the price of the asset and compare the optimal allocations of this planner with those arising from the equilibrium. The planner internalizes the effects of borrowing on asset prices and on the borrowing

constraint but cannot choose asset prices directly or commit to future policies. Hence, the planner acts according to asset prices being consistent with equilibrium conditions. To prevent "overborrowing" the planner takes into account the interaction between the incidence and the severity of future crises and reduces these two aspects of crises by keeping asset prices depressed relative to those that arise in the competitive equilibrium.

We solve the model numerically using global methods and analyze the response of the competitive equilibrium's policy functions to external shocks as well as the dynamics generated by the model around crisis episodes. The use of global methods is critical for this type of model in order to fully characterize the nonlinearities that arise in the region where the collateral constraint binds. We first show that the response of the competitive equilibrium's policy functions to shocks to dividends and levels of interest rates is numerically sizable, but that such effects are not present when we analyze the response to shocks to the volatility of external interest rates. We argue that this unresponsiveness arises because the change in optimal decisions due to precautionary motives associated with volatility shocks is absorbed by changes in prices rather than in aggregate allocations.[15] We show how the model generates nonlinear and asymmetric impulse reposes to interest rate shocks precisely because of the collateral constraint and argue that this is a key feature when thinking of external shocks. Lastly, we simulate the model and show that the dynamics it generates endogenously around typical sudden stop events given our estimated stochastic process for the evolution of interest rates are in line with what the empirical literature has documented for EMEs. Low and stable interest rates precede sudden stops that lead to a large drop in consumption and a reversal of capital flows concurrently with a persistent increase in the level and volatility of interest rates.[16]

In the last and main part of the paper, we provide a detailed study of the implications of external shocks for optimal policy. We show that a state-contingent tax on debt implements the planner's optimal allocations and that the size of this tax is shaped by the incidence and severity of potential future crises. Hence, we proceed to focus

[15] Our simple model does not incorporate some mechanisms that could play a relevant role in amplifying the effects of external volatility shocks on business cycles. For instance, Fernández-Villaverde et al. (2011) emphasize the role of investment, which we do not incorporate, but discuss in detail later in the paper. However, we see the simplicity of our model as an advantage in order to clarify the central mechanism underlying the answer to one key question in this paper: Does greater external volatility call for higher taxes on capital flows? Hence, our results could be interpreted as a lower bound on the responsiveness of the social planner's optimal policy to external shocks.

[16] Ricardo Reyes-Heroles and Tenorio (2017) provide an empirical account of the evolution of interest rates around sudden stops for a large sample of EMEs.

our analysis on the response of the entire optimal tax schedule to external shocks.[17] Our initial analysis shows that the optimal tax is contingent on both types of external shocks. Even though this result was expected for the case of shocks to the level of interest rates—precisely because of our previous result showing that the equilibrium's policy function responds to the same type of shocks—the fact that the optimal tax also responds to shocks to external volatility highlights the role of the pecuniary externality. Shocks to volatility translate into large changes in asset prices, instead of allocations, whose effects on the collateral are internalized by the planner but not by households. No previous work has highlighted other shocks leading to macroprudential policy primarily through this price effect, and we see this finding as an important contribution of this paper. The second result of this analysis is that the level of the tax on capital flows is non-monotonic with respect to external shocks. In other words, the tax schedule does not shift monotonically for different magnitudes of external shocks. For instance, it is optimal to increase the tax on international borrowing when interest rates decrease, but not if the level of debt is very high given a high interest rate.[18] This is a novel insight that we also see as an important contribution of this paper. One particular corollary that follows from this result should be underscored: Very simple intuition would suggest that higher volatility should lead to higher capital controls because capital flows become more volatile, thus increasing the probability of a binding collateral constraint. However, as we carefully explain in this paper, this intuition is flawed because it does not take into account the effects of external shocks on household's precautionary motives, asset prices and their interaction manifested on the incidence and severity of potential future crises. Thus, in the last section of the paper, we provide a decomposition of the optimal tax that provides a detailed explanation of the main factors driving the implementation of optimal policy: the incidence and severity of potential future crises, and the interaction of these two.

This paper is most closely related to two literatures in international macroeconomics. First, this paper is related to the recent literature that explores optimal macroprudential policy to mitigate the risks associated with "overborrowing" and large and volatile capital flows across countries, which is summarized in detail by Korinek and Mendoza (2013). The pecuniary externality mechanism leading to "overborrowing" is emphasized in Jeanne and Korinek (2010), Bianchi (2011), Bianchi and Mendoza (2011), and

[17] By the tax schedule we refer to the tax as a function of current debt given a profile of exogenous shocks.

[18] This statement will be made clearer in subsection 3.3.2 where the detailed analysis is provided.

Bianchi and Mendoza (2013).[19,20] The model we consider is closest to those by Jeanne and Korinek (2010) and Bianchi and Mendoza (2013). We build on the simplified framework of Jeanne and Korinek (2010) but focus on solving for time-consistent optimal macroprudential policies, as do Bianchi and Mendoza (2013), and allow the interest rate at which the economy borrows to follow a stochastic process with time-varying volatility. Bianchi et al. (2016) also consider the design and implementation of optimal policy when the mean of world interest rates follows a regime-switching process, but they do not consider stochastic volatility. In addition, they carry out their analysis in a different framework in which the collateral is a nontradable good rather than an asset, which does not incorporate the forward-looking component of asset prices that are affected by shocks to interest rates.

The second literature to which this paper is closely related studies the effects of external shocks on EMEs business cycles. Multiple studies including Mackowiak (2007) and Ahmed and Zlate (2013) have documented the effects of external shocks on EMEs. However, the literature relying on open-economy business-cycle models has focused on shocks to the interest rate at which EMEs borrow, which are assumed to be independent of countries' fundamentals, as a potential source of variation in real economic activity. Uribe and Yue (2006) and Neumeyer and Perri (2005) show that shocks to world interest rates are an important driver of EME's business cycles. Fernández-Villaverde et al. (2011) show that not only the first, but also the second moment of the shocks to interest rates have implications on real economic activity in EMEs. Ricardo Reyes-Heroles and Tenorio (2017) focus on sudden stops and document the empirical association between sudden capital flow reversals and external interest rate volatility for a large sample of EMEs. Two of their main findings are that (i) sudden stops are preceded by periods of below-normal interest rates, which rise when a sudden stop occurs and revert to their normal levels in the following years; and that (ii) sudden stops follow periods of low interest rate volatility that increases sharply at the beginning of the sudden stop and remains persistently high for multiple periods.[21] We adopt the approach taken in

[19]This amplification mechanism was initially introduced to the positive study of sudden stops by Mendoza (2002), Mendoza and Smith (2002), Mendoza and Smith (2006) and Mendoza (2010).

[20]The literature has focused on two different aspects of optimal policy, either its "prudential" features, in the sense that policy is undertaken ex ante in order to reduce the probability of a crisis, as we do in this paper, or its ex post characteristics, after the crisis has occurred. Benigno et al. (2011) and Benigno et al. (2013b) focus on the ex post policies. Most recently, other studies like Jeanne and Korinek (2013) and Benigno et al. (2013a) have focused on the use of both ex ante as well as ex post policies in order to mitigate the risks associated to capital flows.

[21]Multiple papers document the independence of interest rates from countries' fundamentals. Longstaff et al. (2011) show that the majority of sovereign credit risk can be linked to global factors.

these studies to introduce external shocks into a model of endogenous sudden stops and macroprudential policy. This extension allows us to study and characterize optimal macroprudential policy in response to both the level and the volatility of external interest rates.

The rest of the paper is organized as follows. In Section 2 we introduce the theoretical model of a small open economy facing domestic and external risks that are amplified by the effects of a collateral constraint. We describe the competitive equilibrium and discuss the presence of a pecuniary externality that motivates the intervention of a social planner to increase welfare in the economy. In Section 3 we present the results of our numerical exercises. We show that the dynamics of interest rates around episodes of sudden stop in the model are consistent with their empirical counterparts. Moreover, we explain how the optimal response of the planner is shaped by incidence and severity of potential future crises. In Section 4 we conclude.

2 A model of endogenous sudden stops with external interest rate risk

2.1 Framework

Our framework is closely related to those of Jeanne and Korinek (2010) and Bianchi and Mendoza (2013). Consider an open economy inhabited by a continuum of unit measure of identical households that have preferences for streams of a consumption good, c_t, given by

$$E_0 = \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where u is an increasing, concave, and differentiable function that satisfies the usual Inada conditions.

There is a Lucas tree that yields a stochastic flow of consumption goods of $d_t = d\exp(z_t)$ per period. The flow of goods provided by the tree can be traded period by period with the rest of the world, but the stocks of the tree can only be held by domestic households. A possible explanation is that this arrangement arises from drastic asymmetries of information between domestic managers and international investors

Carrière-Swallow and Cépedes (2013) further emphasize that global uncertainty has important effects on real economic activity in EMEs. Johri et al. (2015) argue that global factors drive an important part of fluctuations stochastic volatility of interest rates and study the implications of these factors in a model of sovereign default.

that impede foreigners from earning profit by holding stocks of the tree. We denote by q_t the market value of the tree at time t, and by s_t the holdings of the asset chosen by the representative household.

Households have access to debt financing in international financial markets in order to smooth their consumption and fund their stock purchases. The bonds issued by households in international markets have a maturity of one period, and they pay an exogenous gross return of $R_t = R\exp(r_t)$. We let the external interest rate have a stochastic transition, but debt contracts are locally risk free: A household knows at time t the interest rate that it must pay next period for its outstanding bonds, but it does not know the interest rate that it will face next period if it decides to refinance its stock of debt.

Following the approach by Ricardo Reyes-Heroles and Tenorio (2017) to study the evolution of external interest rates around sudden stops, we allow for contemporaneous correlation and dynamic feedback between the exogenous output and interest rate processes. The random vector $(z_t, r_t)'$ has the following VAR specification:

$$\begin{pmatrix} z_t \\ r_t \end{pmatrix} = A_0 + A_1 \begin{pmatrix} z_{t-1} \\ r_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_t^z \\ \epsilon_t^r \end{pmatrix}. \tag{1}$$

The draws of the shock vector $(\epsilon_t^z, \epsilon_t^r)'$ are independent across time, and they have a Gaussian distribution with zero mean and a covariance matrix that has itself a stochastic evolution:

$$\Sigma_t = \begin{pmatrix} (\sigma^z)^2 & \rho \cdot \sigma^z \cdot \sigma_t^r \\ \rho \cdot \sigma^z \cdot \sigma_t^r & (\sigma_t^r)^2 \end{pmatrix}.$$

As in Ricardo Reyes-Heroles and Tenorio (2017), we allow the volatility of the external interest rate to take on two values, $\sigma_t^r \in \{\sigma_L^r, \sigma_H^r\}$, with $\sigma_H^r > \sigma_L^r > 0$. The switching between these regimes is governed by a first-order Markov process with transition matrix Π. Introducing stochastic volatility in interest rates is the first element that differentiates this paper from previous work on optimal macroprudential policy.

Let us denote by b_t the face value of bonds that are held by the households at the beginning of period t. Throughout the paper, we follow the convention that a positive b_t represents savings of the households overseas, whereas negative positions represent external household debt. The time t budget constraint faced by a household is

$$c_t + q_t s_{t+1} + \frac{b_{t+1}}{R_t} = (q_t + d_t)s_t + b_t. \tag{2}$$

The key friction in this economy is that the amount of borrowing that households can undertake is limited by the value of their asset holdings. More specifically, the market value of debt issued by a representative household at time t, $-\frac{b_{t+1}}{R_t}$, is constrained to be less than or equal to the value of their holdings of stocks of the tree, $q_t^c s_{t+1}$, multiplied by a constant κ that determines how stringent the financial frictions are:

$$-\frac{b_{t+1}}{R_t} \leq \kappa q_t^c s_{t+1}. \tag{3}$$

Notice that this collateral constraint explicitly takes into account the fact that the price used to value asset holdings as collateral at time t, q_t^c, is not necessarily the same as the market price, q_t. This difference arises because the risky asset cannot be traded across borders, but it is still used as collateral by foreign lenders. In Appendix A.1, we provide a microeconomic foundation of the collateral constraint that is based on contractual imperfections, as is common in the literature of financial frictions (for example, Kiyotaki and Moore, 1997; Bernanke et al., 1999).[22] The main idea is that within each period, there is a time in which households can divert a fraction $(1-\kappa)$ of the assets previously posted as collateral, sell them off at the prevailing price q_t^c, and default on their outstanding loans. After default, the foreign lender is entitled to the remaining fraction κ of collateral assets, which must be sold in the domestic market at the prevailing price q_t^c. In the appendix, we show that the market price of the tree and its resale value need not be the same, and we also derive the relationship that has to hold in equilibrium between them.

[22]The literature on macroprudential policy that considers assets as collateral has assumed that (i) the price used to value the asset as collateral is the same as the equilibrium price in domestic asset markets—that is, $q_t^c = q_t$ in equation (3)—and that (ii) the amount of the asset relevant in the collateral constraint is the one held at the beginning of the period, s_t, rather than at the end of the period, s_{t+1}. For instance, Bianchi and Mendoza (2013) show that a collateral constraint of the form $-\frac{b_{t+1}}{R_t} \leq \kappa q_t s_t$ can be derived as an implication of incentive-compatibility constraints on borrowers if limited enforcement prevents lenders from collecting more than a fraction κ of the value of the assets owned by a defaulting debtor. Notice that assumption (i) overlooks the asymmetry that arises in the framework in these papers from the fact that the tree can only be held by domestic owners, but foreign lenders view it as collateral. Hence, rather than starting by assuming (i), we begin with the microfoundation described in Appendix A.1 and derive an equilibrium relationship between q_t and q_t^c. Regarding assumption (ii), considering s_{t+1} as the amount of the asset that is relevant in the collateral constraint is closer to the literature on sudden stops arising from this type of collateral constraint (Mendoza, 2010; Mendoza and Smith, 2006).

2.2 Competitive equilibrium

A competitive equilibrium is a sequence of allocations $\{c_t, b_{t+1}, s_{t+1}\}_{t=0}^{\infty}$ for every household and a prices of the tree $\{q_t, q_t^c\}_{t=0}^{\infty}$ (market and collateral valuations) such that households optimize their utility, subject to the budget and borrowing constraints, and the market for stocks of the tree clears. Given that all the households are identical and they only face aggregate shocks, market clearing implies that $s_t = 1$ in every period.

We rewrite the problem of the representative household in recursive form in order to highlight the role of pecuniary externalities in the competitive equilibrium. The aggregate states in the household's problem are the aggregate level of savings B and the current realization of the stochastic shocks, which we denote $X \equiv (z, r, \sigma^r)$. The individual states of a household are its holdings of bonds b and stocks of the tree s. We denote by $V(b, s, B, X)$ the value of the problem for a household with portfolio (b, s) when the aggregate states are B and X. Households take as given a perceived law of motion for aggregate bonds, $B' = \mathcal{B}(B, X)$, in order to form expectations on future prices. Then, the Bellman equation of the problem is

$$V(b, s, B, X) = \max_{c, b', s'} u(c) + \beta \mathbb{E}[V(b', s', B', X')|X] \tag{4}$$

subject to

$$\begin{aligned} c + \mathcal{Q}(B, X)s' + \frac{b'}{R(X)} &= [\mathcal{Q}(B, X) + d(X)]s + b, \\ -\frac{b'}{R(X)} &\leq \kappa \mathcal{Q}^c(B, X)s', \\ B' &= \mathcal{B}(B, X). \end{aligned}$$

In the previous expression, $\mathcal{Q}(B, X)$ is the market value of the tree, and $\mathcal{Q}^c(B, X)$ is the value of the asset when employed as collateral. These two prices are determined in equilibrium and depend on the aggregate states of the economy. In a recursive competitive equilibrium, it must be the case that $\mathcal{B}$ is consistent with optimal individual decision rules and that $\mathcal{Q}$ and $\mathcal{Q}^c$ ensure the clearing of the market for stocks of the tree in the different trading cycles described in Appendix A.1.

In Appendix A.2 we show that the solution to the household's problem satisfies the

following Euler equations for bonds and stocks of the tree, respectively:

$$u'(c(b,s,B,X)) - \mu(b,s,B,X) \tag{5}$$
$$= R(X)\beta\mathbb{E}\{u'(c(b',s',\mathcal{B}(B,X),X'))|X\},$$
$$\mathcal{Q}(B,X)u'(c(b,s,B,X)) \cdot \left(1 + \frac{\kappa\mu(b,s,B,X)}{u'(c(b,s,B,X))}\right)^{-1} \tag{6}$$
$$= \beta\mathbb{E}\{u'(c(b',s',\mathcal{B}(B,X),X'))[\mathcal{Q}(\mathcal{B}(B,X),X') + d(X')]|X\},$$

where $\mu \geq 0$ is the multiplier on the borrowing constraint.[23] The left-hand side of the Euler equation for bonds is the marginal cost of saving an additional unit of consumption good at time t: the household loses utility $u'(c_t)$ in the margin, and, if the borrowing constraint is binding, an additional unit of saving relaxes the constraint, with a shadow value of μ_t, thus reducing the marginal cost of saving. The right-hand side represents the gains obtained by the household next period: For the additional unit saved in the margin, the household gets R_t goods in the next period, which are valued at the expected marginal utility $\mathbb{E}_t[u'(c_{t+1})]$ and discounted by the subjective discount factor β.

Similarly, the left-hand side of the Euler equation for stocks shows the marginal cost faced by a household that is buying additional shares of the tree: For each stock, the household must pay a price of q_t, and it has a marginal utility loss of $q_t u'(c_t)$. The factor at the end of the left-hand side is the wedge between the market price of stocks of the tree and their collateral value (see Appendix A.1). This wedge is non-zero only when the borrowing constraint is binding, which means that the household values the additional service that their asset holding brings by increasing its borrowing opportunities. In turn, the right-hand side is the expected benefit received by the household, which is the resale value of the stock, q_{t+1}, and the dividend, d_{t+1}, as valued by the marginal utility of the household, $u'(c_{t+1})$, and discounted by β.[24]

[23] We have expressed the solution to the household's problem in terms of the equilibrium price $\mathcal{Q}(B,X)$ only by relying on the equilibrium condition between $\mathcal{Q}^c(B,X)$ and $\mathcal{Q}(B,X)$. If we had assumed from the beginning that $\mathcal{Q}^c(B,X) = \mathcal{Q}(B,X)$, then the left-hand side of the second equation would become $\mathcal{Q}(B,X)u'(c(b,s,B,X))\,(1 - \kappa\mu(b,s,B,X)/u'(c(b,s,B,X)))$. Notice that the qualitative implications of a binding collateral constraint for the second condition are the same under our microfoundation of the constraint described in Appendix A.1.

[24] Alternative specifications of the household's problem, such as Jeanne and Korinek (2010), assume that the household's borrowing is constrained by the aggregate number of stocks in the economy rather than the household's individual holdings. This assumption eliminates the effect of relaxing the borrowing constraint through an increase of the value of collateral in the Euler equation for stocks (that is, the wedge between the market and collateral values of the tree).

In our framework, a sudden stop in external financing arises endogenously as a consequence of the households' borrowing decisions. For high levels of leverage, if the borrowing constraint binds, households are forced to have a fast reduction of debt, which is only possible through drastic declines in consumption. This decline causes falls in asset prices by increasing today's marginal utility of consumption and discounting more heavily future cash flows. In turn, the value of collateral is reduced, which further tightens the borrowing constraint, and induces more deleveraging. The feedback between asset price reductions, forced deleveraging, and drops in consumption follows ad infinitum, generating a sudden reversal of the capital flows into the country.

Korinek and Mendoza (2013) highlight that when the external borrowing rate is lower than the households' discount factor, the households face a fundamental trade-off between impatience and insurance. They have an incentive to borrow from overseas in order to consume in advance because interest rates are low. Nonetheless, for high levels of borrowing, a sudden stop is more likely to happen and, given that it is accompanied by a drastic decline in consumption, households have the incentive to save out of the sudden stop-region. In the next section, we illustrate the interaction between these two motives by solving the model numerically.

Changes in the level of external interest rates affect the marginal cost of borrowing as can be appreciated in the right-hand side of equation (5). Low interest rates imply low marginal costs of borrowing, equivalent to a high expected stochastic discount factor, that incentivize households to acquire more debt and increase consumption in the current period, c_t.[25] Concurrently, changes in the interest rate have implications for asset prices through its effect on the stochastic discount factor, as can be seen in equation (6), and its effect on how future dividends are discounted by households. Everything else constant, low interest rates increase current asset prices, q_t, because the present value of future dividends increases. Hence, shocks to external interest rates lead to more volatile capital flows and domestic asset prices. Notice that as long as changes in interest rates are somewhat persistent, these changes will also affect the likelihood of the collateral constraint binding in future periods through two effects: a direct effect on the marginal cost of debt and an indirect effect on the future value of collateral. These additional implications of changes in world interest rates also affect households' optimal decisions.

Changes in the volatility of the world interest rate while keeping the level constant

[25] Conditional on the collateral constraint not binding in the current period, t, the stochastic discount factor is given by $\beta \frac{u'(c_{t+1})}{u'(c_t)}$ which in equilibrium must, in expectation, be equal to R_t^{-1}.

affect households' optimal decisions by increasing the volatility of future consumption. An increase in the volatility implies that debt becomes a worse instrument to hedge against future income shocks: it basically increases the risk to refinance debt in the future. Therefore, households reduce their debt, which leads to a reduction in current consumption, c_t.[26] However, notice that this change in volatility also has implications for asset prices as households try to liquidate their assets to smooth consumption, leading to a drop in asset prices. Hence, shocks to the volatility of external interest rates also lead to more volatile capital flows and domestic asset prices. We provide a more detailed description of these mechanisms and how they shape optimal policy in later sections of the paper.

The fact that shocks to the level and volatility of interest rates have implications for borrowing decisions—which shape the volatility of capital flows—and asset prices implies that these shocks have implications for the pecuniary externality and, therefore, for the incidence and severity of potential future crises. In the next section we provide a formal definition of the incidence and severity of crises in the context of our model and show how a planner takes these two aspects of crises into account by internalizing the pecuniary externality.

2.3 Constrained efficient allocation

The fact that the aggregate level of debt determines asset prices and that this, in turn, affects the borrowing capacity of the households creates a pecuniary externality in the economy. Individual households do not internalize the effect of their indebtedness on the borrowing possibilities of all households, which results in Pareto-inefficient allocations. In this section, we study the problem of a social planner that internalizes the effect of external indebtedness on the value of collateral and, hence, on the borrowing capacity of the country. In particular, we consider a social planner that can only choose the level of aggregate debt, subject to the economy's borrowing constraint. The planner cannot directly intervene in the trading of the asset that takes place between households, so it tries to affect the equilibrium value of collateral indirectly by altering the economy's borrowing decisions. Following Bianchi and Mendoza (2013), we assume that the planner cannot commit to future policies, and we solve for the constrained efficient allocation that he would implement through time-consistent policies.

We follow Klein et al. (2005) in laying out the social planner's problem and in

[26] More volatile consumption in the future implies that $\mathbb{E}_t u'(c_{t+1})$ increases (by Jensen's inequality), which in turn implies that current consumption must decline for (5) to hold for fixed R_t.

finding its time-consistent solution. In particular, we restrict attention to the case in which policy rules only depend on the current state variables of the economy. This restriction implies that the policy rule of the planner is given by a simple function of the current states, (B, X), that maps them into levels of aggregate bonds, $B' = \Psi(B, X)$. In Appendix A.3, we show that the problem that is being solved by the social planner can be stated as follows. Given an arbitrary future policy rule, $\Psi(B, X)$, and the associated asset pricing function, $\mathcal{Q}(B, X)$, the social planner chooses c and B' that solves the following Bellman equation:

$$W(B, X) = \max_{c, B'} \{u(c) + \beta \mathbb{E}[W(B', X') | X]\}$$

subject to

$$\begin{aligned} c + \frac{B'}{R(X)} &= d(X) + B, \\ -\frac{B'}{R(X)} &\leq \kappa \bar{\mathcal{Q}}(B, B', X), \end{aligned}$$

and the valuation of collateral is consistent with the household's trading of the stocks of the tree:

$$\bar{\mathcal{Q}}(B, B', X) = \beta \mathbb{E}\left[\frac{u'\left(d(X') + B' - \frac{\Psi(B', X')}{R(X')}\right)[\mathcal{Q}(B', X') + d(X')]}{u'\left(d(X) + B - \frac{B'}{R(X)}\right)} \middle| X \right]. \tag{7}$$

In the appendix we prove that (7) the relevant equilibrium pricing condition that the planner faces, given the microeconomic foundations that give rise to our collateral constraint.[27,28]

The planner's decision now internalizes the fact that increasing households' savings affects equilibrium asset prices, which in turn alters the value of collateral in the borrowing constraint. In particular, the functions that solve the planner's problem,

[27]The planner's problem has been defined in various ways in previous studies. Jeanne and Korinek (2010) and Bianchi and Mendoza (2011) consider variations of the problem that are time-consistent by construction. For instance, Bianchi and Mendoza (2011) use the competitive equilibrium price schedule $\mathcal{Q}(B, X)$ and do not allow it to satisfy the frictionless asset pricing condition of the households. Jeanne and Korinek (2010) make assumptions on the equilibrium pricing function. The planner's problem that we consider is the same as in Bianchi and Mendoza (2013), thus allowing for the issue of time inconsistency to arise.

[28]Following the literature on optimal taxation under commitment, this condition has been referred to as an implementability constraint.

$c = \hat{\mathcal{C}}(B, X)$ and $B' = \hat{\Psi}(B, X)$, must satisfy the following condition:[29]

$$\begin{aligned} u'(\hat{\mathcal{C}}(B, X)) - \mu(B, X)\left[1 + \kappa R(X)\xi(B, X)\right] \\ = R(X)\beta\mathbb{E}\left[u'(\mathcal{C}(B', X')) + \kappa\mu(B', X')\psi(B', X')\right], \end{aligned} \tag{8}$$

where

$$\psi(B, X) = \frac{\partial\bar{\mathcal{Q}}(B, \Psi(B, X), X)}{\partial B}, \quad \xi(B, X) = \frac{\partial\bar{\mathcal{Q}}(B, \Psi(B, X), X)}{\partial B'},$$

and $\mathcal{C}(B, X) = B + d(X) - \frac{\Psi(B,X)}{R(X)}$.

In order to gain some intuition on how the planner internalizes the pecuniary externality, let us first focus on the case in which the collateral constraint is not binding in the current period, $\mu(B, X) = 0$. In this case, equation (8) becomes

$$u'(\hat{\mathcal{C}}(B, X)) = R(X)\beta\mathbb{E}\left[u'(\mathcal{C}(B', X')) - \kappa\mu(B', X')\psi(B', X')\right].$$

The planner's intervention considers not only the possibility of a binding borrowing constraint and how tight it is through the $\mu(B', X')$ term, which formally defines what we refer to as the *incidence* of a crisis, but also the risk associated with the size of the price externality through the $\kappa\psi(B', X')$ term, which we refer to as the *severity* of a crisis. Conditional on today's collateral constraint being non-binding, if the future price schedule were constant with respect to debt, the planner would not intervene, regardless of the possibility of the borrowing constraint being binding. Likewise, if there were an externality from borrowing but the planner did not expect the borrowing constraint to bind in the following period, he would not have a reason to distort the households' borrowing decisions. In the appendix, we show that

$$\psi(B, X) = -\frac{u''(\mathcal{C}(B, X))}{u'(\mathcal{C}(B, X))}\mathcal{Q}(B, X), \tag{9}$$

which implies that the price externality depends on the level of asset prices and the coefficient of absolute risk aversion of the representative household.[30]

Let us now consider the case in which the collateral constraint is binding in the current period. In this case, $\mu(B, X) > 0$, and equation (8) now includes an additional

[29] Klein et al. (2005) call this equation a "generalized Euler equation" because it is a functional equation of an unknown equilibrium object, in this case $\bar{\mathcal{Q}}$.

[30] The fact that $\psi(B, X)$ can be written in terms of unknown functions, rather than partial derivatives of unknown functions, simplifies the analysis of the functional equation.

term related to a partial derivative of an unknown function, $\bar{\mathcal{Q}}$. Notice that, as pointed out by Bianchi and Mendoza (2013), this is the relevant case in which a time inconsistency problem arises for the planner. The term $\xi(B, X) = \frac{\partial \bar{\mathcal{Q}}(B,B',X)}{\partial B'}$ shows that if the borrowing constraint is currently binding, the planner has an incentive to affect current asset prices by making future promises that would not be time consistent for a committed planner.[31] In the problem of the planner, we assumed that an arbitrary future policy rule, $\Psi(B, X)$, and its implied asset pricing function, $\mathcal{Q}(B, X)$, are taken as given. Hence, the current planner can only affect the pricing function by choosing B' and then having the future planner make his decision based on $\Psi(B', X')$, rather than committing to B' and B''. In Appendix A.3, we provide an expression for $\xi(B, X)$ that shows explicitly how it relates to the planner taking future policy rules as given.

Given the characteristics of the social planner's problem, it is straightforward to define a recursive constrained efficient allocation, conditional on arbitrary future planners' policy rules. Our definition of a constrained efficient allocation further requires that these policy rules be time consistent. In other words, we require that the policy that solves the strategic game being played by sequential planners is a fixed point, deriving in a Markov stationary policy rule. We provide further details and formal definitions of these concepts in the appendix.

Shocks to the first and second moments of the world interest rate have important implications for the pecuniary externality. For instance, lower interest rates exacerbate "overborrowing" in the competitive equilibrium because borrowing becomes cheaper which, in turn, increases the incidence and severity of crises. Lower volatility of interest rates also amplifies the problem of "overborrowing" by incentivizing households to acquire more debt. The planner internalizes how these shocks affect asset prices and the value of collateral, as shown in equation (8). In the last part of the paper we focus on how shocks to the level and volatility of interest rates affect the planner's decisions through the influence of these shocks on the incidence and severity of crises by solving the model numerically.

[31] See Bianchi and Mendoza (2013) for a detailed explanation of the difference between a planner with and without commitment.

Table 1: Baseline parameterization

Parameter		Value	Target
Time discount	β	0.96	Standard value
Relative risk aversion	γ	2	Standard value
Dividends	d	1	Normalization
Collateral constraint	κ	0.04	Debt-to-output ratio

3 The dynamics of sudden stops, optimal capital flow management, and external interest rates

3.1 Parameterization and numerical solution

To provide a full analysis of the general equilibrium interaction of the borrowing constraint and external shocks, we now focus on the results derived from solving the model numerically. Hence, we proceed to choose the parameters of the model and estimate the processes of exogenous shocks. To do this, we either consider parameter values in existing literature or use what we consider good data to map to empirical counterparts of the model. However, we remain fully aware of the simplicity of the model and its implied limitations when carrying out our exercises. We will consider a utility function from the constant relative risk aversion family: $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$.

Table 1 presents the baseline parameterization of the model for an annual time frequency. The parameters for preferences are standard in the literature of small open economies. Our choice of the relative risk aversion, $\gamma = 2$, is in the lower end of the values used for emerging economies in the open economy business cycle literature. Hence, the quantitative effects of volatility on real allocations and asset prices that we show are, in principle, conservative. The mean of the dividends process, d, is normalized to one, so we can easily interpret the measurements of consumption, savings, and asset prices relative to the mean annual income. The parameter of the collateral constraint, $\kappa = 0.04$, is chosen to match the ratio of foreign liabilities to GDP observed in a sample of emerging markets over the period from 1990 to 2011, which averaged 66.7%.[32] In the model, the ergodic mean of the debt-to-output ratio is 65.6%.

We estimate the parameters that rule the regime-switching VAR given by (1) for a

[32]These numbers are calculated using data from the updated and extended External Wealth of Nations database of Lane and Milesi-Ferretti (2007). The figure corresponds to the countries in Sample 1 described in Ricardo Reyes-Heroles and Tenorio (2017). As a reference, an alternative calibration target could have been the average net foreign asset to GDP ratio, which amounts to 27.8% of GDP in our sample.

group of emerging markets using the maximum likelihood approach of Ricardo Reyes-Heroles and Tenorio (2017), with the data corresponding to a sample of 10 EMEs. The only difference compared to Ricardo Reyes-Heroles and Tenorio (2017) is that we use annual data, which better correspond to the timing of our model and existing literature. Quarterly GDP figures were annualized and then log-linearly detrended, and monthly interest rate data were averaged arithmetically. The estimated process is

$$\begin{pmatrix} z_t \\ r_t \end{pmatrix} = \begin{pmatrix} 0.0052 \\ 0.0025 \end{pmatrix} + \begin{pmatrix} 0.6079 & -0.1321 \\ 0.1289 & 0.8261 \end{pmatrix} \begin{pmatrix} z_{t-1} \\ r_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_t^z \\ \epsilon_t^r \end{pmatrix}, \tag{10}$$

and the covariance and transition matrices are composed of

$$\begin{aligned} \sigma^z = 0.0312, \quad \rho = -0.4048, \quad \pi_L = 0.9610, \\ \sigma_L^r = 0.0150, \quad \sigma_H^r = 0.0661, \quad \pi_H = 0.7468. \end{aligned}$$

The results of our estimation are in line with those in Ricardo Reyes-Heroles and Tenorio (2017), who estimate a similar process for a large sample of EMEs and provide evidence of the robust existence of multiple regimes in the volatility of interest rates. Ricardo Reyes-Heroles and Tenorio (2017) also study the dynamics of external interest rates around the sudden stop episodes that the literature has identified and show that (i) sudden stops are preceded by periods of below-normal interest rates, which rise when a sudden stop occurs and revert to their normal levels in the following years and that (ii) sudden stops follow periods of low interest rate volatility that increases sharply at the beginning of the sudden stop and remains persistently high for multiple periods. We will show in the following subsection that a typical sudden stop generated by our model occurs precisely when external shocks imply these dynamics for interest rates and their volatility.

The ergodic mean of the output and the interest rate processes can be obtained by inverting the VAR as follows:

$$E \begin{pmatrix} z_t \\ r_t \end{pmatrix} = (I - \hat{A}_1)^{-1} \hat{A}_0 = \begin{pmatrix} 0.0066 \\ 0.0196 \end{pmatrix},$$

where $\hat{A}_0$ and $\hat{A}_1$ denote the estimated matrices in (10). The long-run average of the external interest rate is thus 1.96%, which is considerably below the households' discount rate of $(\beta^{-1} - 1) \approx 4\%$. This difference gives the households an incentive to borrow from the exterior in order to consume up front.

The two regimes of the VAR have considerably different interest rate volatilities. In the low volatility regime, the standard deviation of interest rate shocks is small, $\sigma_L^r = 1.50\%$, leading to a very low refinancing risk for bond holdings. In contrast, in the high volatility state, the standard deviation is 4.4 times higher, $\sigma_H^r = 6.61\%$, which induces a large uncertainty in the future access to debt financing for the economy. The transition matrix between the two volatility states has a high persistence: The mean duration of low and high volatility episodes is 25.6 and 3.9 years, respectively. In the long run, the system spends 86.6% of the time in the low volatility state.

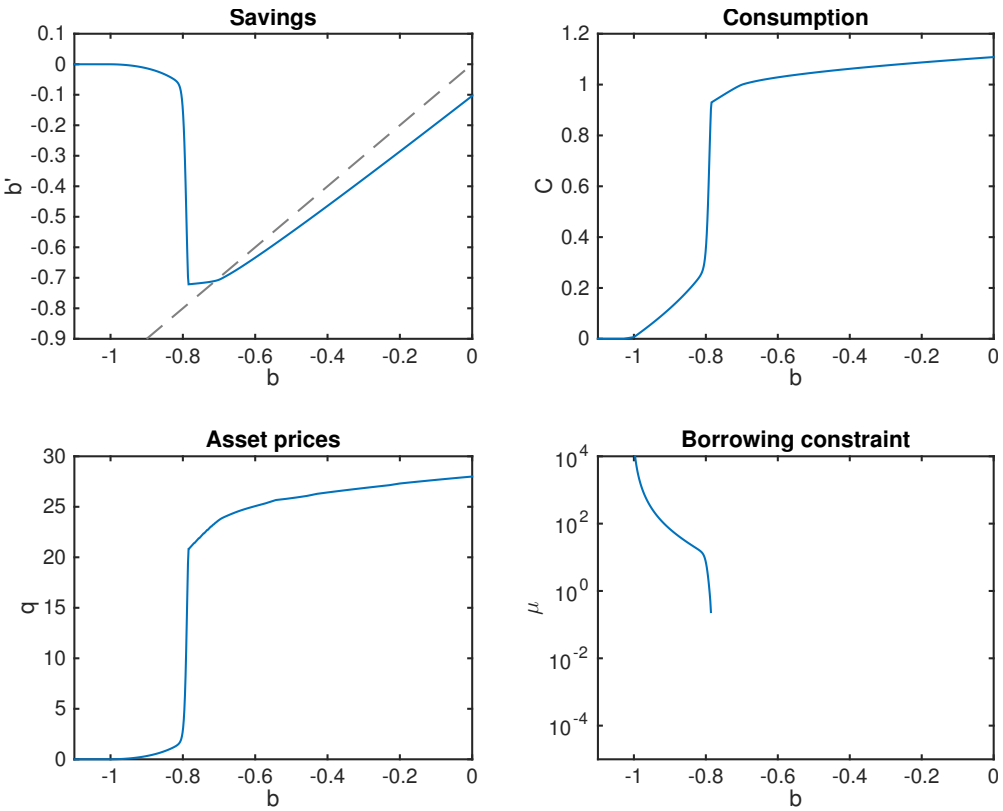
Our estimates for the variance of the external interest rate are consistent with the findings in the literature (for example, Fernández-Villaverde et al., 2011). A limitation in our estimated process is that the shocks to the interest rate are symmetric: When volatility increases, it is equally likely for it to reach high deviations above or below the mean. We opt not to introduce asymmetries in our estimation for the sake of parsimony and simplicity. However, an estimation of the VAR model with additional degrees of freedom can be conducted to assess the quantitative relevance of asymmetric shocks.

We use a global solution method to characterize the recursive competitive equilibrium of the economy in a discretized version of the aggregate state space. We use a grid of 300 points for household savings, placing 80% of them around the region where the borrowing constraint binds in order to better capture the nonlinearities of the model. We discretize the estimated VAR process using a two-dimensional variation of the Tauchen (1986) method that allows for different levels of variance of the shocks. We use a grid of 7 points for output shocks and 15 points for the interest rate to better capture the effects of changing volatility of the latter variable. We truncate the grids in order to include 95% of the probability mass of shocks at the ergodic distribution, which was approximated by simulating the VAR for 1 million periods. To solve the system of rational expectations with occasionally binding constraints, we use an adaptation of the endogenous grid method of Carroll (2006). Appendix B describes in detail our algorithm and its numerical accuracy.

3.2 Description of the competitive equilibrium

Figure 1 depicts the numerical solution to the recursive competitive equilibrium. In the first panel, we show the representative household's savings rule $\mathcal{B}(B, X)$ as a function of the initial level of aggregate savings B. This decision rule is non-monotonic: For high levels of wealth, the savings rule is upward sloping, as expected. Given that the average interest rate is below the households' discount factor, there is an incentive to

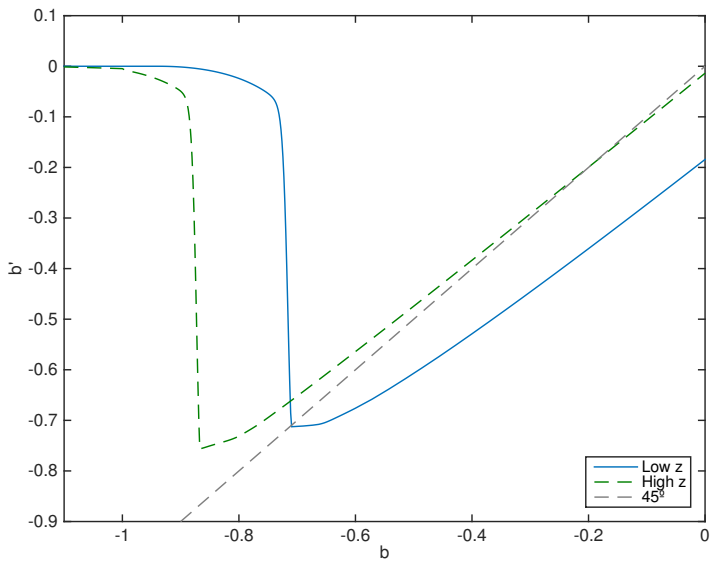
Figure 1: Recursive competitive equilibrium: Savings rule, consumption, asset prices, and multiplier on the borrowing constraint.



increase the economy's indebtedness, which is reflected in the fact that the savings rule lies below the 45 degree line. If the amount of debt reaches high enough levels, the borrowing constraint becomes binding. In this situation, the households must reduce their consumption in order to lower their stock of debt, as displayed in the second panel of the figure. This reduction causes an increase in the marginal utility of contemporaneous consumption, which in turn induces a higher discount of future cash flows and a consequent drop in asset prices. This drop is shown in the third panel, which depicts the equilibrium asset prices, $\mathcal{Q}(B, X)$, as a function of savings B. The sharp drop in the value of collateral forces a large deleveraging, as shown in the first panel, which feeds back into further consumption cuts and asset price falls, ad infinitum.

Episodes with binding borrowing constraints in our economy are accompanied by sharp declines in consumption, and because the households are inelastic in terms of intertemporal substitution, this fast deleveraging entails high utility losses. Therefore, households have a precautionary savings motive around the region in which borrowing constraints bind. The first panel of Figure 1 shows that the rate at which households

Figure 2: Savings rule: Different endowment levels

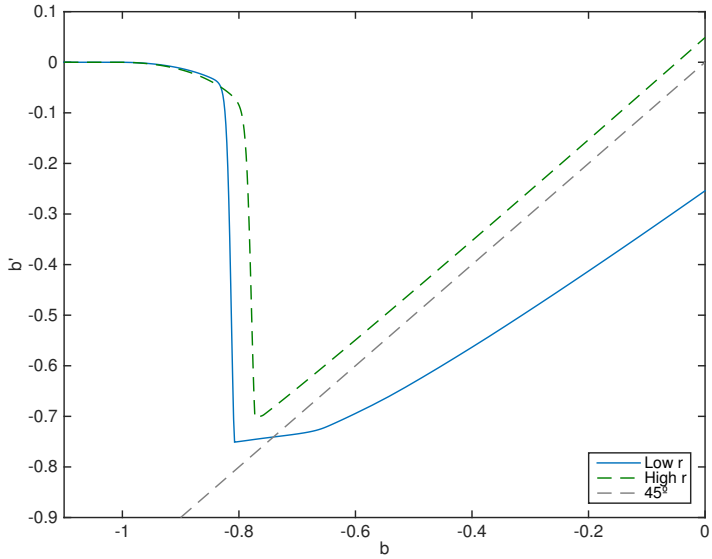


become more indebted is lower around this region: The slope of the savings rule slowly decreases as the level of debt increases before hitting the borrowing constraint. Hence, the precautionary motive gains an increasing importance vis-à-vis the impatience motive in the households' problem.

In Figure 2, we compare the savings decision rule for two different levels of the contemporaneous endowment. When there is a high level of output in the period (green dashed line), there tend to be greater savings from the households in the region where borrowing constraints do not bind. This effect occurs because households wish to smooth consumption across time, and because the process for the endowment is mean reverting, it is likely that in future periods there will be a lower output than in the present. However, because the endowment process is persistent, a high level of contemporaneous output predicts high levels of output in the near future, which in turn increases the value of the Lucas tree for the household. This rise causes an increase in the value of the collateral available in the economy, which raises the borrowing capacity of the households. Hence, the borrowing constraint starts binding at higher levels of debt, as the green dashed line shows.

Figure 3 compares the savings decision rules for two different levels of the external interest rate. Away from the borrowing constraint, the interest rate has the usual effect on the economy: When the country faces a higher cost of borrowing (green dashed line), it tends to increase its savings. However, in the vicinity of the borrowing constraint, changes in the interest rate have an additional effect: An increase in the interest rate

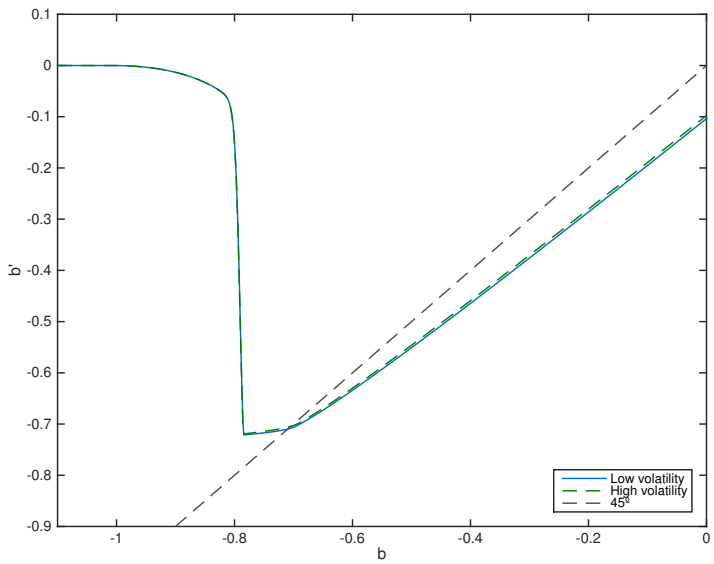
Figure 3: Savings rule: different interest rates



causes a decline in the stochastic discount factor (in expectation), which in turn reduces the value of the tree because its future flows are discounted more heavily. Hence, when the country faces higher interest rates, the value of collateral is lower and the borrowing constraint starts binding for lower levels of debt.

In Figure 4 we compare the decision rules in the economy for two different levels of the variance of the external interest rate, σ^r. We keep the level of interest rates constant and compare the decision rules for the two levels of such variance. The figure shows that the savings rule for the high volatility state lies slightly above the one for low volatility. This outcome was expected because the households should have a higher precautionary saving motive when they face a world with higher uncertainty. Nonetheless, the magnitude of the difference between both decision rules is considerably small, so these shocks do not modify the household's saving substantially. It is important to highlight that the direct effect of more volatile interest rates is different than the direct effect of more volatile income shocks. While the precautionary motive that arises as a result of more volatile shocks to the dividend process is clear, the effect induced by more volatile interest rates affects the hedging properties of bonds. If the interest rate is more volatile, bonds become a worse hedging instrument against dividend shocks, thus leading to a decline in the demand for shares of the tree and, therefore, in their price. As shown in Figure 4, this mechanism does not have significant effects on private agents' borrowing decisions. However, notice that the change in the equilibrium price had an effect on the collateral that is not internalized by private agents, but that plays

Figure 4: Savings rule: Different levels for the variance of the interest rate

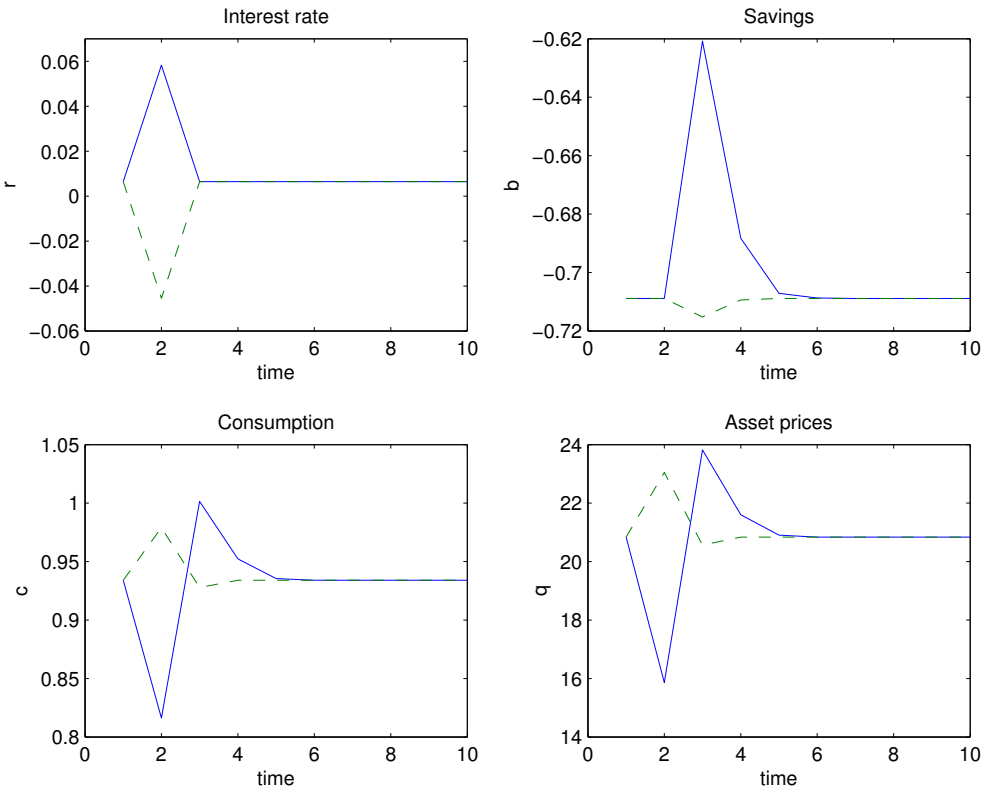


a key role for the decision of the planner.[33]

We now turn to analyzing the asymmetries generated by the nonlinear dynamics of the economy, given shocks to the level of external interest rates. This feature of this type of model plays a relevant role in allowing the model to generate dynamics around sudden stops that are in line with empirical evidence. In Figure 5 we show our simulated impulse-responses around the steady state where the economy would remain if the level of output from the tree remained permanently constant at two standard deviations below its mean, the interest rate remained at 0.6%, and the variance of the interest rate remained permanently at 6.6%, that is, in the high volatility regime.

[33]In our model, the small effect of external volatility on equilibrium allocations arises from the absence of a complete production economy with capital accumulation. Fernández-Villaverde et al. (2011) include capital accumulation in an open economy facing shocks to the volatility of the external interest rates and show a significant response of real activity to these shocks. They also highlight the importance of external debt as a hedge against domestic income shocks and show that the real effects are mainly due to changes in investment decisions. Because most of the risk in households' consumption arises from shocks to the domestic productivity, the external locally-risk-free debt is a good hedge against domestic risk. However, when the rollover risk of external debt increases, foreign bonds are less useful as a hedge, which implies that the economy must cut on their holdings of capital to reduce exposure to domestic risk. The decline in investment causes a decrease in future output which reduces wealth and induces a reduction in consumption and foreign debt. Because we are interested in isolating the policy response of a planner that focuses on the incidence and severity of crises caused by collateral constraints, and given the complexity involved in solving for the time-consistent constrained efficient allocations, we do not incorporate capital accumulation and production in our framework. By introducing investment as in Fernández-Villaverde et al. (2011), we would be potentially increasing the planner's incentives to engage in ex ante and ex post interventions to reduce the incidence and severity of crises. We decide to leave this endeavor for future research.

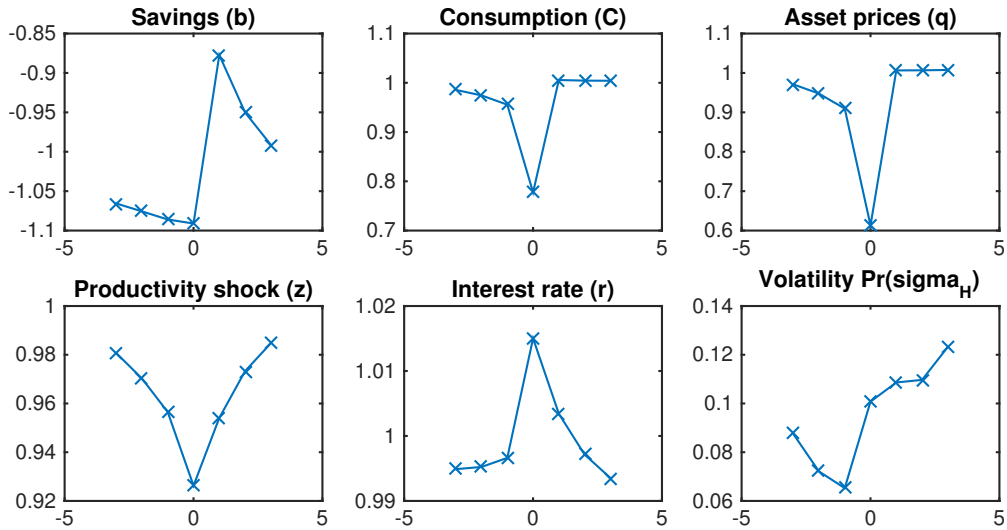
Figure 5: Impulse-response functions



We then give a ±5.2% shock to the interest rate for one period and bring the interest rate to 0.6% thereafter. The idea of this exercise is to simulate a scenario in which changes in interest rates occur during turbulent times, similar to what many EMEs have experienced in distinct occasions. First, we explain the effects of the interest rate decrease on the rest of the economy (green dashed line). The immediate effect is an incentive for the households to consume in advance. Therefore, they increase their consumption 4.8% in the first period, without significant changes in the net savings of the economy. Asset prices show a 10.6% increase in the first period because the households are discounting future cash flows less, but they revert close to their long-run level in the following period.

In contrast, the economy responds very differently to an increase in the interest rate of the same magnitude. The immediate effect of the shock is a decline in asset prices, as shown in the last panel of the figure (blue solid line). The decline in the value of collateral causes the borrowing constraint to bind for a period, which forces a reduction in consumption in order to cut off the level of debt. As mentioned before, the

Figure 6: Event studies of binding borrowing constraints in competitive equilibrium



feedback between deleveraging and the decline of asset prices amplifies the initial shock: Consumption initially falls 12.6%, and asset prices drop 23.9%. This response carries a sharp reduction in foreign debt: It goes from 70.9% of average output to 62.1% in just one period. In addition, as the graphs show, the sudden deleveraging has long-lasting effects: Given that there is a lower level of debt, asset prices remain high because there is a low probability of hitting the borrowing constraint again in the near future. Moreover, because the country has accumulated more savings, the household increases its consumption in the subsequent periods because it remains relatively impatient with respect to the rest of the world, until the stock of debt converges back to its long-run level. This exercise exemplifies the nonlinear and asymmetric dynamics of the model that arise from the presence of an occasionally binding borrowing constraint.

We simulate the model for 100,000 thousand periods to study the prevalence of binding borrowing constraints and their effects around these events. We find that in our baseline parameterization, a binding borrowing constraint is a rare event: It takes place in only 1.82% of the periods. Even in the periods preceding the actual occurrence of a binding constraint, the model assigns conditional probabilities to this event below 10% on average.

In Figure 6, we present event studies by averaging the equilibrium variables around the period in which the borrowing constraint binds. All the variables are divided by their average value in "normal times," that is, in periods in which the borrowing constraint is non-binding. The only exception is the window for interest rate volatility,

which shows the fraction of episodes in which the high volatility regime is prevailing. Each panel shows the normalized average of the variable from $t-3$ to $t+3$, where t is the moment in which the borrowing constraint binds. In the first panel, we see that binding constraints arise from periods in which the economy has a relatively large stock of debt: The average level of debt before sudden stop periods is almost 10% higher than the average debt in non-binding periods. In the panels of the second row, we can see that binding borrowing constraints are typically accompanied by low levels of the endowment, z, and drastic increases in the interest rate, r.

To contrast our model with the empirical evidence, we follow the literature in associating a period in which a borrowing constraint binds in the model with the occurrence of a sudden stop in the data. From this perspective, the prevalence of sudden stops in the model is considerably lower than in the data. Under the typical definition of sudden stops considered in the literature, including in Ricardo Reyes-Heroles and Tenorio (2017), the prevalence of these episodes in the sample of EMEs lies between 14.6% and 15.21% of the periods (measured in months), depending on the countries considered.

Nonetheless, the evolution of the modeled economy around sudden stops is consistent with the empirical evidence presented in Ricardo Reyes-Heroles and Tenorio (2017) regarding the dynamics of the external interest rate. Both in the model and in the data, a sudden stop is associated with a sharp increase in the interest rate: The model predicts that sudden stops happen when the interest rate increases, on average, 1.5 percentage points with respect to the normal times' mean, whereas in the data, the interest rate increases between 1 and 2 percentage points in the 12 months that follow the beginning of such episodes. In addition, the model predicts that sudden stops take place after periods of relatively low interest rate volatility, in the moment in which volatility switches to the high regime, allowing for large upward shocks in the level of the interest rate. Again, this pattern is consistent with the sudden rise in volatility in the year of the sudden stop that we observed in the data.

The fourth panel of Figure 6 shows that a binding borrowing constraint is typically preceded by a sequence of negative output shocks and an abnormally large negative shock in the period in which the constraint binds that brings the level of output almost 8% below its normal times level. These dynamics contrast with the empirical evidence in two respects. First, sudden stops are typically preceded by economic expansions, of around 1% in the sample studied by Ricardo Reyes-Heroles and Tenorio (2017). Second, the empirical output declines after the episode begins are relatively modest, of around 2% relative to its normal times' level. In terms of consumption and asset prices, the

dynamics of the model agree with the empirical patterns of balance of payment crises: These are usually accompanied with sharp declines in consumption and asset prices. However, the fall in consumption that arises in our model, of about 20% below the normal times level, is considerably higher than its empirical counterpart of about 2% or 3% in the countries studied by Korinek and Mendoza (2013).[34]

3.3 The constrained efficient allocation and optimal capital flow management

We now turn to the numerical analysis of the constrained efficient allocation. We maintain the same parameterization as in the previous section to characterize quantitatively the solution to the planner's problem. In this section, we follow Jeanne and Korinek (2010) in postulating that the following condition holds:

Assumption The parameters and stochastic processes of the economy are such that the equilibrium pricing function satisfies

$$1 + \kappa R(X)\xi(B, X) > 0.$$

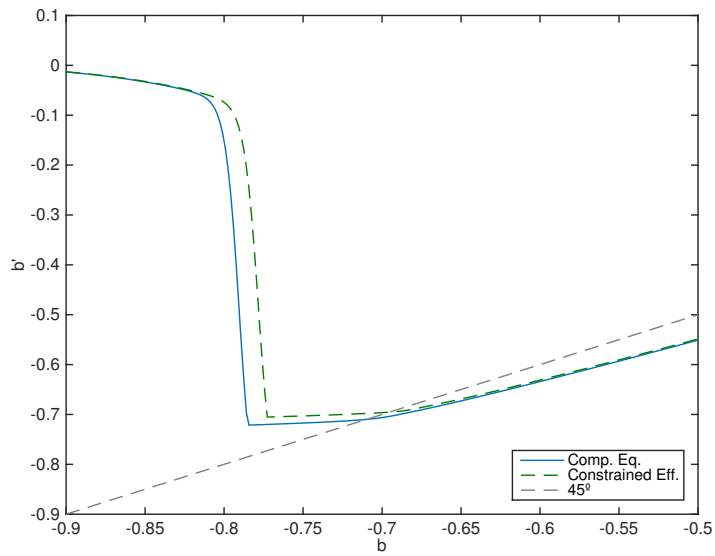
Given our interest in macroprudential policies and how to take actions to prevent and minimize future crises, this condition allows us to delimit our analysis of the effects of the pecuniary externality on future prices rather than on current ones and their implications for time-inconsistency problems. This assumption provides a shortcut to generate time-consistent policies and guarantees that there exists a unique level of future savings in the planner's problem, B', for which the collateral constraint holds with equality. If this condition were not true, it could be the case that an increase in household debt relaxes the constraint by increasing the value of collateral.[35] This outcome is in principle a counterintuitive, but it is possible to have a negative derivative of the $\bar{Q}$ schedule of equation (7) with respect to B' because of to the concavity of the utility function (see Appendix A.3 for an expression of $\xi(B, X)$ based on marginal utilities and equilibrium objects). Jeanne and Korinek (2010) prove that under this assumption, the Euler equation for the planner's problem (8) simplifies to

$$u'(\mathcal{C}(B, X)) - \mu(B, X) = R(X)\beta\mathbb{E}\left[u'(\mathcal{C}(B', X')) + \kappa\mu(B', X')\psi(B', X')\right].$$

[34] However, the drop in consumption in our model is for the marginal household.

[35] Bianchi and Mendoza (2013) provide a detailed explanation of the implications of a similar model without imposing this assumption.

Figure 7: Savings rule: Constrained efficient allocation versus competitive equilibrium



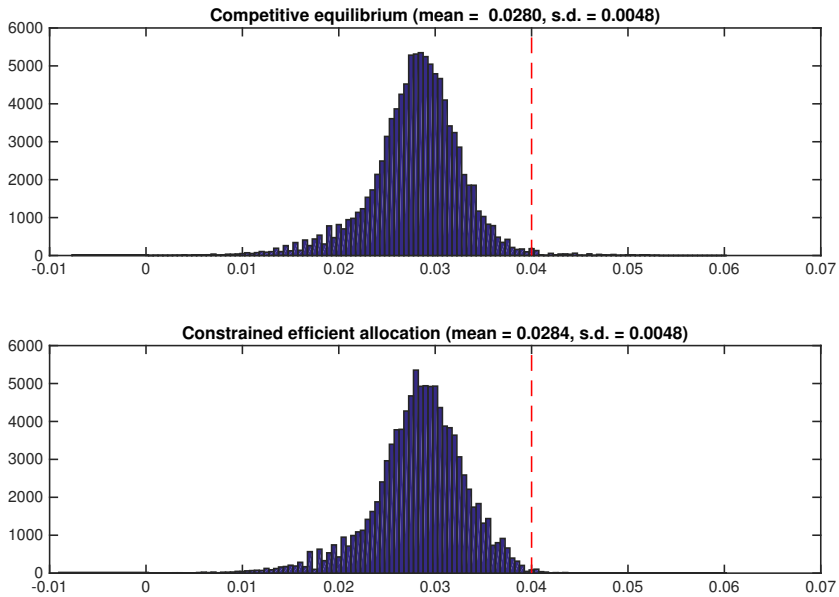
For the remainder of this section, we describe the optimal decision rule of the planner, and the associated equilibrium outcomes, based on this version of the Euler equation.

Figure 7 compares the savings rules for the households in the competitive equilibrium and the solution to the planner's problem. As Bianchi and Mendoza (2013) note, the savings rule in both problems are similar in most of the state space, but they differ considerably in what they call the "high externality region," where the borrowing constraint has a high probability of binding and the asset price schedule becomes steeper as a function of savings.

Even though the savings rules do not show large differences between the competitive equilibrium and the planner's problem, there are indeed important differences in the dynamics of both problems, rather than on steady-state outcomes. For instance, we find that the planner is able to reduce the frequency of sudden stops from 1.82% of the periods in the competitive equilibrium to 1.61% in the constrained efficient allocation. However, as we observe in Figure 8, the amount of leverage in the planner's economy does not change considerably with respect to the competitive equilibrium. Here we define leverage as the discount value of debt divided by the market value of the Lucas tree, $-b_{t+1}/R_t q_t$. The red line marks the level of leverage where the borrowing constraint binds, given by $\kappa = 0.04$ in our numerical example. Both histograms of leverage have a similar mean of around 0.028 and the same dispersion of 0.0048.

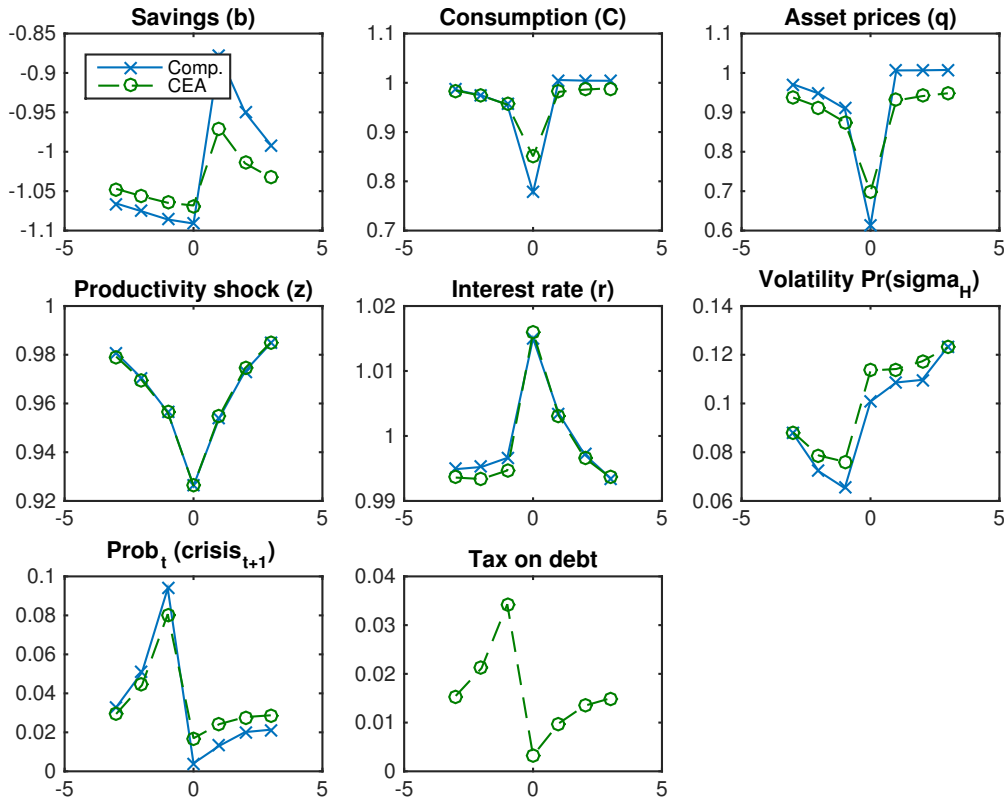
Nevertheless, the planner's actions do have an effect in the severity of the sudden

Figure 8: Histograms of leverage



stops that the economy faces. In Figure 9, we show event studies around the periods in which the borrowing constraint binds in the planner's economy. The outcomes corresponding to the planner's problem are depicted by green dashed lines. We observe that the consequences of a binding constraint are considerably milder in the planner's allocation compared to the competitive equilibrium: Consumption decreases by less, asset prices remain higher, and deleveraging is slower. Even though the average decline in the endowment is roughly the same in both economies, it takes a larger positive interest rate shock to hit a borrowing constraint in the constrained efficient economy. This shock is accompanied by a sudden increase in volatility that enables the interest rate shock to reach high realizations. One of the key features of the policy associated with the constrained efficient allocation is that it depresses asset prices in order to achieve its objective of decreasing the incidence and severity of crises. This feature also holds in Bianchi and Mendoza (2013). By doing so it incentivizes households to decrease their leverage and increase savings. This feature will play a crucial role when we analyze the response of optimal policy to external shocks.

Figure 9: Simulated sudden stops in constrained efficient allocation



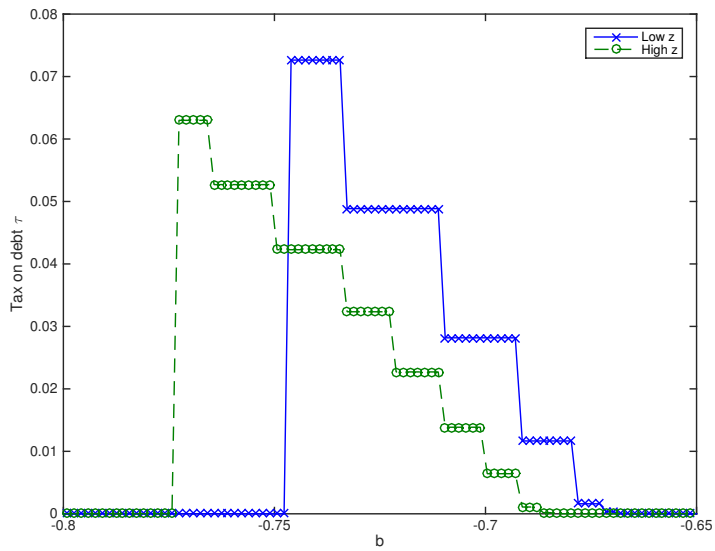
3.3.1 Decentralization

We now explore how the planner responds to the exogenous shocks that the economy faces. We will focus on analyzing the state contingent macroprudential tax on debt that decentralizes the constrained efficient allocation.[36] By comparing the equations that characterize the solution of the competitive equilibrium and the planner's problem it can immediately be seen that the wedge on the households' gross interest rate that implements the allocation of the planner's problem is

$$\tau(B, X) = \frac{\mathbb{E}[\kappa\psi(B', X')\mu(B', X')|X]}{\mathbb{E}[u'(c(B', X'))|X]}, \tag{11}$$

[36]There are other policy instruments that can decentralize the constrained efficient allocation. Bianchi and Mendoza (2013) provide some examples of such instruments; however, they focus on a tax on debt and we do the same here because we believe it provides a very intuitive perspective on how to influence capital flows across countries.

Figure 10: Tax on debt as a function of savings: Different endowment levels

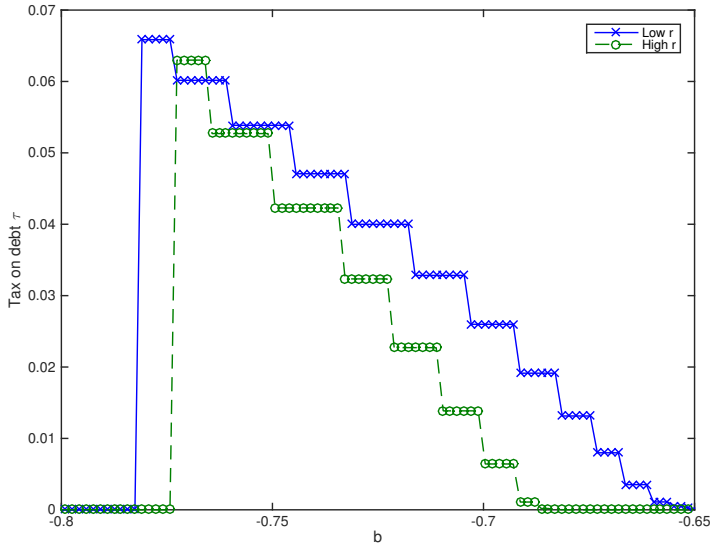


where $B' = \Psi(B, X)$ is the optimal level of savings chosen by the planner when initial savings are B and shocks X are realized. From equation (11) we see that the size of the planner's intervention is determined by the expected marginal welfare gain of reducing households' indebtedness: The value of reducing households' debt by a unit is equal to the increase in the value of collateral, $\kappa\psi(B', X')$, times the marginal value of relaxing the collateral constraint, $\mu(B', X')$.

In Figure 10 we depict the optimal tax on debt, $\tau(B, X)$, as a function of the initial savings of the country, B, for two different levels of the endowment shock. Notice first that, independently of the endowment level, high levels of saving (to the right of the graph) imply that the borrowing constraint is less likely to bind, which makes the planner's intervention small or even null. Then, as debt starts accumulating, we see that the size of the tax increases. Two things happen: (i) The borrowing constraint is more likely to bind, and it becomes tighter, which derives in a higher multiplier $\mu(B', X')$, and (ii) the severity of the pecuniary externality, $\psi(B', X')$, declines. It turns out that the effect of a greater incidence of a potential future crisis dominates the effect of the decline in its severity as debt increases, which is reflected in a higher macroprudential tax.[37] In our numerical example, the tax rate amounts to a few percentage points over the gross interest rate, which considerably increases the after-tax interest rate paid by

[37] We turn to a more detailed analysis of the forces shaping the tax in the next subsection when we proceed to decompose it.

Figure 11: Tax on debt as a function of savings: Different interest rates



the households. In the figure, we also see that for higher levels of debt, the borrowing constraint binds and the households are forced to decrease their leverage drastically by the price-debt mechanisms of the model. This deleveraging brings the stock of debt away from the borrowing constraint for the immediate future. In this case, the tax on debt is zero because the economy is not constrained in the upcoming period. Thus, this model has no space for ex post intervention; the planner's actions to eliminate pecuniary externalities are only necessary before a borrowing constraint binds.[38]

We now analyze how the planner's intervention responds to endowment shocks. The solid blue line in Figure 10 depicts the optimal tax on debt as a function of households' savings for a low realization of the endowment shock, z. In the region where borrowing is unconstrained independently of the endowment shock, the tax on debt is always higher for lower realizations of the endowment, which is explained by the fact that low levels of dividend reduce the value of the Lucas tree, which in turn decreases the value of collateral available and increases the probability of a binding borrowing constraint in the near future.

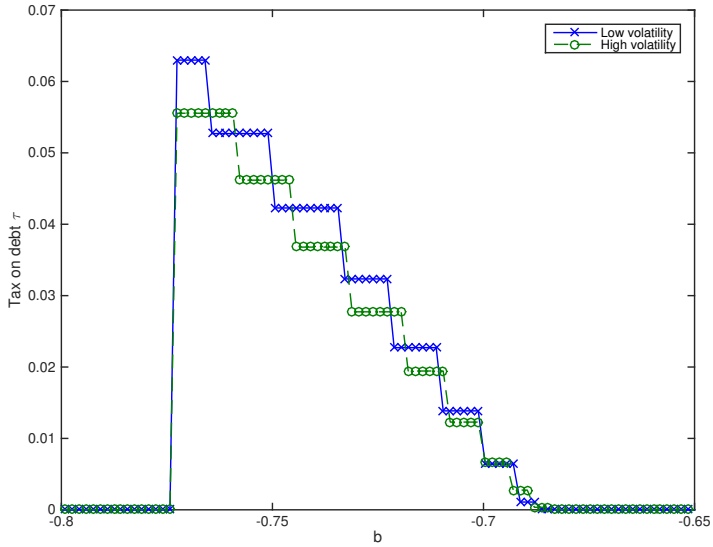
[38]See Benigno et al. (2013b). Figure 10 also shows that the macroprudential tax is always non-negative. The planner taxes the "overborrowing," thus expecting to have positive effects on welfare through the internalization of the price externality. From equation (9), we also see that the size of the pecuniary externality must be non-negative because the utility function is strictly increasing and concave, and asset prices are non-negative throughout the state space. In addition, the effect of relaxing the collateral constraint is non-negative, because it necessarily increases welfare when the constraint binds, and has a null effect otherwise.

Let us now turn to the analysis of the optimal tax for different levels of the external interest rate.[39] Figure 11 shows the dependence of the optimal macroprudential tax on interest rate shocks. The first thing we observe is that the macroprudential tax is uniformly lower for high levels of the interest rate for most levels of debt, which is consistent with the findings of Jeanne and Korinek (2010). The authors make a comparative statics exercise of how the macroprudential tax changes with different values of the external interest rate. In their exercise, the interest rate is a fixed parameter in the planner's problem, and they compare the steady state value of the tax when the value of the endowment is kept constant. The authors find that the steady state level of the macroprudential tax is decreasing with respect to the external interest rate: As the interest rate increases, the planner has a lower need to reduce households' borrowing because they do so themselves as a response of a higher cost of credit and lower value of collateral. Our analysis, in contrast, studies the response of entire schedule of taxes—as a function of the endogenous state variable, debt—to shocks to external interest rates. Figure 11 shows that for some levels of debt, the optimal tax after a shock leading to a high interest rate is larger than the one corresponding to a low interest rate. The intuition behind this result is explained by the fact that higher interest rates depress asset prices and reduce the value of collateral, which increases the probability of a borrowing constraint binding and calls for a larger intervention. This result is also in contrast to the one in Bianchi et al. (2016) who show that, in their environment, the optimal macroprudential tax schedule for a low world interest rate lies above the one corresponding to the high interest rate.

Next, we study whether an increase in the volatility of the external interest rate calls for a larger intervention by the planner, that is, a higher tax. Figure 12 depicts the schedule of tax on debt as a function of household savings for the two different regimes of interest rate variability. Two main conclusions can be drawn from the effect of shocks to interest rate volatility on the planner's problem. First, the planner does indeed have a volatility-contingent optimal policy. This contingency contrasts with the result that the saving rule in the competitive equilibrium does not differ considerably across high and low volatility states (see Figure 4). These two results together imply that shocks to volatility affect the incidence and severity of crises mainly through their effect on the pecuniary externality rather than through their implications for refinancing risks

[39] This analysis is similar to the one by Bianchi et al. (2016), and our results are partially consistent with theirs, as discussed below. However, a crucial difference is that they consider the case in which the real exchange rate is the relevant price driving the pecuniary externality. In addition, they focus in shocks to global liquidity interpreted as regime switches in external interest rates. We consider the case in which the external interest rate follows an AR(1) process with stochastic volatility.

Figure 12: Tax on debt as a function of savings: Different variances of the interest rate

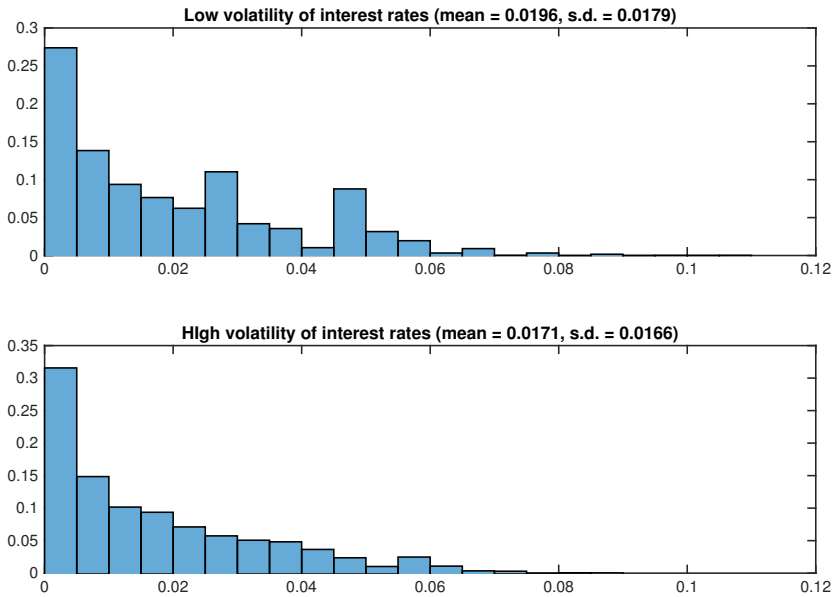


and private decisions. In the constrained efficient allocation, the planner's policy is affected by the volatility of interest rates because this volatility affects asset prices in the future which have implications for how likely the economy is to be in states in which the borrowing constraint binds.

A second result that can be drawn from Figure 12 is that the size of the optimal planner's intervention is non-monotonic with respect to the volatility of the interest rate. Figure 12 shows that for certain levels of savings, the planner intervenes more when the volatility is high, but for other levels of savings the planner has a smaller intervention. This difference follows from the fact mentioned in the previous paragraph that the planner is weighing two criteria while choosing the optimal tax on debt: the Incidence of sudden stops and the size of the pecuniary externality. In the following section we show that the interaction between these two factors shapes the response of the planner to volatility shocks.

Before turning to a detailed numerical analysis of the forces that shape the non-monotonicities across external shocks, we study whether the non-monotonic effect of volatility on taxes is apparent in the simulated economy and which direction of the response to shocks in external volatility prevails. We find that the share of states in which the planner chooses a zero tax on debt is larger when there is high variance than low variance: The planner sets a tax of zero in 59.6% of the periods of high volatility, versus 55.3% of low volatility periods. The difference is in part due to the

Figure 13: Histograms of tax on debt conditional on variance states



fact that the economy is more likely to be hit by very high interest rates when the variance is high, and in those states the planner is unlikely to intervene either because households increase savings or because the collateral constraint binds. In Figure 13 we look at the ergodic distribution of the tax on debt, conditioning on low and high volatility states, and ignoring the periods of zero intervention. We see that the positive interventions in the low volatility state have an average of 1.96%, which is larger than the average positive intervention in high volatility periods of 1.71%. Moreover, the highest interventions in our simulations reach 10.7% and take place only in the low volatility state. In contrast, the highest intervention in the high volatility state is 8.92%. This last result is in line with Figure 12, which shows that the macroprudential tax on debt reaches a higher maximum around high levels of indebtedness for the low volatility state.

Finally, we go back to Figure 9 and observe in the last panel the evolution of the macroprudential tax around the occurrence of a sudden stop. We find that prior to hitting the borrowing constraint, the planner charges, on average, a tax on debt of around 3.5%, which significantly raises borrowing costs for households because the average interest rate they face is just 1.96%. Nonetheless, as we previously discussed,

the planner does not engage in ex post macroprudential policies: The tax on debt when the borrowing constraint binds is close to zero, given the fact that there is a fast deleveraging taking place that makes it unlikely for a subsequent period to observe a binding borrowing constraint. Therefore, there is no motive for the planner to intervene during the period right after the borrowing was binding.

3.3.2 Decomposition of the optimal policy

In this section we study in further detail the planner's response to the different shocks in the modeled economy. The results of our numerical exercises show that changes in the macroprudential tax due to exogenous shocks arise mainly because of differences in the numerator of (11)—the denominator remains fairly constant across different states as a result of to the planner's desire to smooth consumption. Therefore, in this section, we decompose the numerator of the macroprudential tax into a product term and a covariance term in order to better understand the responses.

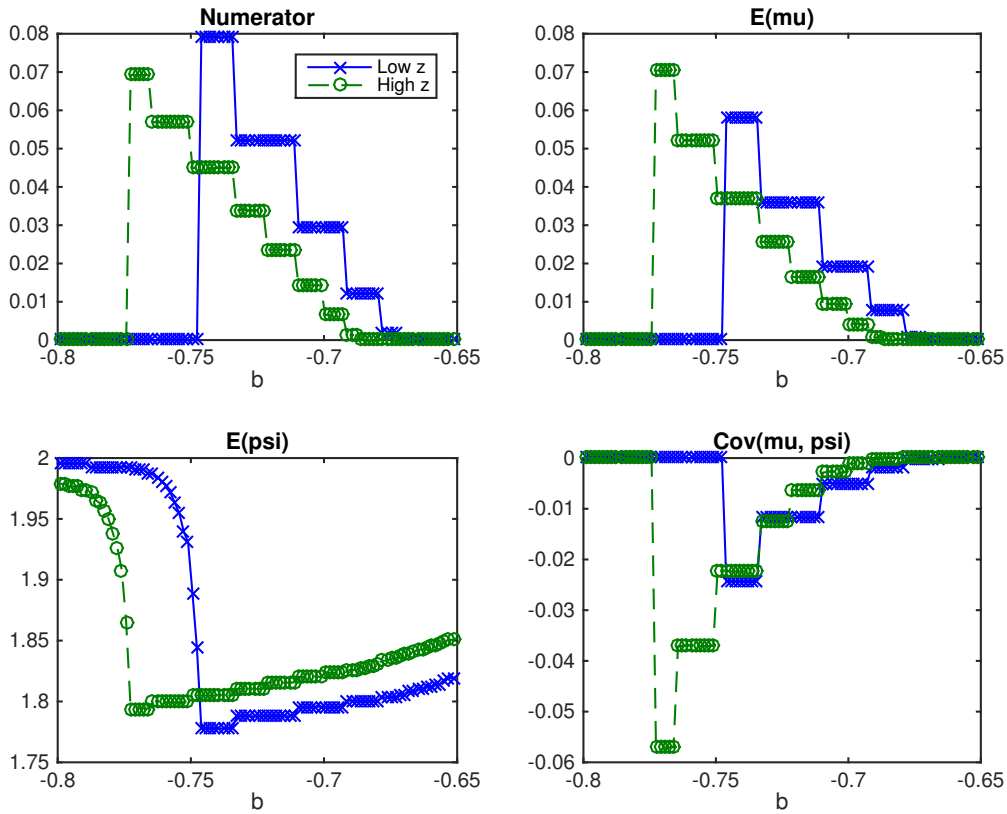
Notice that the denominator of (11) can be decomposed as follows:

$$\mathbb{E}[\kappa\psi(B', X')\mu(B', X')] = \mathbb{E}[\kappa\psi(B', X')] \cdot \mathbb{E}[\mu(B', X')] + Cov(\kappa\psi(B', X'), \mu(B', X')),$$

where all the moments are conditional on the contemporaneous vector of shocks, X. Thus, the planner's intervention is larger when (i) it expects a higher likelihood of a binding constraint and a greater stringency when it does, as manifested in the expectation of $\mu(B', X')$, which we defined as the *incidence* of the crisis; (ii) it expects the size of the pecuniary externality taking place in the following period to be large, as manifested in the expectation of $\kappa\psi(B', X')$, which we defined as the *severity* of the crisis; or (iii) when the planner expects these two factors to be highly correlated in the next period.

The intuition underlying the first two effects is clear, but the covariance term adds a level of complexity to the implementation of the optimal macroprudential tax. The rationale of how the last term affects the tax is also straightforward in the sense that whenever high probabilities are assigned to states in which both the collateral constraint binds and the externality is very large, then the planner should increase the tax on capital flows. Thus, the larger the conditional covariance between the pecuniary externality, $\kappa\psi(B', X')$, and the shadow valuation of the borrowing constraint, $\mu(B', X')$, the larger the planner's efforts will be to reduce the households' borrowing by increasing the tax on debt. As we explain in detail below, the model generates a negative covariance term. Hence, an effective implementation of taxes to increase household welfare should take

Figure 14: Decomposition of the tax on debt as a function of savings: different endowment levels



into account this negative covariance and not reduce borrowing as much as when the incidence and the severity of the externality are seen as independent.

Figure 14 shows the decomposition of the numerator of $\tau(B, X)$ for two different levels of the endowment shock. The planner intervenes more after low endowment realizations because the collateral constraint now binds for lower levels of indebtedness. This issue arises because asset prices decline persistently given the persistence of the shock, which translates into a higher probability assigned to states in which the borrowing constraint binds in future periods and, therefore, an increase in the incidence of a future crisis, $\mathbb{E}_t\left[\mu_{t+1}\right]$. The effect of the severity of the externality, $\mathbb{E}_t\left[\kappa\psi_{t+1}\right]$, goes in the opposite direction precisely because asset prices drop persistently, but consumption does not drop as much.[40] The fourth panel in Figure 14 shows that the covariance

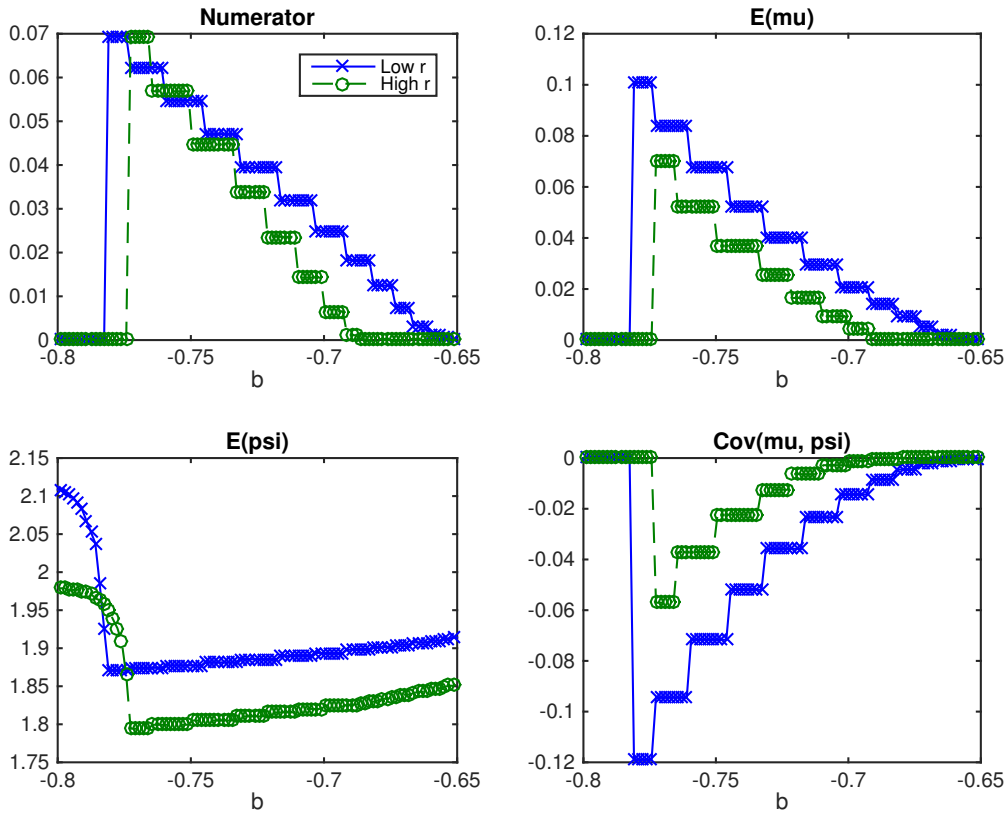
[40]This effect can be appreciated in (9) and the fact that households borrow to smooth consumption, and is precisely the reason why changes in the numerator of the tax explain the changes in the tax as a whole.

effect is negative, that it increases in absolute value for higher levels of indebtedness, and that it does not respond significantly to endowment shocks. The negative sign of the covariance term reflects the fact that states in which crises occur are also those in which asset prices are depressed, thus reducing the severity of the pecuniary externality in these states, as can be appreciated from 9. The covariance term increases in absolute value as the economy is more indebted because a higher debt in the present increases the number of states in the future in which the collateral constraint can bind for certain combination of shocks, thus leading to crisis states together with significantly depressed prices being more likely, that is, higher incidence of a future crisis together with a larger expected drop in prices. The covariance does not vary substantially across endowment levels precisely because it reassigns probabilities across crisis and no-crisis states, but the relationship between μ_{t+1} and ψ_{t+1} remains, as for the other level. Hence, the effect of binding borrowing constraints, $\mathbb{E}_t\left[\mu_{t+1}\right]$, is the one driving the increase in the macroprudential tax after low endowment realizations.

In Figure 15 we perform a similar exercise and decompose the planner's tax on debt for two different levels of the interest rate shock. We want to understand why the planner's intervention is lower when interest rates increase. From the figure we see that both the incidence of future crises and the severity of the associated externalities are expected to be lower when the interest rate increases. The latter effect is the result of a decrease in present and expected future asset prices due to the persistence of the positive shock to interest rates. The change in expected future prices is directly reflected on the severity of the externality in a crisis, as can be seen in (9). With respect to the incidence of the crisis, the increase in present and expected future interest rates incentivizes households to save more and decrease their leverage. These actions reduce the probability of a crisis in the future and its incidence by reducing $\mathbb{E}_t\left[\mu_{t+1}\right]$. These two effects lead households facing a positive shock to the world interest rate to increase savings and decrease consumption marginally, thus leading to a decrease in asset prices, which implies that the collateral constraint binds for lower levels of leverage.

However, notice that covariance term counteracts these two forces and reduces the difference between the two policy schedules. In other words, the covariance between these two effects is higher (or less negative) when the interest rate takes on high levels. Even though over most levels of aggregate savings the two effects dominate the size of the tax, thus leading to lower taxes for high interest rates, the covariance when the interest rate is low is substantially more negative than when the interest rate is high for high levels of indebtedness. This effect implies that for states in which the economy is highly indebted, it is optimal to increase capital controls, given an increase

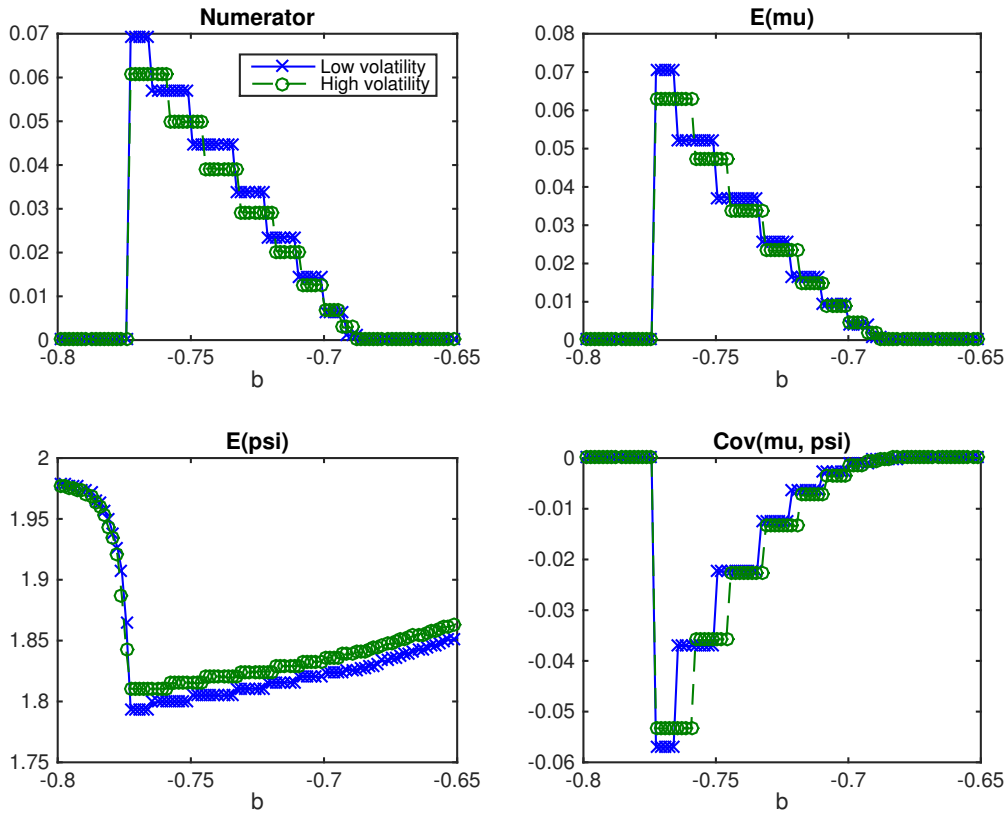
Figure 15: Decomposition of the tax on debt as a function of savings: different interest rates



in interest rates. The increasing difference across interest rates in the covariance terms as the level of indebtedness increases arises from the fact that for high interest rates, asset prices are depressed relative to the case with low interest rates. The depressed prices in turn imply that the size of the externality is even smaller the closer we get to the crisis zone (depressing prices even more), even though the incidence of future binding collateral constraints increase. Therefore, for very high levels of indebtedness it is optimal to increase capital controls for higher interest rates because depressed future prices make the collateral constraint much more likely to bind in the immediate future, before households actually decrease indebtedness.

Finally, in Figure 16, we present the decomposition for the two different levels of interest rate volatility. Disentangling the effects in this case is more complicated because, as can be appreciated in the figure, only the severity of future crises exhibits a uniform difference in the tax for each volatility regime at all levels of indebtedness. However, the incidence of crises and the covariance term show non-monotonic responses

Figure 16: Decomposition of the tax on debt as a function of savings: Different variances of the interest rates



to volatility shocks that lead to the non-uniform response of the macroprudential tax.

With respect to the severity of the pecuniary externality, its greater expectation under the high volatility regime arises from the fact that high volatility leads to lower consumption for all levels of indebtedness. The intuition behind this result comes from the fact that debt becomes riskier when the interest rate is more volatile and, therefore, households want to hold less debt. To do so, households must decrease present consumption.[41] Households would, in principle, like to smooth the drop in consumption by selling some of their asset shares, but this sell leads to a sharp decline in current asset prices (given that their supply is perfectly inelastic) and, therefore, in income. Even though asset prices also fall, the decline in current consumption is greater for

[41] The fact that consumption today must be lower for high volatility can be appreciated in the equilibrium condition $\frac{1}{R_t} = \beta \frac{\mathbb{E}_t[u'(c_{t+1})]}{u'(c_t)}$. Higher volatility in world interest rates leads to an increase in $\mathbb{E}[u'(c_{t+1})]$ by Jensen's inequality, and, given that R_t does not change, then c_t must decrease in order for $u'(c_t)$ to increase and make the equality hold.

higher volatility. The greater decline has to hold given that $\mathbb{E}_t[\kappa\psi_{t+1}]$ is higher for high volatility and given our assumption on preferences together with the fact that ψ_t is as in (9).[42]

The factor driving the non-monotonicity of the macroprudential tax is the incidence of the externality, $\mathbb{E}_t[\mu_{t+1}]$. Hence, now we focus on understanding this factor.[43]To understand the mechanism driving this non-monotonicity and its intuition, let us focus on the region of the state space in which the tax has reached its maximum for the high volatility regime, $b \in [-0.775, -0.076]$, which is also closest to the crisis level of debt.

For levels of debt furthest from the crisis region, high volatility calls for an increase in taxes. The intuition behind this response in optimal policy is easy to grasp and in line with common wisdom because of two mechanisms. First, the increase in volatility generates an increase in the probability of facing a much higher interest rate in the future, thus leading to greater difficulty in refinancing current debt and a higher likelihood of a binding collateral constraint and a crisis in the future. Hence, more volatile capital flows lead to an increase in the incidence of crises. Second, as explained in the previous paragraph, the severity of the pecuniary externality increases for high levels of volatility where consumption is already depressed, given that debt is no longer a good hedge against income shocks. However, the intuition underlying this response is correct only for the levels of debt furthest from the crisis region. Notice that there is a threshold level of debt such that for higher levels of current debt, the planner should set a higher tax given a current state of low volatility in external interest rates. This change in policy emerges from the fact that for levels of debt very close to the crisis region, the change in households' decision to acquire more debt for lower volatility—independently of the amount of additional debt—leads to a large increase in the incidence of the externality that counteracts the decline in the severity of the externality. Consider as an example a highly indebted country under a high volatility regime in which the optimal tax is implemented. Now suppose there is a shock and the volatility levels decrease. If households are already highly indebted, the small amount of extra debt

[42]Notice that the intuition of why asset prices fall relies on the assumption that demand for asset shares declines so as to smooth the current drop in consumption. However, it could be the case that households increase their demand for assets as a precautionary motive to insure against the large shocks to external interest rates and the possibility of hitting the collateral constraint. For our parametrization, the first effect dominates and asset prices are lower for high volatility. This drop in prices is in line with the intuition and parametrization in Fernández-Villaverde et al. (2011), in which investment falls after a positive shock to volatility.

[43]Notice thta changes in this schedule are augmented because they are being multiplied by $\mathbb{E}_t[\psi_{t+1}]$ which is close to 2. This expansion makes these changes more relevant than the corresponding changes in the covariance term.

that households would be willing to acquire can potentially lead to a very large increase in the incidence of a future crisis precisely because households do not internalize the pecuniary externality. Notice that this threshold does not exist for the high volatility regime precisely because this regime leads to precautionary savings that dampen the incidence of a future crisis.

The decomposition of the optimal tax on debt shows that the relations between external shocks and the incentives on the planner's problem are complex. The dynamics of the incidence and severity of crises are determined in general equilibrium and in response to forward-looking factors, and the ultimate policy prescriptions depend on the different forces acting in the economy. One important conclusion derived form the previous exercises is that simple policy prescriptions based on partial equilibrium rationales are insufficient to internalize the effect of "overborrowing" on asset prices and households' borrowing capacity, and they might lead to unintended consequences.

4 Conclusion

The increase in size and volatility of international capital flows in recent years carry inherent risks. The uncertainty regarding policy actions in industrialized economies as well as other underlying institutional and financial risks have made the timing and direction of capital flows unpredictable. Policy makers around the world have grown concerned about the potential consequences of sudden reversals over their domestic financial sectors and ultimately on real economic activity. These concerns has motivated a myriad of unconventional policy tools to moderate the movement and regulate the composition of capital flows across borders. The international community has recognized that the risks carried by the volatility of international flows call for a more thorough analysis of the design, but especially of the implementation, of macroprudential policies (see IMF, 2012). This work contributes to our understanding of the implementation of this type of policies in an economy prone to sudden stop episodes in the face of external risks.

We extend the small open economy framework of Jeanne and Korinek (2010) and Bianchi and Mendoza (2013) to include shocks to the level and volatility of the interest rate faced by the economy, in the spirit of Fernández-Villaverde et al. (2011). We show that the dynamics of interest rates around episodes of sudden stops generated by the model have a similar behavior to that observed empirically in EMEs. The planner's intervention dictates increasing the cost of households' borrowing when it

is likely that both the collateral constraint might be binding in the near future and pecuniary externalities are large. Hence, the planner tends to increase his intervention as a response to low interest rates shocks to offset the increase in the size of pecuniary externalities despite the fact that there is a lower possibility of hitting a borrowing constraint. Moreover, we show that, keeping the level of interest rates constant, the planner has a non-monotonic response to interest rate volatility shocks. The degree of his intervention depends on how the changes in external volatility affect the incidence and severity of crises, as well as the covariance between these two factors.

One simple lesson for policy makers facing a rise in external risks is that multiple factors should be taken into account when implementing any sort of macroprudential policy. Mere spikes in the volatility of external interest rates, like the ones observed in recent years as the international financial markets adjust to expected policy changes in industrialized economies, do not necessarily call for a higher macroprudential taxes and the imposition of more stringent controls on capital flows. Policy makers should not only weigh the possibility of current account reversals to shape their interventions; they should also consider how external shocks affect the size of pecuniary externalities and the borrowing capacity of the country.

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Appendix

A Microeconomic foundations of the model

A.1 The timing of borrowing and asset trading

Deriving the collateral constraint We show that the collateral constraint faced by households in (4) can be derived from incentive compatibility constraints on the borrowers in an environment in which limited enforcement prevents lenders from collecting more than a fraction κ of the value of the asset, s_{t+1}, owned by a defaulting debtor.

We denote the individual and aggregate household choice variables with lowercase and uppercase letters, respectively. We divide any given period in three subperiods: morning, afternoon, and night.

The period begins in the morning with aggregate asset holdings (B, S) carried from the night of the previous period. The realization of the external shocks $X = (z, r, \sigma^r)$ takes place at the beginning of the morning, and individual households receive the dividends from their holdings of the tree, $s \cdot d \exp(z)$. In this subperiod, there is perfect enforcement of debt contracts, so the household fully repays its outstanding debt b. Each household makes an optimal consumption and portfolio decision $(\hat{c}, \hat{b}', \hat{s}')$ subject to its budget constraint, (2), taking the morning price $\mathcal{Q}(B, X)$ and interest rate $R \exp(r)$ as given. At this point, that is, at the end of the morning, the choice of $\hat{c}$ is just a plan. Every household carries the physical goods it has designated to consume into the following subperiods given that consumption occurs at night.

Given the previous assumptions, the recursive problem of a household in the morning is given by

$$V^m(b, s, B, X) = \max_{\hat{c}, \hat{b}', \hat{s}'} \left\{ V^a\left(\hat{c}, \hat{b}', \hat{s}', B, X\right) \right\}$$

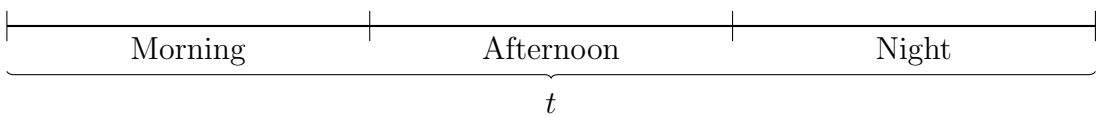
subject to

$$\hat{c} + \mathcal{Q}(B, X)\, \hat{s}' + \frac{\hat{b}'}{R(X)} \quad = \quad [\mathcal{Q}(B, X) + d(X)]s + b$$

where $V^a\left(\hat{c}, \hat{b}', \hat{s}', B, X\right)$ denotes the value of the household in the afternoon.

In the afternoon, an individual household is holding a portfolio of assets $(\hat{b}', \hat{s}')$, and has $\hat{c}$ units of consumption goods to eat at night. At this point, the household has the possibility of diverting current stocks—and therefore the corresponding revenues that

Figure 17: Timing of Actions



will become available when markets reopen at night—and default on his outstanding debt with the foreign lender next period. Therefore, the problem of the household choosing whether to default or not in the afternoon is simply given by

$$V^a\left(\hat{c}, \hat{b}', \hat{s}', B, X\right) \quad = \quad max\left\{V^d\left(\hat{c}, \hat{b}', \hat{s}', B, X\right), V^r\left(\hat{c}, \hat{b}', \hat{s}', B, X\right)\right\},$$

where $V^d\left(\hat{c}, \hat{b}', \hat{s}', B, X\right)$ and $V^r\left(\hat{c}, \hat{b}', \hat{s}', B, X\right)$ denote the values at night of having defaulted or not in the afternoon respectively.

At night, the international lender finds out whether he has been defaulted. The lender is entitled to obtain the fraction $\kappa \in (0, 1)$ of the household's stockholdings—the fraction that is not diverted—and the household regains immediate access to credit markets. The lender, nevertheless, cannot directly receive dividends from the tree, so he must sell it to local households in order to obtain a profit. We denote by $\mathcal{Q}^c(B, X)$ the prevailing price for this transaction in the night market. The lender then proceeds to loan the receipts of the transaction in the international financial markets—which include any households that might have defaulted—at the prevailing risk-free interest rate, $R\exp(r)$.[44]

Households can borrow again from the same lender, buy assets at the prevailing price and exchange consumption goods, but there is no longer the possibility of diverting resources. However, the problem of a household differs depending on if it diverted in the afternoon. Then, if the household decides to default, the value of this decision is

[44]Because the interest rate is positive and the evolution of the stock prices does not in general have a positive trend, the lender has incentives to immediately sell the stocks and lend the revenue in the overnight market. Otherwise, we can assume that the holdings of the tree depreciate overnight when held by the lender, so he has incentives to immediately sell them.

given by

$$\begin{aligned} V^d\left(\hat{c},\hat{b}',\hat{s}',B,X\right) &= \max_{c,b',s'}\left\{u\left(c\right)+\beta\mathbb{E}\left[V\left(b',s',B',X'\right)|X\right]\right\} \text{ s.t.} \\ c+\mathcal{Q}^c\left(B,X\right)s'+\frac{b'}{R\left(X\right)} &= \left(1-\kappa\right)\mathcal{Q}^c\left(B,X\right)\hat{s}'+\hat{c}, \end{aligned}$$

while if it repays, the corresponding value is

$$\begin{aligned} V^r\left(\hat{c},\hat{b}',\hat{s}',B,X\right) &= \max_{c,b',s'}\left\{u\left(c\right)+\beta\mathbb{E}\left[V\left(b',s',B',X'\right)|X\right]\right\} \text{ s.t.} \\ c+\mathcal{Q}^c\left(B,X\right)s'+\frac{b'}{R\left(X\right)} &= \frac{\hat{b}'}{R\left(X\right)}+\mathcal{Q}^c\left(B,X\right)\hat{s}'+\hat{c}, \end{aligned}$$

given that it needs to maintain resources equal to $-\frac{\hat{b}'}{R(X)}$ from its remaining wealth in order to repay tomorrow.

A graphical representation of the timing of borrowing and asset trading in this environment is presented in Figure 17.

Starting from the problem of the household at night, it can be clearly seen that the value of repaying will exceed that of defaulting, that is,

$$V^r\left(\hat{b}',\hat{s}',B,X\right)\geq V^d\left(\hat{b}',\hat{s}',B,X\right)$$

whenever

$$-\frac{\hat{b}'}{R\left(X\right)}\leq\kappa\mathcal{Q}^c\left(B,X\right)\hat{s}',$$

which is the collateral constraint that we use in the model. In words, to avoid losses from household default, lenders constrain the amount that they lend, $-b'/R\exp(r)$, to be less than or equal to the market value of the household's asset holdings that cannot be diverted, $\kappa\mathcal{Q}^c(B,X)s'$. This limit justifies the presence of the borrowing constraint (3) in the problem of the representative household.[45]

Pricing relationships in equilibrium Asset prices in the morning and at night must satisfy particular arbitrage conditions in the equilibrium we consider. First, our assumption that households compete à la Bertrand for the stocks of the tree at

[45] A crucial assumption we need to make in order for there to exist an equilibrium in this environment is that international lenders must lend in both the morning and night markets. Another way to have an equilibrium with such a pricing relationship would be by assuming that the household undertakes borrowing in both subperiods with just one foreign lender. This assumption can be justified by introducing an infinitesimally small fixed cost of borrowing with each additional competitive lender.

night implies that the market price of the tree at night, $\mathcal{Q}^c(B, X)$, is as high as the representative household prices the dividend payouts and resale value next period, according to the following Euler equation:

$$\mathcal{Q}^c(B, X) = \beta \mathbb{E}\left[\frac{u'(\mathcal{C}(\mathcal{B}(B, X), X'))[\mathcal{Q}(\mathcal{B}(B, X), X') + d(X')]}{u'(\mathcal{C}(B, X))} \middle| X \right],$$

where $\mathcal{C}$ and $\mathcal{B}$ denote the aggregate decision rules of the economy.

It only remains to explain the relationship between the morning and the night prices, $\mathcal{Q}$ and $\mathcal{Q}^c$ respectively, in equilibrium. Assume that we impose the collateral constraint on the household's problem in the morning in order to avoid default. Now, consider an equilibrium in which the borrowing constraint is binding, so that $\mu(B, X) > 0$. For every additional stock of the tree that the household buys in the morning, it must sacrifice $\mathcal{Q}(B, X)$ units of consumption that are valued at the marginal utility $u'(\hat{\mathcal{C}}(B, X))$. However, by buying more stocks of the tree, the representative household relaxes the borrowing constraint, and obtains a marginal benefit of $\kappa\mu(B, X)\mathcal{Q}^c(B, X)$, in the same subperiod. Thus, the net marginal cost of saving in stocks of the tree in the morning is

$$\mathcal{Q}(B, X)u'(\hat{\mathcal{C}}(B, X)) - \kappa\mu(B, X)\mathcal{Q}^c(B, X).$$

At night, the household can sell these stocks at the prevailing price, $\mathcal{Q}^c(B, X)$, which is valued at the marginal utility of consumption $u'(\mathcal{C}(B, X))$. Thus, for the household demand of stocks to be optimal, it must be the case that the marginal cost in the morning equates the marginal benefit in the afternoon:

$$\mathcal{Q}(B, X)u'(\hat{\mathcal{C}}(B, X)) - \kappa\mu(B, X)\mathcal{Q}^c(B, X) = \mathcal{Q}^c(B, X)u'(\mathcal{C}(B, X)).$$

However, notice that from the envelope conditions for the household's problems in the morning and at night, it must be the case that $u'(\hat{\mathcal{C}}(B, X)) = u'(\mathcal{C}(B, X))$. Hence, we obtain the relationship between both asset prices which implies that whenever the borrowing constraint binds, the value of the tree in the morning will be higher than at night because holding the asset in the morning helps the households relax the borrowing constraint and increase their debt. The decrease in prices from the morning to the night is perfectly foreseen by every agent in the economy, but there are no opportunities of arbitrage because it is forbidden to hold the asset in short positions.

The planner's intervention The social planner understands that the current aggregate level of debt, B, and the choice of future indebtedness B' affect the value of collateral available in the economy and thus constrains the borrowing possibilities of the households. In order to internalize this pecuniary externality, the planner can control the households' borrowing decisions that take place in the morning, B.

Nonetheless, the planner cannot overcome the fact that households can divert their asset holdings in the afternoon and default on their outstanding debt. Moreover, the planner cannot intervene in the night stock market, in which the defaulted foreign lenders sell the remaining fractions of the diverted asset and regain access to credit markets. Thus, the planner faces the same borrowing constraint as the households (3), and the price of the assets must be consistent with the household Euler equation of stocks:

$$\mathcal{Q}(B,X) = \beta E\left[\frac{u'(\mathcal{C}(\mathcal{B}(B,X),X'))[\mathcal{Q}(\mathcal{B}(B,X),X') + d(X')]}{u'(\mathcal{C}(B,X))}\middle| X\right].$$

In this case, the market price of the stocks is the same throughout the day because households do not internalize the effect of their savings in stocks on the borrowing possibilities for the planner's problem.

A.2 Competitive equilibrium

Consider the recursive formulation of the household's problem, expressed in program (4). The solution to the household's problem is characterized by a pair of optimal decision rules for bonds and stocks, $\hat{b}(b,s,B,X)$ and $\hat{s}(b,s,B,X)$ respectively, that satisfy the following set of equations:

$$\begin{aligned} u'(c) &= \mu(b,s,B,X) + \beta R(X)\,\mathbb{E}\left[u'(c')\,|X\right], \text{ and} \\ \mathcal{Q}(B,X)\,u'(c) &= \beta\mathbb{E}\left[u'(c')\left(\mathcal{Q}(\mathcal{B}(B,X),X') + d(X')\right)|X\right] \\ &+ \mathcal{Q}^c(B,X)\,\mu(b,s,B,X)\,\kappa, \end{aligned}$$

the budget constraint of the household in each period, and the collateral constraint

$$-\frac{\hat{b}(b,s,B,X)}{R(X)} \leq \kappa\mathcal{Q}^c(B,X)\,\hat{s}(b,s,B,X).$$

We now proceed to define a recursive competitive equilibrium.

Definition A recursive competitive equilibrium of this economy consists of pricing

functions $\hat{\mathcal{Q}}(B,X)$ and $\hat{\mathcal{Q}}^c(B,X)$, a perceived law of motion for aggregate bond holdings, $\hat{\mathcal{B}}(B,X)$, and decision rules for households, $\hat{b}(b,s,B,X)$ and $\hat{s}(b,s,B,X)$, with associated value function $\hat{V}(b,s,B,X)$ such that:

1. Given $\hat{\mathcal{Q}}(B,X)$, $\hat{\mathcal{Q}}^c(B,X)$ and $\mathcal{B}(B,X)$, households' decision rules, $\hat{b}(b,s,B,X)$ and $\hat{s}(b,s,B,X)$, and the associated value function $\hat{V}(b,s,B,X)$ solve the recursive problem of the household given by (4).

2. $\hat{\mathcal{B}}(B,X)$ is consistent with the actual law of motion for bond holdings; $\hat{\mathcal{B}}(B,X) = \hat{b}(B,1,B,X)$.

3. Markets must clear. In particular, $\hat{\mathcal{Q}}(B,X)$ and $\hat{\mathcal{Q}}^c(B,X)$ are such that $\hat{s}(B,1,B,X) = 1$.

Given the definition of the equilibrium, notice that the equilibrium level of bonds can be characterized by a simple function of the aggregate state variables, $B' = \hat{\mathcal{B}}(B,X)$, which together with the resource constraint defines consumption as a function of aggregate state variables, $\hat{\mathcal{C}}(B,X)$.

A.3 Social planner's recursive problem

We consider a social planner that lacks commitment and that can only choose aggregate bond holdings for households but is still subject to the borrowing constraint. Following Klein et al. (2005), in order to solve for the time consistent policy, we focus on Markov stationary policy rules that only depend on the current state of the economy. In particular, they only depend on the aggregate state of the economy, (B,X). We solve for the constrained efficient allocation following the three steps described in Klein et al. (2005): (i) We first define a recursive competitive equilibrium for arbitrary policy rules; (ii) we then proceed to define a constrained-efficient allocation for arbitrary policy rules of future planners; and (iii) we define the constrained efficient allocation for the case in which such policies are time consistent, that is, we solve for the fixed point of the game being played by successive planners. In this problem, the social planner makes the borrowing decisions for the households, so he is the one facing the collateral constraint. Households are allowed to trade stocks of the tree freely without government intervention.

Let us consider a planner who chooses an arbitrary sequence of state-contingent lump-sum transfers, $\{T_t\}_{t=0}^{\infty}$. Given this sequence of transfers, we can write down the

Bellman equation for the household's problem as follows:

$$V^A(s,T,X) = \max_{c,s'} \left\{ u(c) + \beta \mathbb{E}\left[V^A(s',T',X') \,|X \right] \right\}$$

subject to

$$c + \mathcal{Q}^A(T,X)\, s' = \left[\mathcal{Q}^A(T,X) + d(X) \right] s + T.$$

When solving this problem, the household takes the pricing function, $\mathcal{Q}^A(T,X)$, and the sequence of transfers as given. The solution to this problem is characterized by a policy rule for stock holdings, $s^A(s,T,X)$, such that Euler equation for stock holdings holds,

$$\mathcal{Q}^A(T,X) = \frac{\beta \mathbb{E}\left[u'(c') \left(\mathcal{Q}^A(T',X') + d(X') \right) |X \right]}{u'(c)},$$

where

$$c + \mathcal{Q}^A(T,X)\, s^A(s,T,X) = \left[\mathcal{Q}^A(T,X) + d(X) \right] s + T.$$

Notice that the resource constraint of the economy implies that $T = B - \frac{B'}{R(X)}$. Hence, given B, the planner actually chooses T by choosing B'. Therefore, we can rewrite the planner's policy rule as one that dictates B' as a function of the current aggregate state, (B,X). Call this policy rule $\Psi(B,X)$, and define the following functions:

$$\begin{aligned}
\mathcal{Q}(B,X) &\equiv \mathcal{Q}^A\left(B - \frac{\Psi(B,X)}{R(X)}, X \right), \\
s(s,B,X) &\equiv s^A\left(s, B - \frac{\Psi(B,X)}{R(X)}, X \right), \text{ and} \\
V(s,B,X) &\equiv V^A\left(s, B - \frac{\Psi(B,X)}{R(X)}, X \right).
\end{aligned}$$

Hence, we can rewrite the optimality conditions for the household's problem as follows:

$$\mathcal{Q}(B,X) = \frac{\beta \mathbb{E}\left[u'(c') \left(\mathcal{Q}(B',X') + d(X') \right) |X \right]}{u'(c)},$$

where

$$c + \mathcal{Q}(B,X)\, \hat{s}(s,T,X) = \left[\mathcal{Q}(B,X) + d(X) \right] s + B - \frac{\Psi(B,X)}{R(X)}.$$

Definition A recursive competitive equilibrium for an arbitrary policy rule $\Psi(B,X)$ consists of a pricing function, $\hat{\mathcal{Q}}(B,X)$, and decision rules for households, $\hat{s}(s,B,X)$, with associated value function $\hat{V}(s,B,X)$ such that:

1. Given $\Psi(B,X)$ and $\hat{\mathcal{Q}}(B,X)$, households' decision rules, $\hat{s}(s,B,X)$, and the associated value function $\hat{V}(s,B,X)$ solve the recursive problem of the household.

2. Markets clear: $\hat{\mathcal{Q}}(B,X)$ is such that $\hat{s}(s,B,X)=1$ and the resource constraint holds, $c+\frac{B'}{R(X)}=B+d(X)$, where $B'=\Psi(B,X)$.

Therefore, in such an equilibrium, we have that the following set of equations must be satisfied:

$$\begin{aligned}\hat{\mathcal{Q}}(B,X) &= \frac{\beta\mathbb{E}\left[u'\left(B'+d(X)-\frac{B''}{R(X)}\right)\left[\hat{\mathcal{Q}}(B',X')+d(X')\right]\middle| X\right]}{u'\left(B+d(X)-\frac{B'}{R(X)}\right)},\\ B' &= \Psi(B,X) \text{ and } B''=\Psi(\Psi(B,X),X').\end{aligned}$$

Given that the planner we consider can only affect the allocation of bond holdings but cannot directly intervene in the markets for stocks, the pricing condition for $\hat{\mathcal{Q}}(B,X)$ has to hold in a constrained efficient allocation; in particular, this condition defines the price at which lenders value collateral in the current period borrowing constraint. Taking into account this kind of implementability constraint for the planner, we can now define the problem to be solved by a planner that takes as given the policy functions of future planners. Given future policy rules, $\Psi(B,X)$, associated pricing function $\hat{\mathcal{Q}}(B,X)$, and consumption rule $\mathcal{C}(B,X)$, the current planner chooses current consumption, c, and future bond holdings to solve the following Bellman equation:

$$W(B,X)=\max_{c,B'}\left\{u(c)+\beta\mathbb{E}\left[W(B',X')\,|X\right]\right\}$$

subject to

$$\begin{aligned}c+\frac{B'}{R(X)} &= d(X)+B,\\ -\frac{B'}{R(X)} &\leq \kappa\tilde{\mathcal{Q}}(c,B',X),\end{aligned}$$

where

$$\tilde{\mathcal{Q}}(c,B',X)=\frac{\beta\mathbb{E}\left[u'\left(\mathcal{C}(B',X')\right)\left(\mathcal{Q}(B',X')+d(X')\right)|X\right]}{u'(c)},$$

and $\mathcal{C}(B',X')=d(X')+B'-\frac{\Psi(B',X')}{R(X')}$.

Definition A constrained efficient allocation given a policy rule for future planners $\Psi(B,X)$, with associated pricing function $\hat{\mathcal{Q}}(B,X)$ and consumption rule $\mathcal{C}(B,X)$, con-

sists of an optimal policy rule, $\hat{\Psi}(B,X)$, such that given functions $\Psi(B,X)$, $\hat{\mathcal{Q}}(B,X)$ and $\mathcal{C}(B,X)$, the current policy rule $B' = \hat{\Psi}(B,X)$ and associated value function, $\hat{W}(B,X)$, solve the recursive problem of the current planner.

Let us define the following function,

$$\bar{\mathcal{Q}}(B,B',X) = \beta\mathbb{E}\left[\frac{u'\left(B' + d(X') - \frac{\Psi(B',X')}{R(X')}\right)\left[\hat{\mathcal{Q}}(B',X') + d(X')\right]}{u'\left(d(X) + B - \frac{B'}{R(X)}\right)}\middle| X\right].$$

Then, $\hat{\Psi}(B,X)$ has to be such that the generalized Euler equation holds:

$$\begin{aligned} u'(\hat{\mathcal{C}}(B,X)) - \hat{\mu}(B,X)\left[1 + \kappa R(X)\xi(B,X)\right] \\ = R(X)\beta\mathbb{E}\left[u'(\mathcal{C}(B',X')) + \kappa\hat{\mu}(B',X')\psi(B',X')|X\right], \end{aligned} \tag{12}$$

where $\psi(B,X) = \frac{\partial\bar{\mathcal{Q}}(B,\Psi(B,X),X)}{\partial B}$, $\xi(B,X) = \frac{\partial\bar{\mathcal{Q}}(B,\Psi(B,X),X)}{\partial B'}$, and $\hat{\mathcal{C}}(B,X) = B + d(X) - \frac{\hat{\Psi}(B,X)}{R(X)}$. The multiplier on the collateral constraint is given by

$$\begin{aligned} \hat{\mu}(B,X) = \max\Bigg\{0, \frac{1}{1+\kappa R(X)\xi(B,X)}\Bigg[u'\left(B + d(X) - \frac{\hat{\Psi}(B,X)}{R(X)}\right) \\ - \beta R(X)\mathbb{E}\left[u'(\mathcal{C}(B',X')) + \kappa\mu(B',X')\psi(B',X')|X\right]\Bigg]\Bigg\}, \end{aligned}$$

where $\hat{\Psi}(B,X) = -R(X)\kappa\bar{\mathcal{Q}}(B,\Psi(B,X),X)$. After this characterization of the allocation, we can now define a recursive constrained efficient allocation as follows.

Definition The recursive constrained efficient allocation consists of functions $\Psi(B,X)$, $\hat{\mathcal{Q}}(B,X)$, $\mathcal{C}(B,X)$, and $\hat{\Psi}(B,X)$ with associated value function, $\hat{W}(B,X)$, such that

1. $\hat{\mathcal{Q}}(B,X)$, $\mathcal{C}(B,X)$, $\hat{\Psi}(B,X)$, and the associated value function $\hat{W}(B,X)$, constitute a constrained efficient allocation, given a policy rule for future planners, $\Psi(B,X)$.

2. The planner's plans are time-consistent: $\hat{\Psi}(B,X) = \Psi(B,X)$ and $\bar{\mathcal{Q}}(B,\hat{\Psi}(B,X),X) = \hat{\mathcal{Q}}(B,X)$.

Non-binding current collateral constraint: $\mu(B,X)=0$ Let us consider first the case in which $\mu(B,X)=0$. Given our definition of $\bar{\mathcal{Q}}(B,B',X)$, notice that

$$\begin{aligned}\frac{\partial\bar{\mathcal{Q}}(B,B',X)}{\partial B} &= \beta\mathbb{E}\left\{-\frac{u'(\mathcal{C}(B',X'))\left(\hat{\mathcal{Q}}(B',X')+d(X')\right)}{u'(c)}\frac{u''(c)}{u'(c)}\right\}\\ &= -\frac{u''(c)}{u'(c)}\bar{\mathcal{Q}}(B,B',X),\end{aligned}$$

which implies that

$$\psi(B,X)=-\frac{u''(\mathcal{C}(B,X))}{u'(\mathcal{C}(B,X))}\hat{\mathcal{Q}}(B,X).$$

Therefore, when $\mu(B,X)=0$, condition (12) becomes a regular Euler equation (with a μ wedge):

$$u'\left(\hat{\mathcal{C}}(B,X)\right)=R(X)\beta\mathbb{E}\left[u'\left(\mathcal{C}(B',X')\right)-\kappa\hat{\mu}(B',X')\frac{u''(\mathcal{C}(B',X'))}{u'(\mathcal{C}(B',X'))}\hat{\mathcal{Q}}(B',X')\,|X\right].$$

Binding current collateral constraint: $\mu(B,X)>0$ Let us first notice that the current planner has to choose B' subject to the collateral constraint

$$\frac{B'}{R(X)}+\kappa\bar{\mathcal{Q}}(B,B',X)\geq 0.$$

If the left-hand side of the previous inequality is strictly increasing in B', then, given B, there is a unique B' such that this equation holds with equality. Hence, when the current collateral constraint is binding, the optimal policy rule by the current planner must solve $\frac{\hat{\Psi}(B,X)}{R(X)}+\kappa\bar{\mathcal{Q}}(B,\hat{\Psi}(B,X),X)=0$, and this policy rule is unique. Notice that that left-hand side if strictly increasing if and only if

$$1+\kappa R(X)\xi(B,X)>0.$$

In equilibrium, $\xi(B, X) < 0$, therefore we expect this condition to hold whenever κ is a small number.[46] Given the definition of $\bar{Q}(B, B', X)$, notice that

$$\frac{\partial \bar{\mathcal{Q}}(B, B', X)}{\partial B'} = \frac{\beta \mathbb{E}[\Omega(B, B', X)]}{u'(c)} + \frac{u''(c)}{u'(c)} \frac{\bar{\mathcal{Q}}(B, B', X)}{R(X)} \tag{13}$$

where

$$\Omega(B, B', X) = u''(\mathcal{C}(B', X'))\frac{\partial \mathcal{C}(B', X')}{\partial B}[\mathcal{Q}(B', X') + d(X')] + u'(\mathcal{C}(B', X'))\frac{\partial \mathcal{Q}(B', X')}{\partial B}.$$

This last expression shows how the current planner takes into account how his decision affect future planners' actions by changing B'.

B Numerical solution of the model

B.1 Competitive equilibrium

Let us denote by B the aggregate equilibrium savings of the economy, and by $X = (z, r, \sigma^r)$ the realization of exogenous shocks. We wish to find functions $\mathcal{B}(B, X)$, $\mathcal{C}(B, X)$, $\mathcal{Q}(B, X)$, $\mathcal{Q}^c(B, X)$, and $\mu(B, X)$ that satisfy

$$u'(\mathcal{C}(B, X)) = \beta R(X)\mathbb{E}[u'(\mathcal{C}(\mathcal{B}(B, X), X'))|X] + \mu(B, X), \tag{14}$$

$$\mathcal{C}(B, X) + \frac{\mathcal{B}(B, X)}{R(X)} = d(X) + B, \tag{15}$$

$$-\frac{\mathcal{B}(B, X)}{R(X)} \leq \kappa \mathcal{Q}(B, X), \tag{16}$$

$$\mathcal{Q}^c(B, X) = \beta \mathbb{E}\left[\frac{u'(\mathcal{C}(\mathcal{B}(B, X), X'))\left[\mathcal{Q}(\mathcal{B}(B, X), X') + d(X')\right]}{u'(\mathcal{C}(B, X)) - \kappa\mu(B, X)}\middle| X\right], \tag{17}$$

$$\mathcal{Q}(B, X) = \left(1 + \frac{\kappa\mu(B, X)}{u'(\mathcal{C}(B, X))}\right)\mathcal{Q}^c(B, X). \tag{18}$$

We extend the endogenous grid method (EGM) of Carroll (2006) to our framework where there is a borrowing constraint that binds occasionally:

1. For each $\sigma^r \in \{\sigma^r_L, \sigma^r_H\} \equiv \mathcal{S}$, calculate the transition matrix for a discrete approximation to the VAR(1) process of (z, r) over $\mathcal{Z} \times \mathcal{R}$, with $\mathcal{Z} = \{z_1, \ldots, z_{N_z}\}$ and $\mathcal{R} = \{r_1, \ldots, r_{N_z}\}$.

[46] Notice that when κ is small enough, the term $\kappa\mu(B', X')\psi(B', X')$ also becomes very small and $\hat{\Psi}(B, X)$ is also unique in the case in which $\mu(B, X) = 0$.

2. Generate a grid $\bar{\mathcal{B}} = \{b_1, b_2, \ldots, b_N\}$, and an extended grid

$$\bar{\bar{\mathcal{B}}} = \bar{\mathcal{B}} \cup \{b_{N+1}, b_{N+2}, \ldots, b_{N+M}\},$$

where b_{N+M} is chosen such that the resulting $\max_X \mathcal{B}(b_N, X) \leq b_{N+M}$ (to be verified in the end).

3. Guess functions $\mathcal{C}_1(B, X)$, $\mathcal{Q}_1(B, X)$ and $\mathcal{Q}_1^c(B, X)$, for every $(B, X) \in \bar{\bar{\mathcal{B}}} \times \mathcal{Z} \times \mathcal{R} \times \mathcal{S}$. The initial guess we use is

$$\mathcal{C}_1(B, X) = d(X) + B\left(1 - \frac{1}{R(X)}\right),$$
$$\mathcal{Q}_1(B, X) = \frac{\beta}{1-\beta} d(X),$$

and $\mathcal{Q}_1^c(B, X) = \mathcal{Q}_1(B, X)$, which corresponds to the assumption that $\mathcal{B}(B, X) = B$, $z' = z$ and $r' = r$ for all (B, X).

4. Set $\mathcal{C}_0(B, X) = \mathcal{C}_1(B, X)$, $\mathcal{Q}_0(B, X) = \mathcal{Q}_1(B, X)$ and $\mathcal{Q}_0^c(B, X) = \mathcal{Q}_1^c(B, X)$ for each $(B, X) \in \bar{\bar{\mathcal{B}}} \times \mathcal{Z} \times \mathcal{R} \times \mathcal{S}$.

5. Assume that (16) does not bind. Use (14) and (15) to calculate

$$\hat{\mathcal{C}}(B', X) = u'^{-1}\left(\beta R(X) \mathbb{E}\left[u'(\mathcal{C}_0(B', X'))|X\right]\right),$$
$$\hat{\mathcal{B}}(B', X) = \hat{\mathcal{C}}(B', X) + \frac{B'}{R(X)} - d(X).$$

Notice that $\hat{\mathcal{B}}$ is the level of contemporaneous savings that yield an optimal savings decision B' when the realization of shocks is X and the borrowing constraint does not bind.

6. For each X, let us denote by $\hat{\bar{\mathcal{B}}}(X)$ the endogenous grid of points generated by $\hat{\mathcal{B}}(B', X)$. For every X, interpolate B' from $\hat{\mathcal{B}}(B', X)$ to $\bar{\mathcal{B}}$, and denote the resulting function $\check{B}(B, X)$.

7. Calculate $\tilde{\mathcal{B}}(B, X) = \max\{\check{B}(B, X), -\kappa R(X) \mathcal{Q}_0^c(B, X)\}$, and the corresponding consumption:

$$\tilde{\mathcal{C}}(B, X) = d(X) + B - \frac{\tilde{\mathcal{B}}'(B, X)}{R(X)}.$$

8. Find $\mathcal{B}^*(B,X) = \min\{B \in \bar{\bar{\mathcal{B}}} : B \geq \tilde{\mathcal{B}}(B,X)\}$. Using (14), (17) and (18), find

$$\begin{aligned}
\tilde{\mu}(B,X) &= u'(\tilde{\mathcal{C}}(B,X)) - \beta R(X)\mathbb{E}\left[u'(\mathcal{C}_0(\mathcal{B}^*(B,X),X'))|X\right], \\
\tilde{\mathcal{Q}}^c(B,X) &= \beta\mathbb{E}\left[\frac{u'(\mathcal{C}_0(\mathcal{B}^*(B,X),X'))\left[\mathcal{Q}_0(\mathcal{B}^*(B,X),X') + d(X')\right]}{u'(\tilde{\mathcal{C}}(B,X)) - \kappa\tilde{\mu}(B,X)}\middle| X\right], \\
\tilde{\mathcal{Q}}(B,X) &= \left(1 + \frac{\kappa\tilde{\mu}(B,X)}{u'(\tilde{\mathcal{C}}(B,X))}\right)\tilde{\mathcal{Q}}^c(B,X).
\end{aligned}$$

9. For every $(B,X) \in \bar{\mathcal{B}} \times \mathcal{Z} \times \mathcal{R} \times \mathcal{S}$, update

$$\begin{aligned}
\mathcal{C}_1(B,X) &= \alpha\tilde{\mathcal{C}}(B,X) + (1-\alpha)\mathcal{C}_0(B,X), \\
\mathcal{Q}_1(B,X) &= \alpha\tilde{\mathcal{Q}}(B,X) + (1-\alpha)\mathcal{Q}_0(B,X) \\
\mathcal{Q}_1^c(B,X) &= \alpha\tilde{\mathcal{Q}}^c(B,X) + (1-\alpha)\mathcal{Q}_0^c(B,X).
\end{aligned}$$

for some $\alpha \in (0,1]$. For $B \in \bar{\bar{\mathcal{B}}}\backslash\bar{\mathcal{B}}$, set $\mathcal{C}_1(B,X) = \mathcal{C}_1(b_N,X)$, $\mathcal{Q}_1(B,X) = \mathcal{Q}_1(b_N,X)$ and $\mathcal{Q}_1^c(B,X) = \mathcal{Q}_1^c(b_N,X)$.

10. Repeat steps 4-9 until convergence.

B.2 Constrained efficient allocation

The constrained efficient allocation satisfies

$$\begin{aligned}
u'(\mathcal{C}(B,X)) - \mu(B,X)\left[1 + \kappa R(X)\xi(B,X)\right] \\
= R(X)\beta\mathbb{E}\left[u'(\mathcal{C}(B',X')) + \kappa\mu(B',X')\psi(B',X')|X\right], \qquad (19) \\
\mathcal{Q}(B,X) = \beta\mathbb{E}\left[\frac{u'(\mathcal{C}(\mathcal{B}(B,X),X'))\left[\mathcal{Q}(\mathcal{B}(B,X),X') + d(X')\right]}{u'(\mathcal{C}(B,X)) - \kappa\mu(B,X)}\middle| X\right], \qquad (20)
\end{aligned}$$

together with (15) and (16). Some steps of the EGM algorithm change with respect to the solution of the competitive equilibrium:

3. Guess functions $\mathcal{C}_1(B,X)$, $\mathcal{Q}_1(B,X)$ and $\mu_1(B,X)$ for every $(B,X) \in \bar{\bar{\mathcal{B}}} \times \mathcal{Z} \times \mathcal{R} \times \mathcal{S}$. The initial guess we use is: $\mu_1(B,X) = 0$.

4. Set $\mathcal{C}_0(B,X) = \mathcal{C}_1(B,X)$, $\mathcal{Q}_0(B,X) = \mathcal{Q}_1(B,X)$ and $\mu_0(B,X) = \mu_1(B,X)$ for each $(B,X) \in \bar{\bar{\mathcal{B}}} \times \mathcal{Z} \times \mathcal{R} \times \mathcal{S}$.

   Calculate:
$$\psi(B,X) = -\frac{u''(\mathcal{C}_0(B,X))}{u'(\mathcal{C}_0(B,X))}\mathcal{Q}_0(B,X).$$

Use the numerical derivatives of C_0 and Q_0 with respect to B to calculate $\xi(B, X)$ using equation (13) of Appendix A.3.

5. Assume that (16) does not bind. Use (19) and (15) to calculate:

$$\hat{\mathcal{C}}(B', X) = u'^{-1}\left(\beta R(X)\mathbb{E}\left[u'(\mathcal{C}_0(B', X')) + \kappa\mu_0(B', X')\psi(B', X')|X\right]\right),$$
$$\hat{\mathcal{B}}(B', X) = \hat{\mathcal{C}}(B', X) + \frac{B'}{R(X)} - d(X).$$

8. Find $\mathcal{B}^*(B, X) = \min\{B \in \bar{\bar{\mathcal{B}}} : B \geq \tilde{\mathcal{B}}(B, X)\}$. Using (19) and (20), find:

$$\tilde{\mu}(B, X) = \frac{1}{1 + \kappa R(X)\xi(B, X)}\Big\{u'(\tilde{\mathcal{C}}(B, X))$$
$$- \beta R(X)\mathbb{E}\left[u'(\mathcal{C}_0(\mathcal{B}^*(B, X), X')) + \kappa\mu_0(\mathcal{B}^*(B, X), X')\psi(\mathcal{B}^*(B, X), X')|X\right]\Big\},$$
$$\tilde{\mathcal{Q}}(B, X) = \beta\mathbb{E}\left[\frac{u'(\mathcal{C}_0(\mathcal{B}^*(B, X), X'))\left[\mathcal{Q}_0(\mathcal{B}^*(B, X), X') + d(X')\right]}{u'(\tilde{\mathcal{C}}(B, X)) - \kappa\tilde{\mu}(B, X)}\Bigg| X\right],$$

9. For every $(B, X) \in \bar{\mathcal{B}} \times \mathcal{Z} \times \mathcal{R} \times \mathcal{S}$, update:

$$\mathcal{C}_1(B, X) = \alpha\tilde{\mathcal{C}}(B, X) + (1 - \alpha)\mathcal{C}_0(B, X),$$
$$\mathcal{Q}_1(B, X) = \alpha\tilde{\mathcal{Q}}(B, X) + (1 - \alpha)\mathcal{Q}_0(B, X)$$
$$\mu_1(B, X) = \alpha\tilde{\mu}(B, X) + (1 - \alpha)\mu_0(B, X).$$

for some $\alpha \in (0, 1]$. For $B \in \bar{\bar{\mathcal{B}}}\backslash\bar{\mathcal{B}}$, set $\mathcal{C}_1(B, X) = \mathcal{C}_1(b_N, X)$, $\mathcal{Q}_1(B, X) = \mathcal{Q}_1(b_N, X)$ and $\mu_1(B, X) = \mu_1(b_N, X)$.

10. Repeat steps 4-9 until convergence.

B.3 Accuracy of the approximation

We compute the Euler equation errors following Aruoba et al. (2006) to assess the accuracy of our solution. The histograms in Figure 18 show that the errors remain below 10^{-2} units of consumption in most of the state space. The maximum levels of the errors are reached around the region where the borrowing constraint binds. The errors are modestly higher in the solution to the constrained efficient allocation, but they remain within a reasonable level.

Figure 18: Euler equation errors: Ergodic distributions