Managing Capital Flows in the Presence of External Risks

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Introduction
External Risks and Policy

1. External shocks affect economic activity (independent of countries’ fundamentals).
   - Generate large and volatile capital flows and affect the real economy
     → Reminded by Global Financial Crisis
   - Significant risks: 1st and 2nd moments of world interest rates matter

2. Policy prescriptions to prevent and reduce the effects of large and volatile capital flows.
   - Policy makers and international institutions have justified capital account intervention as a response to perceived increase in external risks (volatility), e.g. uncertainty generated by “Taper Tantrum.”
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IMF (2012): “Capital flows have grown significantly in both size and volatility [...] (these) carry risk. Because capital flows have a bearing on economic and financial stability in both individual economies and globally, an important challenge for policy makers is to develop a coherent approach to capital flows and the policies that affect them.”
Motivation and Question

Theoretical Framework: Silent on External Risks

2. ⇒ Theoretical literature on macroprudential policy in small open economies

   ▶ Pecuniary externalities → overborrowing → scope for intervention based on welfare.
   ▶ Optimal policy response to domestic (output) shocks.
   ▶ Financial crises rely on size of capital flows, not volatility.
Motivation and Question
Theoretical Framework: Silent on External Risks

2. ⇒ Theoretical literature on macroprudential policy in small open economies
   → **Benchmark theoretical framework:** Lorenzoni (2008), Bianchi (2010), Jeanne (2012), Korinek and Mendoza (2014)
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   ▶ Financial crises rely on size of capital flows, not volatility.

→ However, literature silent on policy response to shocks to external risk.
   ▶ Environment in which external shocks affect asset prices driving pecuniary externality.
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**Question**: How should optimal macroprudential policy respond to external shocks (international interest rates)?
Methodology

What do we do?

1. Study response of optimal policy to shocks to 1st and 2nd moments of international interest rates in a benchmark SOE framework with external borrowing constraints.
   - Estimate stochastic process for international interest rates with regime-switches in volatility.

2. Model: SOE subject to endowment + interest rate shocks and collateral constraint that depends on asset prices:
   - Endogenous financial crises nested within business cycles; and pecuniary externalities ⇒ ex ante policy intervention
   - Microfoundation of collateral constraint.

3. Numerical analysis of time-consistent optimal policy across interest rate levels and volatility regimes.
Findings

1. Simulations of financial crises the evolution of external shocks are consistent with the data.
   - Reyes-Heroles and Tenorio (2017)
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2. In the competitive equilibrium, allocations and prices are quantitatively sensitive to external interest rate shocks, but not to their volatility.

3. The borrowing decisions that solve the time-consistent constrained efficient allocation depend on the level and volatility of external shocks.
   - Incidence and severity of crises shape optimal policy → Shocks to volatility affect asset prices.
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3. The borrowing decisions that solve the time-consistent constrained efficient allocation depend on the level and volatility of external shocks.
   - Incidence and severity of crises shape optimal policy → Shocks to volatility affect asset prices.

4. No monotone relation between macroprudential tax on external debt and external shocks.
   - Tax schedule as a function of current debt does not shift in one single direction when external risks change.
   - “Volatility paradox” contrary to conventional wisdom in policy circles.
Related Literature

- **Capital Flows, Financial Crises and Optimal Policy:**
  - Positive analysis: Mendoza and Smith (2002) and Mendoza (2010).
  - Optimal capital controls: Schmitt-Grohé and Uribe (2016a,b)

- **Emerging Market Business Cycles and Global Shocks:**
  - Neumeyer and Perri (2005), Uribe and Yue (2006), Fernández-Villaverde et al. (2011)
  - Mackowiak (2007), Chang and Fernández (2013), Eichengreen and Gupta (2016) [capital reversals]
  - Sovereign default: Longstaff et al. (2011), Johri et al. (2015)
The Model
Small open economy subject to collateral constraint [similar to JK(2010) & BM(forth)]

- SOE with an infinitely lived unit continuum of identical households that consume a single traded good $c_t$.
  - Access to international bonds markets and domestic asset markets.
    - Period $t$ divided into Morning (M), Afternoon (A) and Night (N).
    - Access to financial markets: M and N.
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- Sources of risk:
  - Stochastic external interest rate $R_t = R \times \exp(r_t)$.
  - Variance of interest rate process depends on regime: $\sigma_t^r$.
  - Stochastic endowment (Lucas tree) pays a dividend $d_t = d \times \exp(z_t)$. 

Financial frictions → Collateral constraint
- Fraction $\kappa$ of value of assets as collateral with foreign lenders.
  - A: Households can divert resources and default on existing debt. Lenders do not observe actions.
  - N: Lenders sell confiscated asset.
- Collateral constraint ⇒ No default.

Financial crises occur when collateral constraint binds.
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The Model

Exogenous Shocks

- $(z_t, r_t)'$ follows the VAR specification
  
  $\begin{pmatrix} z_t \\ r_t \end{pmatrix} = A_0 + A_1 \begin{pmatrix} z_{t-1} \\ r_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon^z_t \\ \epsilon^r_t \end{pmatrix}.$

- $(\epsilon^z_t, \epsilon^r_t)' \sim N(0, \Sigma_t)$ where

  $\Sigma_t = \begin{pmatrix} (\sigma^z)^2 & \rho \cdot \sigma^z \cdot \sigma^r_t \\ \rho \cdot \sigma^z \cdot \sigma^r_t & (\sigma^r_t)^2 \end{pmatrix}.$

- Regime-switching: $\sigma^r_t \in \{\sigma^r_L, \sigma^r_H\}$, with $0 < \sigma^r_L < \sigma^r_H$, and switching between regimes governed by first-order Markov process with transition matrix $\Pi.$
The Model

Household’s Problem (implied by no default)

- Given prices, each household solves:

$$\max_{c_t, b_{t+1}, s_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$c_t + q_t s_{t+1} + \frac{b_{t+1}}{R_t} = (q_t + d_t) s_t + b_t$$

$$- \frac{b_{t+1}}{R_t} \leq \kappa q_t^c s_{t+1},$$

where

- $b_t$: face value of bonds held at beginning of period $t$.
- $s_t$: share of the asset held at the beginning of period $t$ (only trades domestically).
- $q_t$: market value of the asset.
- $q_t^c$: price at which collateral is valued at $N$.  

Derivation of CC
The Model

Competitive Equilibrium

Definition

Sequences \( \{c_t, b_{t+1}, s_{t+1}\}_{t=0}^{\infty} \) for each household, and prices \( \{q_t, q_t^c\}_{t=0}^{\infty} \) such that given prices households’ problems are solved, and there are no arbitrage opportunities and markets for stocks clear, \( s_{t+1} = 1 \), in each interim period for all \( t = 0, 1, \ldots \).

Lemma

The optimality conditions that characterize the competitive equilibrium are

\[
q_t u'(c_t) \left( 1 + \frac{\kappa \mu_t}{u'(c_t)} \right)^{-1} = E_t \left[ \beta u'(c_{t+1})(q_{t+1} + d_{t+1}) \right] \quad \text{and} \quad u'(c_t) - \mu_t = R_t E_t \left[ \beta u'(c_{t+1}) \right]
\]

where \( q_t^c \) is such that \( q_t u'(c_t) - \kappa \mu_t q_t^c = q_t^c u'(c_t) \).
The Model

Competitive Equilibrium

Definition

Sequences \( \{ c_t, b_{t+1}, s_{t+1} \}_{t=0}^{\infty} \) for each household, and prices \( \{ q_t, q^c_t \}_{t=0}^{\infty} \) such that given prices households’ problems are solved, and there are no arbitrage opportunities and markets for stocks clear, \( s_{t+1} = 1 \), in each interim period for all \( t = 0, 1, \ldots \)

Lemma

The optimality conditions that characterize the competitive equilibrium are

\[
q_t u' (c_t) \left( 1 + \frac{\kappa \mu_t}{u' (c_t)} \right)^{-1} = \mathbb{E}_t \left[ \beta u' (c_{t+1}) (q_{t+1} + d_{t+1}) \right] \quad \text{and} \quad u' (c_t) - \mu_t = R_t \mathbb{E}_t \left[ \beta u' (c_{t+1}) \right]
\]

where \( q^c_t \) is such that \( q_t u' (c_t) - \kappa \mu_t q^c_t = q^c_t u' (c_t) \).

- Fundamental trade-off between impatience and insurance when \( \beta R_t < 1 \).
- Crisis: constraint binds (\( \mu_t > 0 \)) \( \rightarrow \) \( c_t \downarrow, q_t \downarrow \) and tightens constraint.
  - Feedback effect not internalized in competitive equilibrium
- External shocks \( \implies \) volatile capital flows.
The Model
Recursive Competitive Equilibrium

![Graphs showing savings, consumption, asset prices, and borrowing constraint against a parameter b.]

Estimation and Parameters
Competitive Equilibrium

Finding 1

1. Simulations of sudden stop episodes and the evolution of external shocks are consistent with the data.
   - Reyes-Heroles and Tenorio (2016)
Competitive Equilibrium
Finding 2

2. In the competitive equilibrium, allocations and prices are sensitive to external interest rate shocks, but not to their volatility.
   - Fernández-Villaverde et al. (2011)
The Model
Constrained-Efficient Allocation

- Consider a social planner that internalizes externality on borrowing capacity and:
  1. Can choose aggregate debt, subject to economy's borrowing constraint,
  2. Cannot commit to future policies.

- Solve for constrained efficient allocations that a social planner would implement through time-consistent policies:
  - Following Klein et al. (2005, 2008) we restrict attention to time-consistent Markov policies: \( B' = \Psi(B, X) \), where \( B \) is current aggregate debt and \( X \) is the vector of current exogenous shocks.
  - Focus on recursive formulation.
The Model

Constrained-Efficient Allocation

- **Assumption [Jeanne & Korinek (2010)]** Parameters and stochastic processes are such that the equilibrium pricing function satisfies \( 1 + \kappa R(X) \psi(B, X) > 0 \) where \( \psi(B, X) \equiv \partial \bar{Q}(B, \Psi(B, X), X)/\partial B \).

**Lemma**

The optimality condition that characterizes the constrained-efficient allocation is

\[
u'(C(B, X)) - \mu(B, X) = R(X) \beta \mathbb{E} \left[ u'(C(B', X')) - \kappa \mu(B', X') \psi(B', X') \right]
\]

where \( \psi(B, X) = \partial \bar{Q}(B, \Psi(B, X), X)/\partial B \) and \( \mu(B, X) \) is the multiplier on the borrowing constraint.

- Solution to the planner’s problem \( \iff Q(B, X) = \bar{Q}(B, \Psi(B, X), X) \).
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- **Assumption [Jeanne & Korinek (2010)]** Parameters and stochastic processes are such that the equilibrium pricing function satisfies $1 + \kappa R(X) \psi(B, X) > 0$ where $\psi(B, X) \equiv \partial \bar{Q}(B, \Psi(B, X), X) / \partial B$.

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The optimality condition that characterizes the constrained-efficient allocation is

$$u'(C(B, X)) - \mu(B, X) = R(X) \beta \mathbb{E} [u'(C(B', X'))] - \kappa \mu(B', X') \psi(B', X')$$

where $\psi(B, X) = \partial \bar{Q}(B, \Psi(B, X), X) / \partial B$ and $\mu(B, X)$ is the multiplier on the borrowing constraint.

- Solution to the planner’s problem $\iff Q(B, X) = \bar{Q}(B, \Psi(B, X), X)$.
- Implementation through macroprudential tax on external borrowing:

$$\tau(B, X) = \frac{\mathbb{E} [\kappa \psi(B', X') \mu(B', X')] | X]}{\mathbb{E} [u'(C(B', X'))] | X}.$$  

- Considers interaction of severity, $\kappa \psi(B, X)$, and incidence, $\mu(B, X)$, of potential future crises.
The Model

Constrained-Efficient Allocation
Constrained-Efficient Allocation

Findings 3 and 4

- Tax increasing in debt.

![Graph showing the tax on debt (τ(B, r, σL)) as a function of b. The graph illustrates the relationship between tax and debt, with a notable decrease in tax as debt increases. The legend indicates "Low r."](image-url)
Constrained-Efficient Allocation

Findings 3 and 4

- Tax increasing in debt. Across SSs, independent $\mu$ and $\psi$ effects dominate (JK (2010), BM (2016)), but in regions of state-space $\psi - \mu$ interaction dominates.
Constrained-Efficient Allocation

Findings 3 and 4

- Tax increasing in debt. Across SSs, independent $\mu$ and $\psi$ effects dominate (JK (2010), BM (2016)), but in regions of state-space $\psi - \mu$ interaction dominates.

$$\mathbb{E}[\kappa \psi(B', X') \mu(B', X')] = \mathbb{E}[\kappa \psi(B', X')] \cdot \mathbb{E}[\mu(B', X')] + \text{Cov}(\kappa \psi(B', X'), \mu(B', X'))$$
Policy response to volatility shocks is non-monotonic → Changes in $\mu$ effects are key: precautionary motives vs. price effects.

$$E[\kappa\psi(B', X')\mu(B', X')] = E[\kappa\psi(B', X')] \cdot E[\mu(B', X')] + Cov(\kappa\psi(B', X'), \mu(B', X'))$$

Tax on debt $\tau(B, r, \sigma')$
Conclusions

- Increases in external risks by themselves do not justify greater macroprudential intervention (e.g. capital controls) ⇒ *Important policy lesson*
  - *Shocks to interest rate levels:* Clear message → consider effect of shocks on asset prices in crisis regions.
  - *Volatility shocks:* “Volatility paradox”
    - Relevant effect of volatility on asset prices (mechanism)
    - Individual precautionary saving motives have effects on particular regions of the state space

- Importance of considering the effects of external shocks on asset prices and their real implications (e.g. borrowing capacity).
  - Aggregate effects not internalized by private imply more room for macroprudential policy → influence borrowing decisions
Conclusions

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  - Aggregate effects not internalized by private imply more room for macroprudential policy → influence borrowing decisions

Thank You!
Motivation and Question

External Risks

- Neumeyer and Perri (2005), Uribe and Yue (2006) and Fernández-Villaverde et al. (2011)
- Reyes-Heroles and Tenorio (2017) using same data as previous work
  - Longstaff et al. (2011), Johri et al. (2015)

a. Deviation of the interest rate from the normal-times country-specific mean (23 EMEs).

b. Deviation of interest rate volatility from normal-times country-specific mean (23 EMEs). Interest rate volatility is measured as the seven-month centered moving standard deviation. t denotes the month in which the sudden stop begins. Dotted lines represent one standard error intervals.
The Model

Derivation of Collateral Constraint: Timing of Events

- Incentive compatibility constraint from limited enforcement problem.
- Recursive setup: state \((b, s, B, X)\) given. HH’s constraint:

- **Household**: chooses optimally \((\hat{b}', \hat{s}', \hat{c})\) given \(Q\) and \(R\).
  - At this point \(\rightarrow \hat{c}\) is a plan.
- **Lender**: does not observe Household’s actions.
- **Household**: given \((\hat{b}', \hat{s}', \hat{c})\)
  - \(\rightarrow\) can divert \((1 - \kappa)\hat{s}'\) and decide to default.
- **Lender**: actions revealed to.
  - \(\rightarrow\) confiscate \(\kappa s'\) in country and sell for \(Q^c\) and lend at \(R\)
- **Household**: can choose final \(c\), regain access to asset and credit markets.

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Morning | Afternoon | Night
The Model

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- **Household**: can choose final \(c\), regain access to asset and credit markets.

\[
V^m (b, s, B, X) = \max_{\hat{c}, \hat{b}', \hat{s}'} \{ V^a (\hat{c}, \hat{b}', \hat{s}', B, X) \}
\]

\[
\hat{c} + Q (B, X) \hat{s}' + \frac{\hat{b}'}{R (X)} = [Q (B, X) + d (X)] s + b
\]

\[
V^a (\hat{c}, \hat{b}', \hat{s}', B, X) = \max_{c, b', s'} \{ u (c) + \beta \mathbb{E} [ V (b', s', B', X') | X] \}
\]

\[
d: c + Q^c (B, X) s' + \frac{b'}{R (X)} = (1 - \kappa) Q^c (B, X) \hat{s}' + \hat{c}
\]

\[
r: c + Q^c (B, X) s' + \frac{b'}{R (X)} = \frac{b'}{R (X)} + Q^c (B, X) \hat{s}' + \hat{c}
\]
The Model

Derivation of Collateral Constraint: Timing of Events

- Incentive compatibility constraint from limited enforcement problem.
- Recursive setup: state \((b, s, B, X)\) given. HH’s constraint:

- **Household**: chooses optimally \((\hat{b}', \hat{s}', \hat{c})\) given \(Q\) and \(R\).
- **Lender**: does not observe **Household**’s actions.
- **Household**: given \((\hat{b}', \hat{s}', \hat{c})\)
  - can divert \((1 - \kappa)\hat{s}'\) and decide to default.

**Morning**

\[
V^m (b, s, B, X) = \max_{\hat{c}, \hat{b}', \hat{s}'} \left\{ V^a (\hat{c}, \hat{b}', \hat{s}', B, X) \right\}
\]

\[
\hat{c} + Q (B, X) \hat{s}' + \frac{\hat{b}'}{R(X)} = [Q (B, X) + d (X)]s + b
\]

**Afternoon**

\[
V^a (\hat{c}, \hat{b}', \hat{s}', B, X) = \max_{c, \hat{b}', \hat{s}'} \left\{ u (c) + \beta \mathbb{E} \left[ V (b', s', B', X') | X \right] \right\}
\]

\[
d: c + Q^c (B, X) s' + \frac{\hat{b}'}{R(X)} = (1 - \kappa) Q^c (B, X) s' + \hat{c}
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\[
r: c + Q^c (B, X) s' + \frac{\hat{b}'}{R(X)} = \frac{\hat{b}'}{R(X)} + Q^c (B, X) s' + \hat{c}
\]

**Night**

\[
V^a (\hat{c}, \hat{b}', \hat{s}', B, X) = \max_{c, \hat{b}', \hat{s}'} \left\{ u (c) + \beta \mathbb{E} \left[ V (b', s', B', X') | X \right] \right\}
\]

- To avoid diversion and default: 
  \[- \frac{\hat{b}'}{R(X)} \leq \kappa Q^c (B, X) s'.\]
- No arbitrage \(\Leftrightarrow Q (B, X) u' (\hat{C} (B, X)) - \kappa \mu (B, X) Q^c (B, X) = Q^c (B, X) u' (\hat{C} (B, X)).\)
Estimation and Calibration

Table: Baseline parameterization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time discount</td>
<td>$\beta$</td>
<td>0.96</td>
</tr>
<tr>
<td>Relative risk aversion</td>
<td>$\gamma$</td>
<td>2</td>
</tr>
<tr>
<td>Dividends</td>
<td>$d$</td>
<td>1</td>
</tr>
<tr>
<td>Collateral constraint</td>
<td>$\kappa$</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Result of estimation:

$$
\begin{pmatrix}
    z_t \\
    r_t
\end{pmatrix}
= \begin{pmatrix}
    0.0052 \\
    0.0025
\end{pmatrix}
+ \begin{pmatrix}
    0.6079 & -0.1321 \\
    0.1289 & 0.8261
\end{pmatrix}
\begin{pmatrix}
    z_{t-1} \\
    r_{t-1}
\end{pmatrix}
+ \begin{pmatrix}
    \epsilon_t^z \\
    \epsilon_t^r
\end{pmatrix},
$$

and the covariance and transition matrices are composed of:

$$
\sigma^z = 0.0312, \quad \rho = -0.4048, \quad \pi_L = 0.9610,
$$

$$
\sigma'_L = 0.0150, \quad \sigma'_H = 0.0661, \quad \pi_H = 0.7468.
$$
The Model

Constrained-Efficient Allocation

Lemma

Given an arbitrary future policy rule, \( \Psi(B, X) \) and the associated asset pricing function, \( Q(B, X) \), the social planner solves

\[
W(B, X) = \max_{c, B'} \left\{ u(c) + \beta \mathbb{E} [W(B', X) | X] \right\} \text{ s.t.}
\]

\[
c + \frac{B'}{R(X)} = d(X) + B,
\]

\[
\frac{B'}{R(X)} \leq \kappa \bar{Q}(B, B', X)
\]

and the valuation of collateral is consistent with the household's trading of the stocks of the tree

\[
\bar{Q}(B, B', X) = \beta \mathbb{E} \left[ u' \left( B' + d(X') - \frac{\Psi(B', X')}{R(X')} \right) \left( Q(B', X') + d(X') \right) \left| X \right. \right].
\]
Finding 4

- Should the planner intensify his intervention when external volatility increases? → Not necessarily.

Prevalence of $\tau = 0$: Low Volatility $\rightarrow 55.3\%$, High Volatility $\rightarrow 59.6\%$. 
Constrained-Efficient Allocation

Findings 3 and 4

- Decomposition of optimal tax.