

Preliminary Examination

Data Structures

Santa Clara University, COEN, Fall 2006

Do either Problem 1 or Problem 2. Do Problem 3

(If you hand in three solutions, then the highest scoring ones are picked, but trying to do more than three is not a recommended strategy.)

- 1 You are given an array of numbers of length n and a sum s . Create an $O(n)$ algorithm that determines whether there are two numbers in the array that add to the value s . For example, for the array $[1,3,2,5]$ and $s = 8$, the result is true, but for the same array and $s = 10$, the result is false. You should clearly describe the algorithm and you should justify why it is $O(n)$. You may write code or pseudo-code. Solutions that are not $O(n)$ are accepted with some penalty.
- 2 Consider the following implementation of a set. We use a pair of arrays. Both of them are sorted. The length of the shorter array is the square root of the length of the longer array, although it may have some empty entries if it is not full yet. To add an element, we insert it in order in the short array, moving other elements as necessary. If the short array fills up, we allocate larger long and short arrays and *merge* the two old arrays (the old long array and the old short array) into the new long array. The new short array is empty. To find an element, we use binary search on both arrays. Show that this data structure gives $O(\lg n)$ lookup time, but the *amortized* time to add an element is $O(\sqrt{n})$.
- 3 Show that the following problem is in *NP*. Let F be a finite field. That is, F is a finite set and has the operation of addition and multiplication with the same rules of arithmetic as in the more familiar fields of real or complex numbers. Assume that all these operations are done in constant time. In particular, we have the normal notion of powers. A polynomial in n variables x_1, x_2, \dots, x_n is what you might expect, basically a sum of monomials $a \cdot x_1^{m_1} \cdot x_2^{m_2} \cdot \dots \cdot x_n^{m_n}$ with exponents m_i that are non-negative integers. The root problem is given F and a polynomial $f(x_1, x_2, \dots, x_n)$ to find n elements in F that plugged into the polynomial make it zero. Basically, is there a solution to $f(x_1, x_2, \dots, x_n) = 0$.