Robust Decentralized Exciter Control With Linear Feedback

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***Abstract*—A new strategy is developed for the design of robust decentralized exciter control in power systems. The method is computationally attractive and the resulting feedback is linear, which allows for easy implementation. Experiments on the IEEE 39 bus system demonstrate that such a control is robust with respect to the fault location and to variations in the system operating point.**

***Index Terms*—Exciter control, linear matrix inequalities, robustness.**

I. Introduction

ANY successful strategy for the control of large-scale power systems must satisfy two fundamental requirements. In the first place, the control must be *decentralized*, since only local measurements are normally available to any given machine. Secondly, the control needs to be *robust*, in the sense that it must guarantee satisfactory performance over a wide range of operating conditions and disturbances. This requirement is particularly important in a deregulated environment, where the system tends to be more stressed and the load distribution is virtually impossible to anticipate.

The last decade has seen a number of new developments in the design of robust power system control. Although the proposed methods include both decentralized turbine/governor [1]–[5] and decentralized exciter control designs [6]–[11], it is fair to say that the latter approach has received more attention, given the relatively small time constants associated with the excitation system control loop. Much of the recent work related to robust exciter control has been based on the concept of direct feedback linearization, which transforms the original nonlinear model into a linear one. After such a transformation, the control design becomes quite straightforward, but the implementation is complicated by the fact that the resulting controller is nonlinear. Despite this difficulty, relatively few attempts have been made to develop reliable techniques for designing *linear* robust exciter control. Among these, it is of interest to mention the approach proposed in [8], where linear controller design is based on Lyapunov's method. A problem that arises in this context is associated with the quadratic term in the model, which cannot be properly incorporated into the analysis. In [8], this issue was resolved by performing a partial linearization, which amounts to discarding the problematic term from the model. In the following we will take a different approach to this problem, which does not require any approximations.

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The main objective of this paper is to develop a systematic approach for designing robust decentralized exciter control. The resulting controllers are linear and the gain matrix can be obtained directly, with no need for tuning parameters or trial and error procedures. The design is based on linear matrix inequalities (LMI) [12]–[15] and the general framework developed in [5] and [16]. In that respect, this paper represents a continuation of the work in [5], which focused on the design of robust turbine/governor control. We should point out, however, that the extension to exciter control is by no means straightforward, and requires a generalization of the LMI approach that incorporates a wider class of nonlinearities.

II. Control Design Using Linear Matrix Inequalities

To describe robust control design in the context of LMI, let us consider system

$$\dot{x} = Ax + Bu + Gh(x) \tag{1}$$

where $x \in R^n$ is the state of the system, $u \in R^m$ is the input vector, A and B are constant $n \times n$ and $n \times m$ matrices, and $h: R^n \to R^n$ is a piecewise—continuous nonlinear function in x, satisfying $h(0) = 0$. The term $h(x)$ is assumed to be uncertain, but bounded by a quadratic inequality

$$h^T(x)h(x) \leq \alpha^2 x^T H^T H x \tag{2}$$

where H is a constant matrix and α is a scalar parameter that reflects the degree of robustness. If we assume a linear feedback control law $u = Kx$, the closed-loop system takes the form

$$\dot{x} = \hat{A}x + Gh(x) \tag{3}$$

where $\hat{A} = A + BK$. The global asymptotic stability of the equilibrium $x = 0$ can then be established using a Lyapunov function

$$V(x) = x^T P x \tag{4}$$

where P is a symmetric positive definite matrix (denoted $P > 0$). As is well known, a sufficient condition for stability is for the derivative of $V(x)$ to be negative along the solutions of (3). Formally, this condition can be expressed as a pair of inequalities

$$\begin{gathered} x^T P x > 0 \\ \begin{bmatrix} x \\ h \end{bmatrix}^T \begin{bmatrix} \hat{A}^T P + P\hat{A} & PG \\ G^T P & 0 \end{bmatrix} \begin{bmatrix} x \\ h \end{bmatrix} < 0. \end{gathered} \tag{5}$$

Defining matrix $Y = \tau P^{-1}$ (where τ is a positive scalar) and setting $\gamma = 1/\alpha^2$, the control design can be formulated as an LMI problem in Y, L and γ [5], [16]:

Minimize γ, *subject to* $Y > 0$ *and*

$$\begin{bmatrix} AY + YA^T + BL + L^TB^T & G & YH^T \\ G^T & -I & 0 \\ HY & 0 & -\gamma I \end{bmatrix} < 0 \quad (6)$$

where $L = KY$.

If the optimization problem (6) is feasible, the resulting gain matrix stabilizes system (3) for all nonlinearities satisfying (2). We should note, however, that the process outlined above places no restrictions on the size of the gain. To limit the gain and, at the same time, guarantee a desired value $\bar{\alpha}$, we need to apply the following modification of the optimization problem [5], [16]:

Minimize $\gamma + \kappa_Y + \kappa_L$ *subject to*

$$Y > 0; \quad \gamma - \frac{1}{\bar{\alpha}^2} < 0 \quad (7)$$

$$\begin{bmatrix} AY + YA^T + BL + L^TB^T & G & YH^T \\ G^T & -I & 0 \\ HY & 0 & -\gamma I \end{bmatrix} < 0 \quad (8)$$

and

$$\begin{bmatrix} -\kappa_L I & L^T \\ L & -I \end{bmatrix} < 0; \quad \begin{bmatrix} Y & I \\ I & \kappa_Y I \end{bmatrix} > 0 \quad (9)$$

where κ_Y *and* κ_L *are constraints on the gain magnitudes, satisfying*

$$L^T L < \kappa_L I, \quad Y^{-1} < \kappa_Y I. \quad (10)$$

For the purposes of exciter control design, it will be necessary to extend these results to the class of problems (1) where the nonlinearities satisfy a *generalized* constraint of the form

$$h^T(x)h(x) \leq x^T H^T(x) H(x) x. \quad (11)$$

The elements of matrix $H(x)$ in (11) can be unbounded functions of x, and our only assumption is that there exists a constant matrix $\bar{H}$ and a region $\Omega \subset R^n$ such that $0 \in \Omega$, and

$$h^T(x)h(x) \leq \alpha^2 x^T \bar{H}^T \bar{H} x, \quad \forall x \in \Omega \quad (12)$$

for some $\alpha \geq 1$. In order to analyze this type of system, let us consider an arbitrary function $h(x)$ that satisfies condition (12), and associate with it a piecewise-continuous function $\hat{h} : R^n \rightarrow R^n$ defined as

$$\hat{h}(x) = \begin{cases} h(x), & x \in \Omega \\ \rho(x), & x \notin \Omega \end{cases} \quad (13)$$

where

$$\rho^T(x)\rho(x) \leq \alpha^2 x^T \bar{H}^T \bar{H} x, \quad \forall x \in R^n. \quad (14)$$

From the construction of $\hat{h}(x)$, it clearly follows that inequality:

$$\hat{h}^T(x)\hat{h}(x) \leq \alpha^2 x^T \bar{H}^T \bar{H} x \quad (15)$$

holds for *any* $x \in R^n$, which implies that the corresponding system

$$\dot{x} = Ax + Bu + G\hat{h}(x) \quad (16)$$

satisfies the requirements of standard LMI optimization.

If we now perform this optimization for system (16) and replace matrix H in (8) with $\bar{H}$, we will obtain a gain matrix K that *globally* stabilizes the closed-loop system

$$\dot{x} = (A + BK)x + G\hat{h}(x). \quad (17)$$

The LMI procedure also produces a Lyapunov function

$$V(x) = x^T P x \quad (18)$$

which allows us to define a collection of sets

$$\Pi(r) = \{x : V(x) \leq r\}. \quad (19)$$

Denoting the largest set that satisfies $\Pi(r) \subset \Omega$ by $\Pi(r_0)$, it now follows that solutions $x(t; t_0, x_0)$ of (17) originating at $x_0 \in \Pi(r_0)$ must remain in the set $\Pi(r_0)$ at all times; in other words, $x_0 \in \Pi(r_0)$ implies $x(t; t_0, x_0) \subset \Pi(r_0)$ for all $t \geq t_0$, and $\Pi(r_0)$ is an invariant set [17], [18]. Any such trajectory $x(t) = x(t; t_0, x_0)$ satisfies

$$\hat{h}(x(t)) = h(x(t)), \quad \forall t \quad (20)$$

by definition, and is therefore a solution of the closed loop system

$$\dot{x} = (A + BK)x + Gh(x) \quad (21)$$

as well. We can thus conclude that the gain matrix K obtained in this manner stabilizes system (21) *locally*, and that $\Pi(r_0)$ represents an estimate of the region of attraction.

Recalling that $h(x)$ was chosen as an *arbitrary* function that satisfies (12), it follows that the stability of (21) is guaranteed for *any* nonlinearity with this property. In this context, it is important to note an inherent trade-off that exists between the size of the region of attraction and the robustness of the system. Namely, whenever $\alpha > 1$, inequality (12) clearly holds even if there is a degree of uncertainty in $h(x)$. In that sense, a larger α implies a greater degree of robustness. On the other hand, it is also true that the value of α obtained from the LMI process *decreases* as region Ω becomes larger. The following example clearly illustrates this effect.

Example 1: Consider the system

$$\begin{aligned} \dot{x}_1 &= 3x_1 + x_1x_2 + x_2 + u \\ \dot{x}_2 &= -x_1 + x_2^2 + u \end{aligned} \quad (22)$$

which has an unstable equilibrium at the origin when $u = 0$, and a nonlinear term $h(x)$ that satisfies

$$h^T(x)h(x) = x^T \begin{bmatrix} x_2^2 & 0 \\ 0 & x_2^2 \end{bmatrix} x. \quad (23)$$

This system has a pair of additional equilibria—an unstable one at $[0.14 \ -0.38]^T$, and a stable one at $[6.85 \ -2.62]^T$.

If we initially choose region Ω as

$$\Omega_1 = \{x : x_1 \in R, \quad |x_2| \leq 1\} \quad (24)$$

it is obvious that

$$h^T(x)h(x) \leq x^T x, \quad \forall x \in \Omega_1. \quad (25)$$

In this case, the LMI optimization using $\bar{H} = I$ and $\|K\| \leq 25$ produces $\alpha = 1.6$ and gains $K_1 = -20.25$ and $K_2 = 8.31$, respectively. This result indicates that the closed loop system remains locally stable for *any* nonlinearity $\tilde{h}(x)$ that satisfies

$$\tilde{h}^T(x)\tilde{h}(x) \leq \alpha^2 x^T \bar{H}^T \bar{H} x = 2.56\, x^T x, \quad \forall x \in \Omega_1. \quad (26)$$

Comparing (25) and (26), we can conclude that the system is capable of tolerating considerable uncertainty in $h(x)$, provided that the initial conditions remain within set $\Pi(r_0) \subset \Omega_1$.

Let us now consider a larger region Ω, defined as

$$\Omega_2 = \{x : x_1 \in R, \quad |x_2| \le 1.6\}. \tag{27}$$

In this region, $h(x)$ satisfies

$$h^T(x)h(x) \le 2.56\, x^T x, \quad \forall x \in \Omega_2. \tag{28}$$

When the LMI process is repeated with $\bar{H} = 1.6I$ and $\|K\| \le 25$ we obtain $\alpha = 1$, which implies that the closed loop system is locally stable for *any* $\tilde{h}(x)$ such that

$$\tilde{h}^T(x)\tilde{h}(x) \le \alpha^2 x^T \bar{H}^T \bar{H} x = 2.56\, x^T x, \quad \forall x \in \Omega_2. \tag{29}$$

It is readily observed from (28) that $h(x)$ satisfies inequality (29) with *no* margin for uncertainty. This scenario clearly illustrates an inherent trade-off between the size of the region of attraction and the degree of robustness. We should also observe that the gains obtained in this case, $K_1 = -20.15$ and $K_2 = 8.24$, are virtually identical to the ones obtained using $\bar{H} = I$, which suggests that this trade-off has little effect on the gain matrix. An intuitive explanation for this fact (corroborated by numerous experiments) is that the gains computed by LMI optimization tend to depend more on the bounding of $\|K\|$ than on the specific choice of region Ω and matrix $\bar{H}$.

III. Design of Robust Exciter Control

Our objective in the following will be to apply the control strategy developed in the previous section to exciter control design. In order to do this, it is first necessary to consider the appropriate mathematical description of an n—machine power system with two axis generator models. The state space representation for such a system has the form

$$\begin{aligned}
\dot{\delta}_i &= \omega_i \\
M_i \dot{\omega}_i &= P^0_{m_i} - P_{e_i} - D_i \omega_i \\
T'_{d0_i} \dot{E}_{q_i} &= -E_{q_i} - \left(x_{d_i} - x'_{d_i}\right) I_{d_i} + E_{fd_i} \\
T'_{q0_i} \dot{E}_{d_i} &= -E_{d_i} + \left(x_{q_i} - x'_{d_i}\right) I_{q_i}
\end{aligned} \tag{30}$$

for $i = 1, 2, \ldots, n$, with

$$\begin{aligned}
I_{d_i} &= \sum_k [G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}] E_{d_k} \\
&\quad + \sum_k [G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}] E_{q_k}
\end{aligned} \tag{31}$$

$$\begin{aligned}
I_{q_i} &= \sum_k [B_{ik}\cos\delta_{ik} - G_{ik}\sin\delta_{ik}] E_{d_k} \\
&\quad + \sum_k [G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}] E_{q_k}
\end{aligned} \tag{32}$$

and $\delta_{ik} = \delta_i - \delta_k$ (a detailed description of all symbols and quantities can be found in [19]). In the following, we will assume that $E_{fd_i} = E^0_{fd_i} + u_i$ and that the control has the form

$$\begin{aligned}
u_i &= k_{1i}\left(\delta_i - \delta_i^r\right) + k_{2i}\left(\omega_i - \omega_i^r\right) \\
&\quad + k_{3i}\left(E_{q_i} - E^r_{q_i}\right) + k_{4i}\left(E_{d_i} - E^r_{d_i}\right)
\end{aligned} \tag{33}$$

where $\{\delta_i^r, \omega_i^r, E^r_{q_i}, E^r_{d_i}\}$ represent user-defined reference values, and $\{k_{1i}, k_{2i}, k_{3i}, k_{4i}\}$ are the gains. It is important to recognize that the proposed control *does not* require knowledge of the rotor angle of one generator relative to another, or any other external information. All the quantities in (33) are locally available, either by measurement or by calculation.

Defining new states $\{x_{1i}, x_{2i}, x_{3i}, x_{4i}\}$ as deviations from the equilibrium values $\{\delta_i^e, \omega_i^e, E^e_{q_i}, E^e_{d_i}\}$, the model in (30) can be rewritten in the form

$$\begin{aligned}
\dot{x}_{1i} &= x_{2i} \\
M_i \dot{x}_{2i} &= -D_i x_{2i} - \Delta P_{e_i}(x) \\
T'_{d0_i} \dot{x}_{3i} &= -x_{3i} - \left(x_{d_i} - x'_{d_i}\right) \Delta I_{d_i}(x) + u_i \\
T'_{q0_i} \dot{x}_{4i} &= -x_{4i} + \left(x_{q_i} - x'_{d_i}\right) \Delta I_{q_i}(x)
\end{aligned} \tag{34}$$

where $\Delta I_{d_i}(x) = I_{d_i}(x) - I^e_{d_i}$, $\Delta I_{q_i}(x) = I_{q_i}(x) - I^e_{q_i}$, and

$$\Delta P_{e_i}(x) = \left(E_{d_i} I_{d_i} + E_{q_i} I_{q_i}\right) - \left(E^e_{d_i} I^e_{d_i} + E^e_{q_i} I^e_{q_i}\right). \tag{35}$$

A straightforward algebraic manipulation of (31) and (32) allows us to express $\Delta I_{d_i}(x)$ and $\Delta I_{q_i}(x)$ as

$$\begin{aligned}
\Delta I_{d_i}(x) &= -B_{ii} x_{3i} + G_{ii} x_{4i} + \sum_{k \ne i} \alpha_{ik}(\delta) x_{4k} \\
&\quad + \sum_{k \ne i} \beta_{ik}(\delta) x_{3k} + \sum_{k \ne i} \mu_{ik}(\delta) \sin y_{ik}
\end{aligned} \tag{36}$$

and

$$\begin{aligned}
\Delta I_{q_i}(x) &= G_{ii} x_{3i} + B_{ii} x_{4i} + \sum_{k \ne i} \hat{\alpha}_{ik}(\delta) x_{4k} \\
&\quad + \sum_{k \ne i} \hat{\beta}_{ik}(\delta) x_{3k} + \sum_{k \ne i} \hat{\mu}_{ik}(\delta) \sin y_{ik}
\end{aligned} \tag{37}$$

where $y_{ik} = (x_{1i} - x_{1k})/2$ and

$$\begin{aligned}
\alpha_{ik}(\delta) &= G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik} \\
\hat{\alpha}_{ik}(\delta) &= B_{ik}\cos\delta_{ik} - G_{ik}\sin\delta_{ik}
\end{aligned} \tag{38}$$

$$\begin{aligned}
\beta_{ik}(\delta) &= G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik} \\
\hat{\beta}_{ik}(\delta) &= B_{ik}\sin\delta_{ik} + G_{ik}\cos\delta_{ik}
\end{aligned} \tag{39}$$

$$\begin{aligned}
\mu_{ik}(\delta) &= 2\left[G_{ik}E^e_{d_k} - B_{ik}E^e_{q_k}\right]\sin\frac{(\delta^e_{ik} + \delta_{ik})}{2} \\
&\quad + 2\left[G_{ik}E^e_{q_k} + B_{ik}E^e_{d_k}\right]\cos\frac{(\delta^e_{ik} + \delta_{ik})}{2}
\end{aligned} \tag{40}$$

$$\begin{aligned}
\hat{\mu}_{ik}(\delta) &= 2\left[B_{ik}E^e_{d_k} + G_{ik}E^e_{q_k}\right]\sin\frac{(\delta^e_{ik} + \delta_{ik})}{2} \\
&\quad + 2\left[B_{ik}E^e_{q_k} - G_{ik}E^e_{d_k}\right]\cos\frac{(\delta^e_{ik} + \delta_{ik})}{2}.
\end{aligned} \tag{41}$$

Defining constants

$$v_{ik} = G^2_{ik} + B^2_{ik} \tag{42}$$

and

$$r_{ik} = \left[G_{ik}E^e_{d_k} - B_{ik}E^e_{q_k}\right]^2 + \left[G_{ik}E^e_{q_k} + B_{ik}E^e_{d_k}\right]^2 \tag{43}$$

it is easily verified that

$$|\alpha_{ik}(\delta)|, \quad |\hat{\alpha}_{ik}(\delta)|, \quad |\beta_{ik}(\delta)|, \quad \left|\hat{\beta}_{ik}(\delta)\right| \le \sqrt{v_{ik}} \tag{44}$$

and

$$|\mu_{ik}(\delta)|, \quad |\hat{\mu}_{ik}(\delta)| \le 2\sqrt{r_{ik}} \tag{45}$$

for *any* δ.

After some manipulation, (35) can be expressed in the form

$$\begin{aligned}
\Delta P_{e_i}(x) &= x_{4i} I^e_{d_i} + x_{3i} I^e_{q_i} + E^e_{d_i} \Delta I_{d_i}(x) + E^e_{q_i} \Delta I_{q_i}(x) \\
&\quad + x_{4i} \Delta I_{d_i}(x) + x_{3i} \Delta I_{q_i}(x).
\end{aligned} \tag{46}$$

Since the first two terms in (36), (37) and (46) are linear in x, we now obtain an overall state model of the form

$$\dot{x}_i = A_i x_i + B_i u_i + G_i h_i(x) \quad (i = 1, 2, \ldots, n) \tag{47}$$

where

$$B_i = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T'_{d0i}} \\ 0 \end{bmatrix} \tag{48}$$

$$G_i = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{M_i} & 0 & 0 \\ 0 & 0 & -\frac{\Delta x_{di}}{T'_{d0i}} & 0 \\ 0 & 0 & 0 & -\frac{\Delta x_{qi}}{T'_{q0i}} \end{bmatrix} \tag{49}$$

with $\Delta x_{di} = x_{di} - x'_{d_i}$ and $\Delta x_{qi} = x_{qi} - x'_{d_i}$, respectively. Matrix A_i has the structure

$$A_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{bmatrix} \tag{50}$$

with elements

$$\begin{aligned} a_{22} &= -\frac{D_i}{M_i} \\ a_{23} &= -\frac{\left[G_{ii}E^e_{q_i} - B_{ii}E^e_{di} + I^e_{q_i}\right]}{M_i} \\ a_{24} &= -\frac{\left[G_{ii}E^e_{d_i} + B_{ii}E^e_{qi} + I^e_{d_i}\right]}{M_i} \\ a_{33} &= -\frac{1}{T'_{d0_i}} + \frac{B_{ii}\Delta x_{di}}{T'_{d0_i}} \\ a_{34} &= -\frac{G_{ii}\Delta x_{di}}{T'_{d0_i}} \\ a_{43} &= \frac{G_{ii}\Delta x_{qi}}{T'_{q0_i}} \\ a_{44} &= -\frac{1}{T'_{q0_i}} + \frac{B_{ii}\Delta x_{qi}}{T'_{q0_i}}. \end{aligned} \tag{51}$$

The nonlinear term $h_i(x)$ can be expressed as a vector $h_i(x) = [0 \;\; h_{i2}(x) \;\; h_{i3}(x) \;\; h_{i4}(x)]^T$, with components

$$\begin{aligned} h_{i2}(x) = &\sum_{k\neq i} \lambda_{ik}(\delta)x_{4k} + \sum_{k\neq i} \rho_{ik}(\delta)x_{3k} \\ &+ \sum_{k\neq i} \xi_{ik}(\delta)\sin y_{ik} + \psi_i(x) \end{aligned} \tag{52}$$

$$\begin{aligned} h_{i3}(x) = &\sum_{k\neq i} \alpha_{ik}(\delta)x_{4k} + \sum_{k\neq i} \beta_{ik}(\delta)x_{3k} \\ &+ \sum_{k\neq i} \mu_{ik}(\delta)\sin y_{ik} \end{aligned} \tag{53}$$

$$\begin{aligned} h_{i4}(x) = &\sum_{k\neq i} \hat{\alpha}_{ik}(\delta)x_{4k} + \sum_{k\neq i} \hat{\beta}_{ik}(\delta)x_{3k} \\ &+ \sum_{k\neq i} \hat{\mu}_{ik}(\delta)\sin y_{ik}. \end{aligned} \tag{54}$$

The quantities $\lambda_{ik}(\delta)$, $\rho_{ik}(\delta)$, and $\xi_{ik}(\delta)$ are defined as

$$\begin{aligned} \lambda_{ik}(\delta) &= E^e_{d_i}\alpha_{ik}(\delta) + E^e_{q_i}\hat{\alpha}_{ik}(\delta) \\ \rho_{ik}(\delta) &= E^e_{d_i}\beta_{ik}(\delta) + E^e_{q_i}\hat{\beta}_{ik}(\delta) \\ \xi_{ik}(\delta) &= E^e_{d_i}\mu_{ik}(\delta) + E^e_{q_i}\hat{\mu}_{ik}(\delta) \end{aligned} \tag{55}$$

and the term $\psi_i(x)$ is given by

$$\psi_i(x) = x_{4i}\Delta I_{d_i}(x) + x_{3i}\Delta I_{q_i}(x). \tag{56}$$

It is important to recognize at this point that $\psi_i(x)$ is *quadratic* in x, and therefore requires a bound of the form (11). Problems associated with this type of nonlinearity were recognized in [8], where a partial linearization of the model was used to eliminate the undesirable terms. We now proceed to show how the technique proposed in Section II can resolve these difficulties without any simplification of the system model.

Before applying the LMI approach, we should observe that the system in (47) has an input-decentralized structure [17], which is suitable for applying local control laws of the form $u_i(x_i) = K_i x_i (i = 1, \ldots, n)$, where $x_i \in R^{n_i}$ are the subsystem states and K_i are the corresponding $m_i \times n_i$ gain matrices. To incorporate this feature into the framework of convex optimization, let us first assume that the nonlinearities in each subsystem are bounded as

$$h_i^T(x)h_i(x) \leq \alpha_i^2 x^T \bar{H}_i^T \bar{H}_i x, \quad \forall x \in \Omega \tag{57}$$

which is the decentralized equivalent of (12). Defining matrices $K_D = \text{diag}\{K_1, \ldots, K_n\}$ and $L_D = K_D Y_D$, where Y_D is a block diagonal version of matrix Y in (8), the optimization can now be reformulated as [16].

Minimize $\sum_{i=1}^n \gamma_i + \sum_{i=1}^n \kappa_{Y_i} + \sum_{i=1}^n \kappa_{L_i}$ *subject to:*

$$Y_D > 0; \quad \gamma_i - \frac{1}{\bar{\alpha}_i^2} < 0 \tag{58}$$

$$\begin{bmatrix} W_D & G_D & Y_D H_1^T & \cdots & Y_D H_n^T \\ G_D^T & -I & 0 & \cdots & 0 \\ H_1 Y_D & 0 & -\gamma_1 I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ H_n Y_D & 0 & 0 & \cdots & -\gamma_n I \end{bmatrix} < 0 \tag{59}$$

and

$$\begin{bmatrix} -\kappa_{L_i} I & L_i^T \\ L_i & -I \end{bmatrix} < 0; \quad \begin{bmatrix} Y_i & I \\ I & \kappa_{Y_i} I \end{bmatrix} > 0 \tag{60}$$

where $W_D = A_D Y_D + Y_D A_D^T + B_D L_D + L_D^T B_D^T$, $\gamma_i = 1/\alpha_i^2$ *and* κ_{Y_i} *and* κ_{L_i} *are constraints on the magnitudes of the decentralized gains, satisfying*

$$L_i^T L_i < \kappa_{L_i} I, \quad Y_i^{-1} < \kappa_{Y_i} I. \tag{61}$$

To obtain an explicit bound of the form (57), let us observe that the coefficients $\lambda_{ik}(\delta)$, $\rho_{ik}(\delta)$ and $\xi_{ik}(\delta)$ in (55) can be bounded as

$$\begin{aligned} |\lambda_{ik}(\delta)|, \quad |\rho_{ik}(\delta)| &\leq \sqrt{w_{ik}} \\ |\xi_{ik}(\delta)| &\leq \sqrt{c_{ik}^2 + d_{ik}^2} = u_{ik} \end{aligned} \tag{62}$$

where

$$w_{ik} = \left[G_{ik}E^e_{d_k} + B_{ik}E^e_{q_k}\right]^2 + \left[B_{ik}E^e_{d_k} - G_{ik}E^e_{q_k}\right]^2 \quad (63)$$

and

$$\begin{aligned} c_{ik} =& 2E^e_{q_k}\left[G_{ik}E^e_{q_k} + B_{ik}E^e_{d_k}\right] \\ &+ 2E^e_{d_k}\left[G_{ik}E^e_{d_k} - B_{ik}E^e_{q_k}\right] \\ d_{ik} =& 2E^e_{q_k}\left[B_{ik}E^e_{q_k} - G_{ik}E^e_{d_k}\right] \\ &+ 2E^e_{d_k}\left[G_{ik}E^e_{q_k} + B_{ik}E^e_{d_k}\right]. \end{aligned} \quad (64)$$

Using bounds (44) and (45) in conjunction with (62), it can now be shown that

$$\begin{aligned} h_i^T(x)h_i(x) \le& x^T\left[F_{i1}^T(D_{i1} + 2D_{i3})F_{i1}\right]x \\ &+ x^T\left[F_{i1}^T 2\phi_i(x)D_{i2}F_{i1} + 2\phi_i^2(x)F_{i2}^T F_{i2}\right]x \end{aligned} \quad (65)$$

where

$$\phi_i(x) = \sum_{k\ne i}\sqrt{v_{ik}}\left(|x_{4k}| + |x_{3k}|\right) + \sum_{k\ne i}2\sqrt{r_{ik}}\left|\sin\frac{(x_{1i} - x_{1k})}{2}\right|. \quad (66)$$

The constant matrices $D_{ij}(j = 1,2,3)$ and $F_{ij}(j = 1,2)$ are easily constructed; details of this process are provided in the Appendix.

IV. Experimental Results

The LMI—based control design proposed in the previous sections was applied to the IEEE 39 bus system [20], with $\|K\| \le 650$ as a constraint on the gain sizes. Two axis generator models with IEEE Type I exciters were used in all the simulations, with field voltage limits $-3 \le E_{fd_i} \le 6$ $(i = 1, 2, \ldots, n)$.

In our experiments we considered a variety of short circuit faults, and simulated the transient responses and critical clearing times. Different fault locations were modeled using the scalar quantity σ, which represents the fraction of the transmission line between the lower-numbered bus and the fault. In all cases, the following fault sequence was assumed:

1) The system is in the prefault state.
2) The fault occurs at $t = 0.1$ s.
3) The fault is cleared by removing the line.

In Figs. 1–4 we show the four states corresponding to generator 8 for a fault on line (25), (26) with $\sigma = 0.05$ (the fault was cleared after 0.15 s). These responses indicate that the proposed exciter control performs well from the standpoint of transient stability.

The terminal voltage V_t and the field voltage E_{fd} associated with generator 8 are provided in Figs. 5 and 6 respectively. It should be noted that V_t remains within 10% of its prefault value $V_t^0 = 1.0278$ after the fault is cleared; the voltage is restored to its nominal value after approximately 4 s. This is an important performance measure, since terminal voltage regulation is the principal function of exciter control.

To further evaluate the robustness of the control, we computed critical clearing times (CCT) for the following three disturbances:

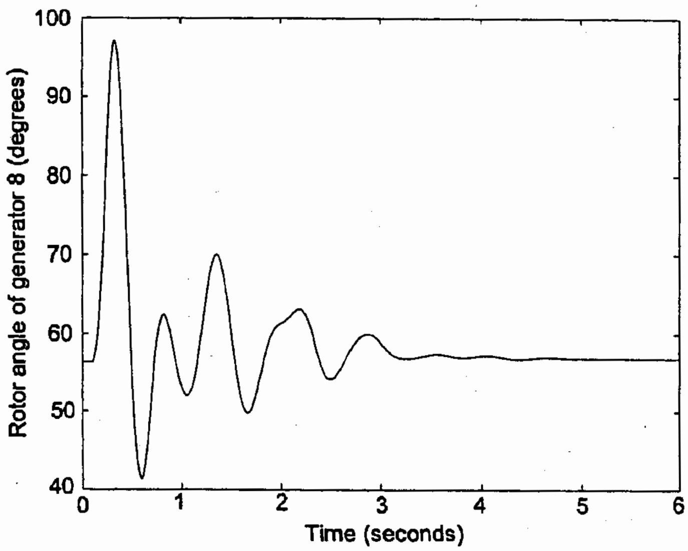

Fig. 1. Rotor angle of generator 8.

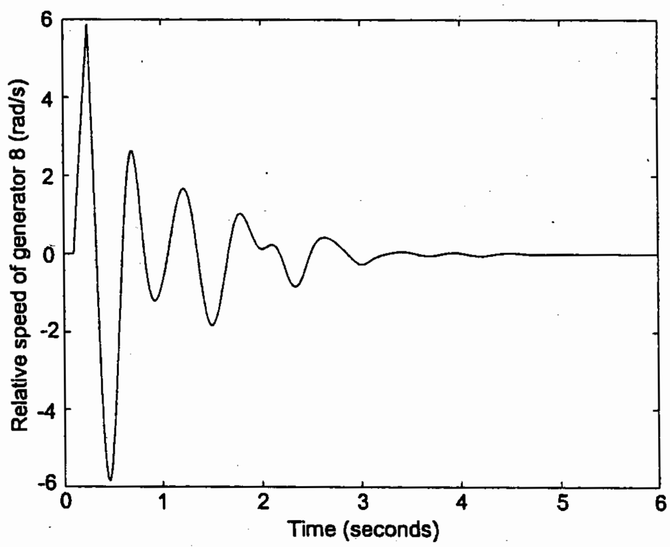

Fig. 2. Relative speed of generator 8.

Disturbance 1. Three phase fault on line (25, 26), with $\sigma = 0.05$.;

Disturbance 2. Three phase fault on line (5, 6), with $\sigma = 0.95$.;

Disturbance 3. Three phase fault on line (22, 23), with $\sigma = 0.05$.

The critical clearing times corresponding to these disturbances were computed under nominal loading conditions and for an overall load increase of 15%. In the latter case, we considered several load distributions—a uniform one, where each load is increased by 15%, and two different random distributions (referred to in the following as Cases 1 and 2). The results of these experiments are summarized in Tables I–IV; they indicate that the proposed control is robust with respect to load changes and different load distributions.

In addition to the simulation results, it is also necessary to discuss a number of practical issues related to the implementation of the proposed control. We begin by recalling that the feedback scheme in (33) requires no remote information, and

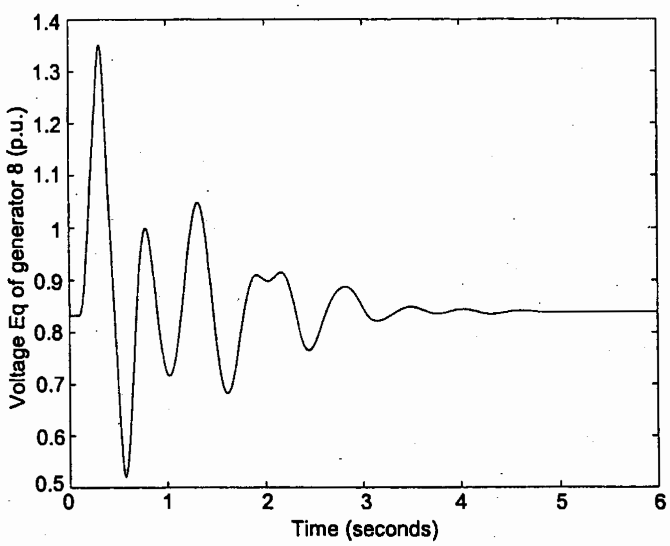

Fig. 3. Voltage E_q of generator 8.

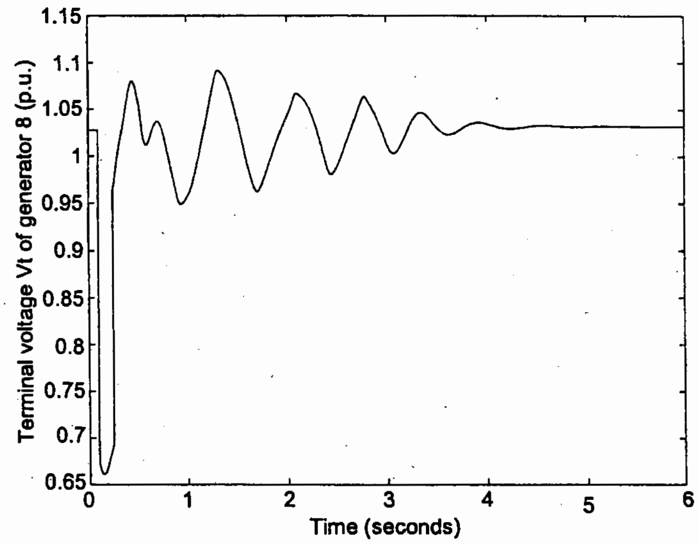

Fig. 5. Terminal voltage V_t of generator 8.

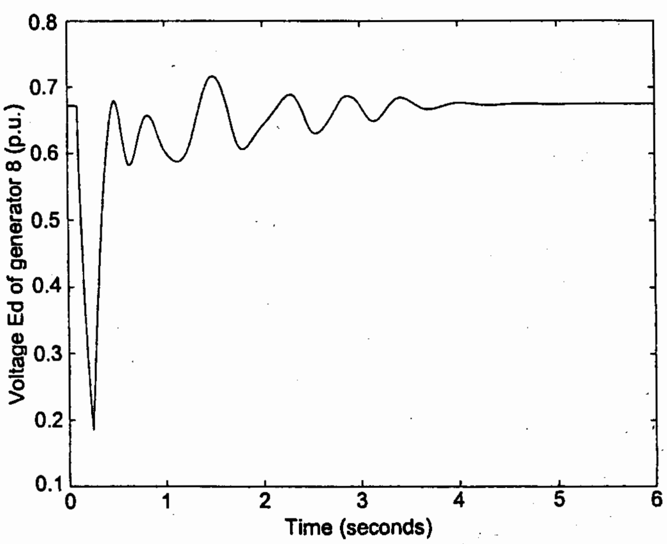

Fig. 4. Voltage E_d of generator 8.

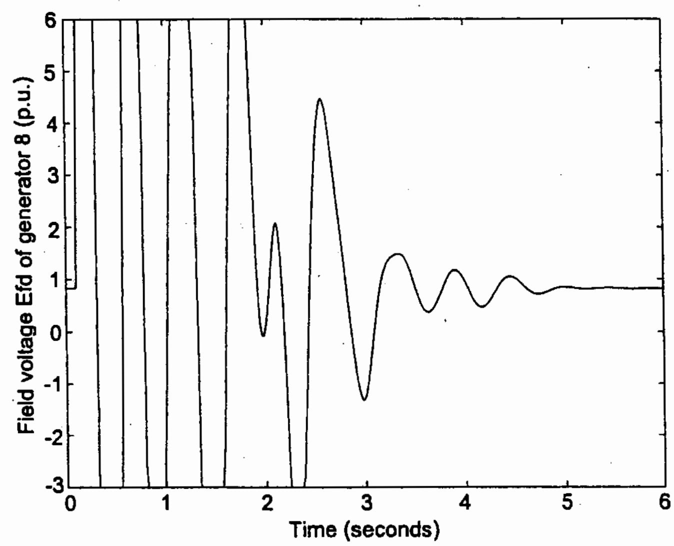

Fig. 6. Field voltage E_{fd} of generator 8.

uses the deviation of the local states with respect to arbitrarily chosen reference values $\{\delta_i^r, \omega_i^r, E_{q_i}^r, E_{d_i}^r\}$. A natural way to select these reference values is to equate them to the prefault steady state values obtained from the most recent power flow; this is the way they were computed in all our experiments. Since the operating point of the system varies continuously due to load fluctuations and topological changes in the system, the reference values will typically differ from the system equilibrium. We should add, however, that the equilibrium is implicitly defined by these values through (34).

Several further remarks need to be made regarding the relationship between reference values and the equilibrium.

1) The proposed LMI-based control guarantees that all states tend to their equilibrium values $\{\delta_i^e, \omega_i^e, E_{q_i}^e, E_{d_i}^e\}$, although these values needn't be known explicitly.
2) Since the main objective of the proposed control is to preserve stability following a large disturbance, the postfault equilibrium should not be expected to be optimal. This is not a serious limitation, since the system operator can subsequently move the system to a desired operating point.
3) From a practical standpoint, it is prudent to periodically update the local reference values, based on the most recent power flow results. Such a strategy would reduce the discrepancy between the equilibrium and the reference values. It is important to recognize, however, that this is only an added convenience, and that there is no need for frequent updates.
4) The gains obtained using the LMI optimization are computed for a nominal operating point. Since the control strategy is robust, the same gains can be used for a range of operating condition. If the system were to significantly deviate from its nominal configuration, the performance of the controller could be improved by recomputing the gains. It is possible to do this offline, based on load forecasts and/or contingency studies.

Another issue of practical interest is the performance of the proposed controller with respect to postfault oscillatory

TABLE I
CRITICAL CLEARING TIMES FOR NOMINAL LOADING CONDITIONS

	CCT (seconds)
Disturbance 1	0.178
Disturbance 2	0.184
Disturbance 3	0.188

TABLE II
CRITICAL CLEARING TIMES FOR A UNIFORMLY DISTRIBUTED LOAD INCREASE OF 15%

	CCT (seconds)
Disturbance 1	0.167
Disturbance 2	0.171
Disturbance 3	0.173

TABLE III
CRITICAL CLEARING TIMES FOR A RANDOMLY DISTRIBUTED LOAD INCREASE OF 15% (CASE 1)

	CCT (seconds)
Disturbance 1	0.168
Disturbance 2	0.172
Disturbance 3	0.171

TABLE IV
CRITICAL CLEARING TIMES FOR A RANDOMLY DISTRIBUTED LOAD INCREASE OF 15% (CASE 2)

	CCT (seconds)
Disturbance 1	0.166
Disturbance 2	0.169
Disturbance 3	0.174

stability. We should point out in this context that the model we have used does not explicitly consider exciter dynamics or the regulation of terminal voltages. In order to properly account for these effects, it would be necessary to apply a larger model in which the field voltage appears as a state variable. Additional states associated with a conventional power system stabilizer would also need to be included. Although the type of analysis proposed in this paper is applicable to such a model, it is by no means trivial. This is one of our research objectives for the near future.

V. CONCLUSIONS

In this paper, we presented a new strategy for exciter control design. The proposed method represents a generalization of the LMI-based approach formulated in [5] and [16], which allows for the inclusion of a wider class of nonlinearities. The resulting control law is linear and the gain matrix can be obtained directly, using standard convex optimization techniques. Experimental results indicate that the obtained exciter control is robust with respect to load fluctuations, and can maintain stability for a variety of short circuit faults.

APPENDIX

Let us consider the quadratic form $x^T A(x) y$ where $x \in R^n$, $y \in R^m$ and $A(x)$ is an $n \times m$ matrix whose elements satisfy $|a_{ij}(x)| \leq \bar{a}_{ij}, \forall i, j$. It is easily verified that any such form can be bounded as $x^T A(x) y \leq z^T D z$ where $z = [x^T y^T]^T$ and D is a diagonal $(n+m) \times (n+m)$ matrix with entries

$$d_{ii} = \begin{cases} \sum_{j=1}^{n} \bar{a}_{ij}, & i = 1, \ldots, n \\ \sum_{j=1}^{m} \bar{a}_{ji}, & i = n+1, \ldots, n+m \end{cases}. \tag{67}$$

Using this result and the bounds obtained in (44), (45), and (62), we now proceed to construct matrices $D_{ij} (j = 1, 2, 3)$ and $F_{ij} (j = 1, 2)$ that appear in (65). To that effect, let us define a $1 \times 3n$ vector $P_i = [p_1^{(i)} \ \ldots \ p_{3n}^{(i)}]$ with elements

$$p_k^{(i)} = \begin{cases} \sqrt{w_{ik}}, & 1 \leq k \leq n \\ \sqrt{w_{i,k-n}}, & n+1 \leq k \leq 2n \\ u_{i,k-2n}, & 2n+1 \leq k \leq 3n \end{cases} \tag{68}$$

and a matrix $M_i = P_i^T P_i$, whose entries are denoted by $m_{kj}^{(i)}$. The diagonal matrix D_{i1} is now formed as

$$d_{kk}^{(1)} = \sum_{j=1}^{3n} m_{kj}^{(i)} \tag{69}$$

and D_{i2} is a diagonal matrix with elements

$$d_{kk}^{(2)} = \begin{cases} p_k^{(i)}, & k \neq i, \ n+i \\ p_k^{(i)} + \frac{1}{2} \sum_{j=1}^{n} p_j^{(i)}, & k = i \\ p_{n+1}^{(i)} + \frac{1}{2} \sum_{j=1}^{n} p_j^{(i)}, & k = n+i \end{cases}. \tag{70}$$

To compute matrix D_{i3}, we define a $1 \times 3n$ vector $Q_i = [q_1^{(i)} \ \ldots \ q_{3n}^{(i)}]$ where

$$q_k^{(i)} = \begin{cases} \sqrt{v_{ik}}, & 1 \leq k \leq n \\ \sqrt{v_{i,k-n}}, & n+1 \leq k \leq 2n \\ 2\sqrt{r_{i,k-2n}}, & 2n+1 \leq k \leq 3n \end{cases} \tag{71}$$

and a matrix $L_i = Q_i^T Q_i$, whose elements are denoted $l_{kj}^{(i)}$. The diagonal matrix D_{i3} is then formed as

$$d_{kk}^{(3)} = \sum_{j=1}^{3n} l_{kj}^{(i)}. \tag{72}$$

To interpret matrices F_{i1} and F_{i2}, we should first note that the state vector in (34) has the form

$$x = [x_{11} x_{21} x_{31} x_{41} \ \ldots \ x_{1n} x_{2n} x_{3n} x_{4n}]^T. \tag{73}$$

If we now introduce vectors $x_3 = [x_{31} \ \ldots \ x_{3n}]^T$, $x_4 = [x_{41} \ \ldots \ x_{4n}]^T$ and $y_i = [y_{i1} \ \ldots \ y_{in}]^T$ (with $y_{ik} = (x_{1i} - x_{1k})/2$, as defined in Section III), matrices F_{i1} and F_{i2} are uniquely determined by the relationships

$$\begin{bmatrix} x_3 \\ x_4 \\ y_i \end{bmatrix} = F_{i1} x; \quad \begin{bmatrix} x_{3i} \\ x_{4i} \end{bmatrix} = F_{i2} x. \tag{74}$$

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